On the Deceleration of Relativistic Jets (Photon Drag and Particle Loading)

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with

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(R.R.Rafikov, N.Zakamska)

Plan

- Thanks
- AGN Jets – internal structure (observations)
- AGN Jets – internal structure (theory)
- Possible mechanism(s) of deceleration
  - drag
  - loading
- Thanks again

[1] VB, A.V.Chernoglazov, "On the deceleration of relativistic jets in active galactic nuclei I: Radiation drag". MNRAS, **463**, 3398-3408 (2016)

[2] E.E.Nokhrina, VB, "On the deceleration of relativistic jets in active galactic nuclei II: Particle loading". MNRAS, **469**, 3840-3850 (2017)

History

Two-fluid MHD

[0] VB, R.R.Rafikov, “On the particle acceleration near the light surface of radio pulsars”, MNRAS, **313**, 433-444 (2000)

[0] VB, N.Zakamska, H.Sol, “Radiation drag effects on magnetically dominated outflows around compact objects”, MNRAS, **347**, 587 (2004)

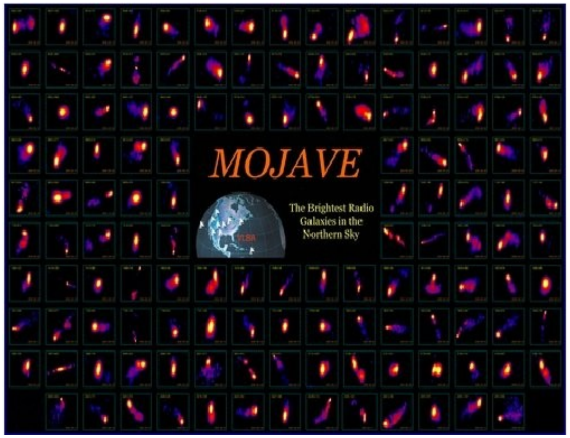
$$\Gamma = C\sqrt{\frac{R_{\rm c}}{r_{\perp}}}$$

[1] VB, A.V.Chernoglazov, "On the deceleration of relativistic jets in active galactic nuclei I: Radiation drag". MNRAS, **463**, 3398-3408 (2016)

[2] E.E.Nokhrina, VB, "On the deceleration of relativistic jets in active galactic nuclei II: Particle loading". MNRAS, **469**, 3840-3850 (2017)

Observational reason – AGN

New possibilities

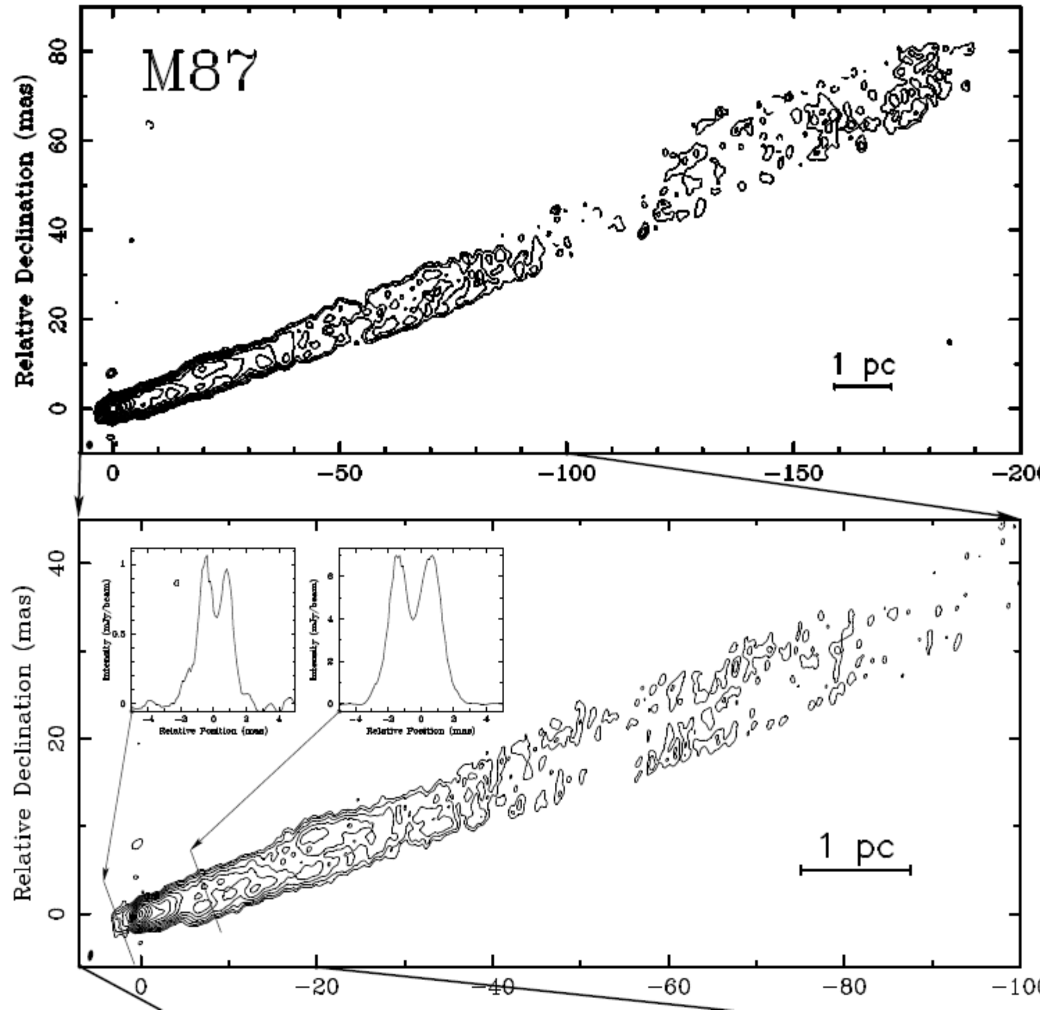


MOJAVE team (time)

Radioastron (base)

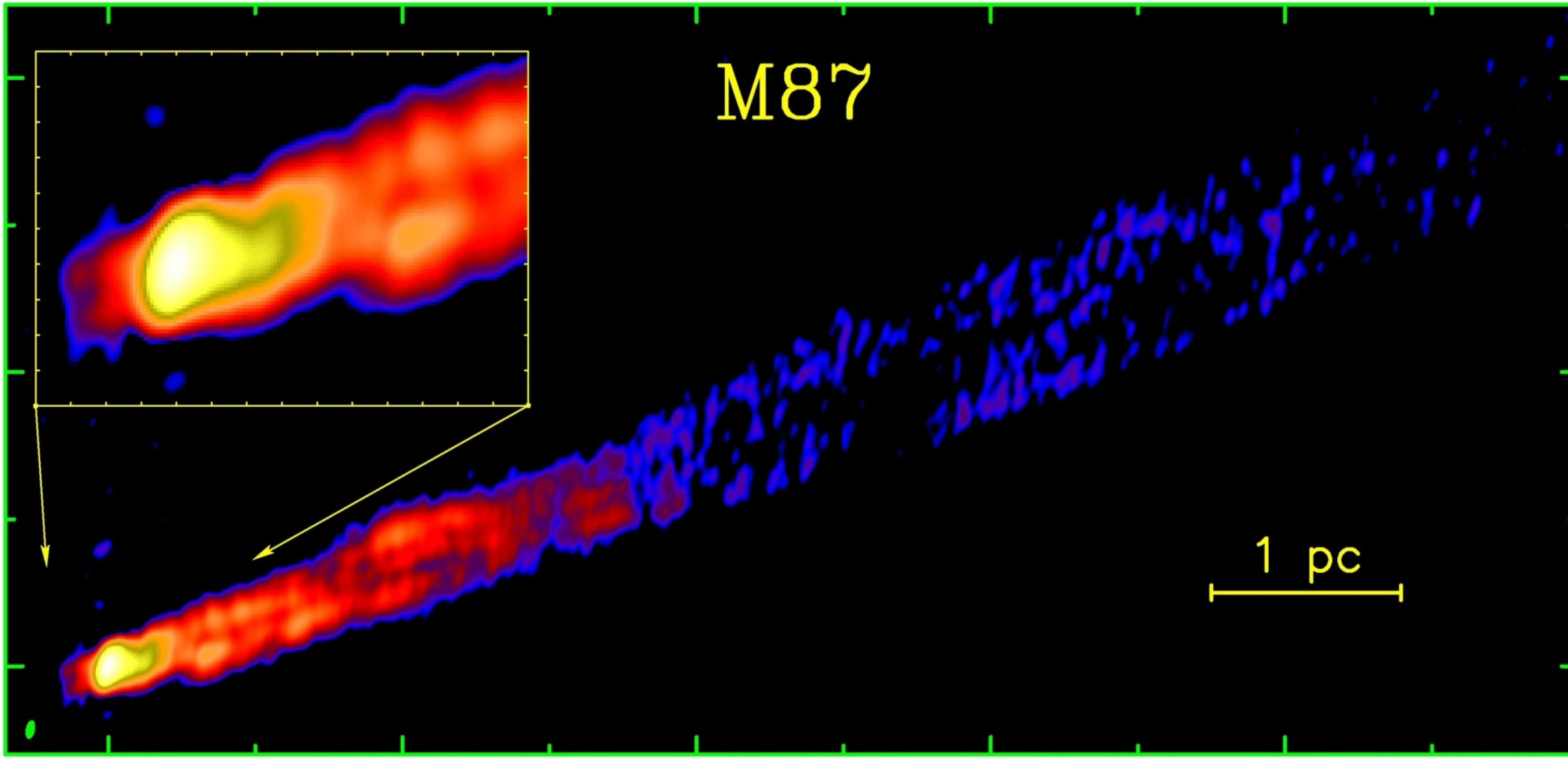
Internal structure – AGN

Y.Y.Kovalev et al, ApJ, **668**, L27 (2007)



VLBA+VLA1, 15 GHz

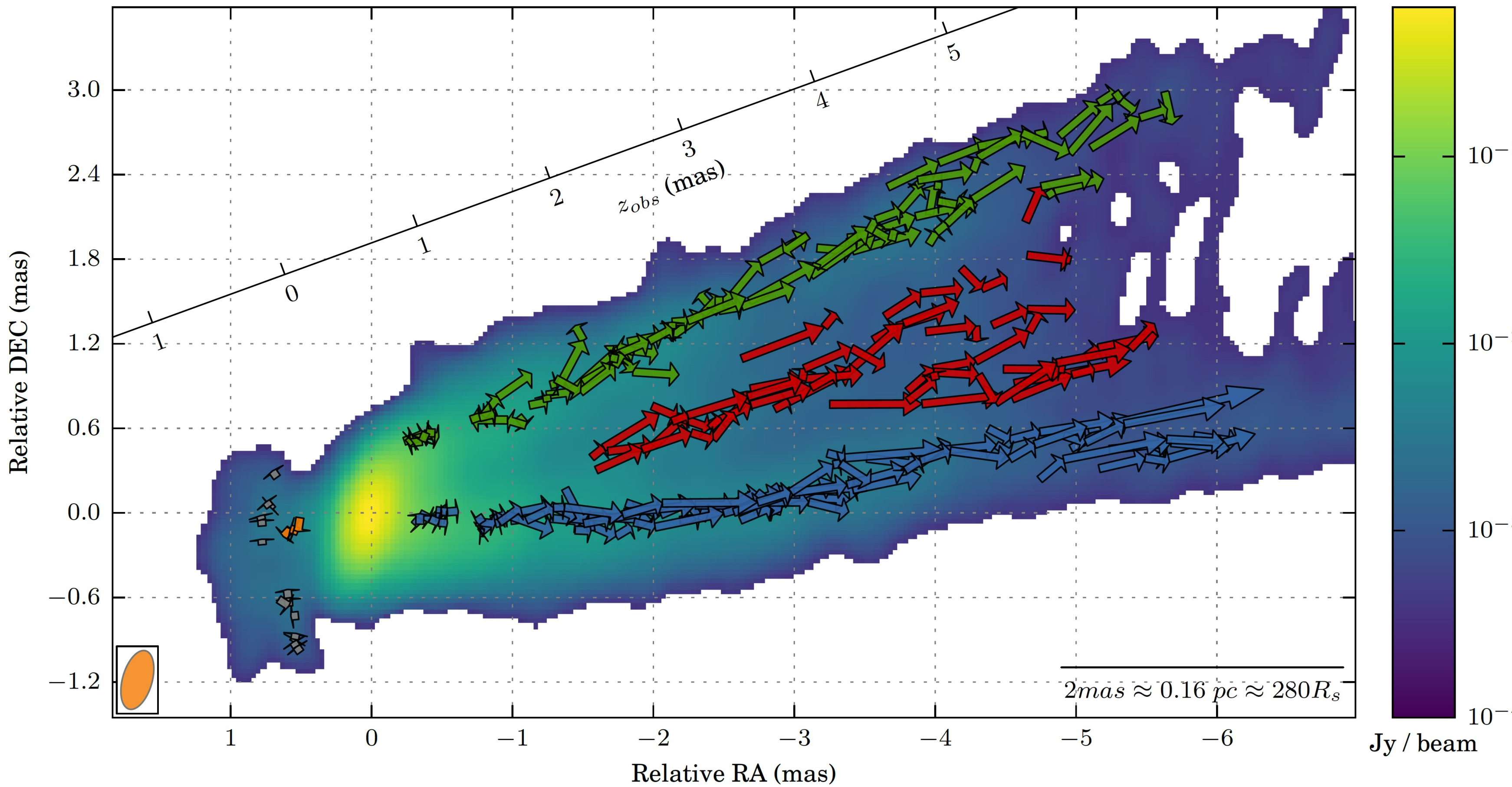
The inner jet structure is clearly resolved, a short counter jet is detected



Y.Y.Kovalev et al, ApJ, **668**, L27 (2007)

Internal structure – AGN

F. Mertens A.P.Lobanov, R.C.Walker, P.E.Hardee, A&A, **595**, A54 (2016)



Internal structure – AGN

F. Mertens A.P.Lobanov, R.C.Walker, P.E.Hardee, A&A, **595**, A54 (2016)

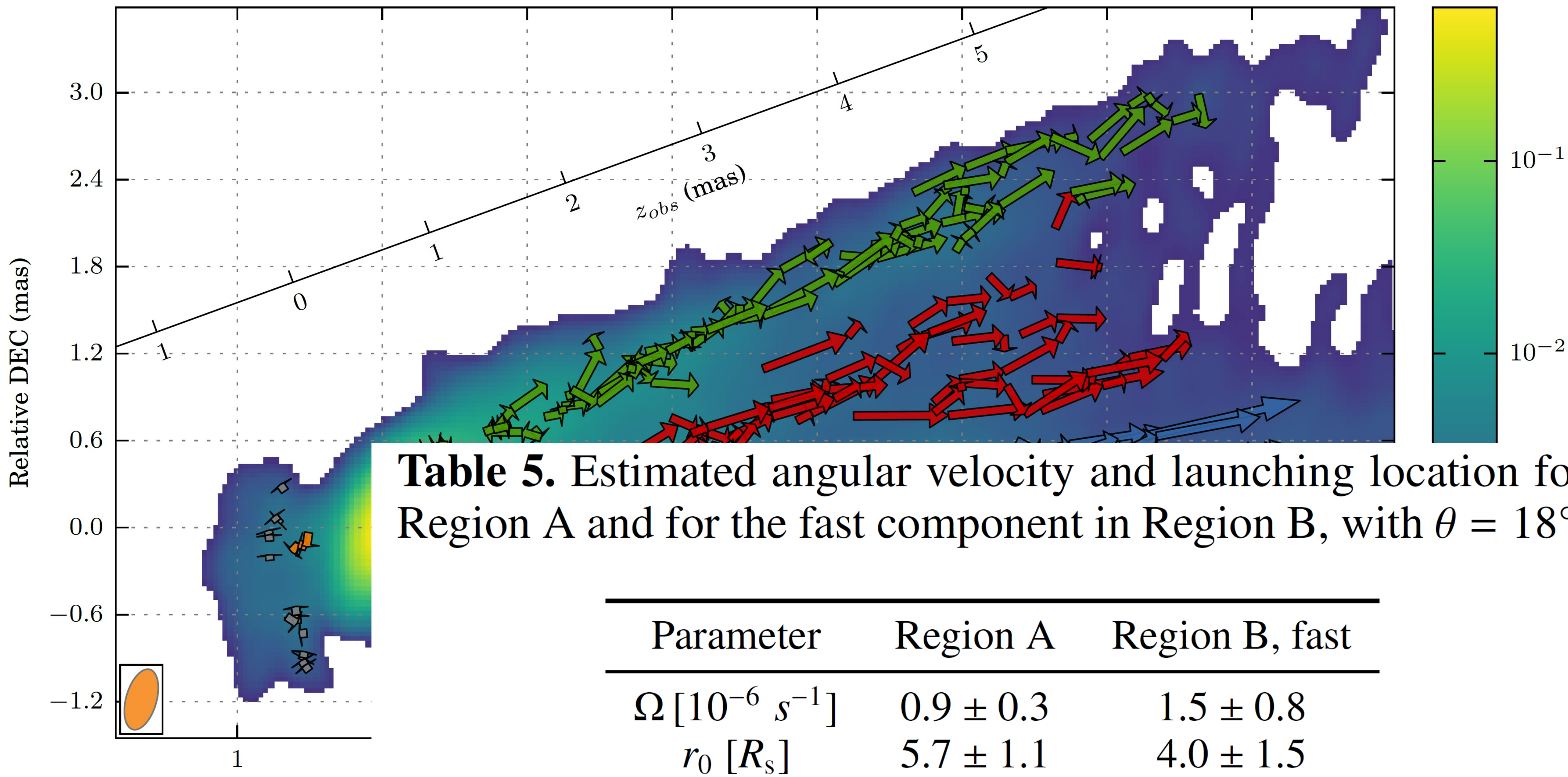


Table 5. Estimated angular velocity and launching location for Region A and for the fast component in Region B, with $\theta = 18°$.

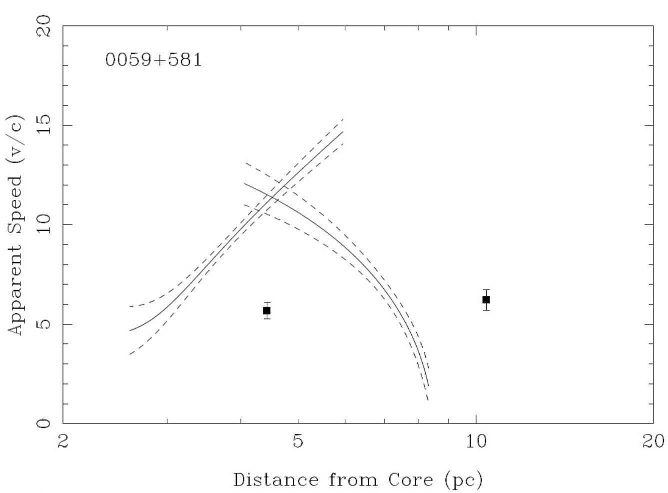
Parameter	Region A	Region B, fast
$\Omega\,[10^{-6}\ s^{-1}]$	0.9 ± 0.3	1.5 ± 0.8
$r_0\ [R_s]$	5.7 ± 1.1	4.0 ± 1.5
$r_{lc}\ [R_s]$	19 ± 5	11 ± 6

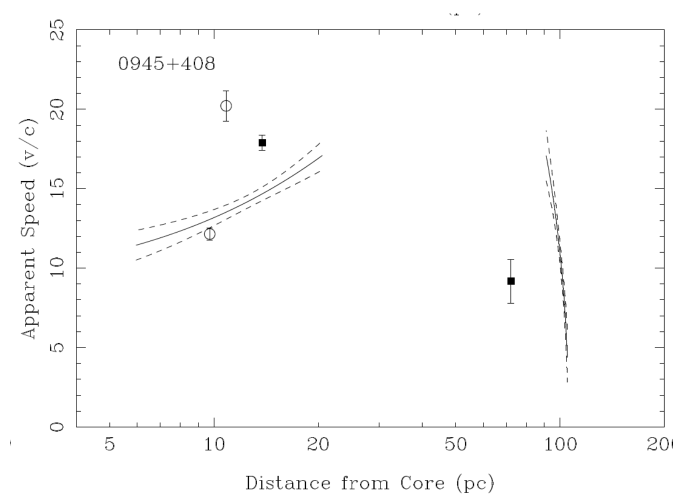
Internal structure – AGN

Homan D. C. et al, ApJ, **789**, 134 (2015)

Acceleration at small distances,
decceleration at large distances.

$$\dot{\Gamma}/\Gamma = 10^{-3}\ \mathrm{yr}^{-1}$$





Internal structure – AGN

Homan D. C. et al, ApJ, **789**, 134 (2015)

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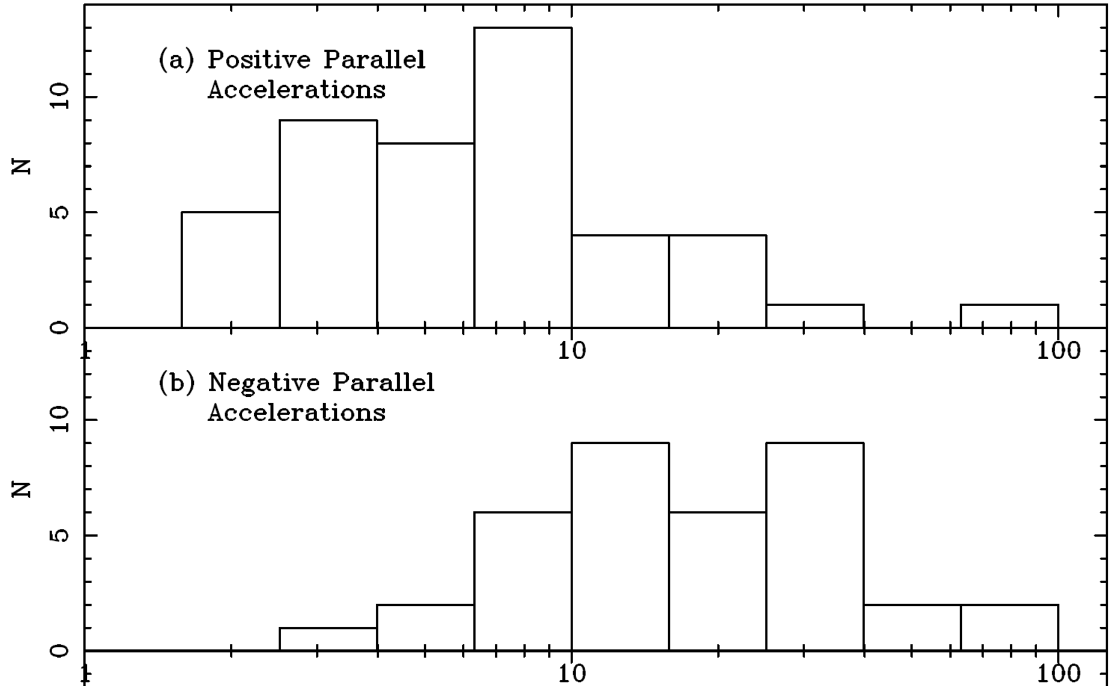


pc (projection)

Jets – theory

- It is necessary to include external media into consideration. It is the ambient pressure that determines jet transverse scale and particle energy.

- Simple asymptotic solutions for the bulk Lorentz-factor.

- Transverse profile of the poloidal magnetic field.

- Magnetization – multiplication connection.

Jets – theory

Main parameters

- Michel magnetization parameter (maximal bulk Lorentz-factor)

$$\boxed{\sigma_{\rm M} = \frac{\Omega_0 e B_0 r_{\rm jet}^2}{4\lambda m_{\rm e} c^3}}$$

μ now

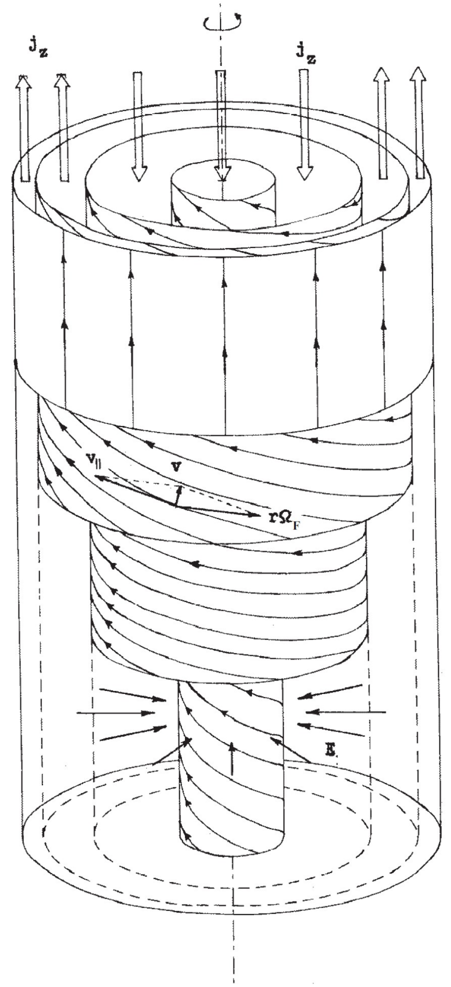
- Multiplicity parameter

$$\boxed{\lambda = \frac{n^{(\rm lab)}}{n_{\rm GJ}}}$$

$$\rho_{\rm GJ} = -\frac{\mathbf{\Omega}\cdot\mathbf{B}}{2\pi c}$$

- Total potential drop

$$\lambda\sigma_{\rm M} \sim \frac{e E_r r_{\rm jet}}{m_{\rm e} c^2}$$



Jets – theory

Magnetization – multiplication connection

$$\sigma_{\mathrm{M}} = \frac{\Omega_0 e B_0 r_{\mathrm{jet}}^2}{4\lambda m_{\mathrm{e}} c^3}$$

$$\lambda = \frac{n^{(\mathrm{lab})}}{n_{\mathrm{GJ}}}$$

MHD 'central engine' energy losses

$$W_{\mathrm{tot}} \approx \left(\frac{\Omega R_0}{c}\right)^2 B_0^2 R_0^2 c$$

After some algebra

$$\boxed{\sigma_{\mathrm{M}} \sim \frac{1}{\lambda}\left(\frac{W_{\mathrm{tot}}}{W_{\mathrm{A}}}\right)^{1/2}}$$

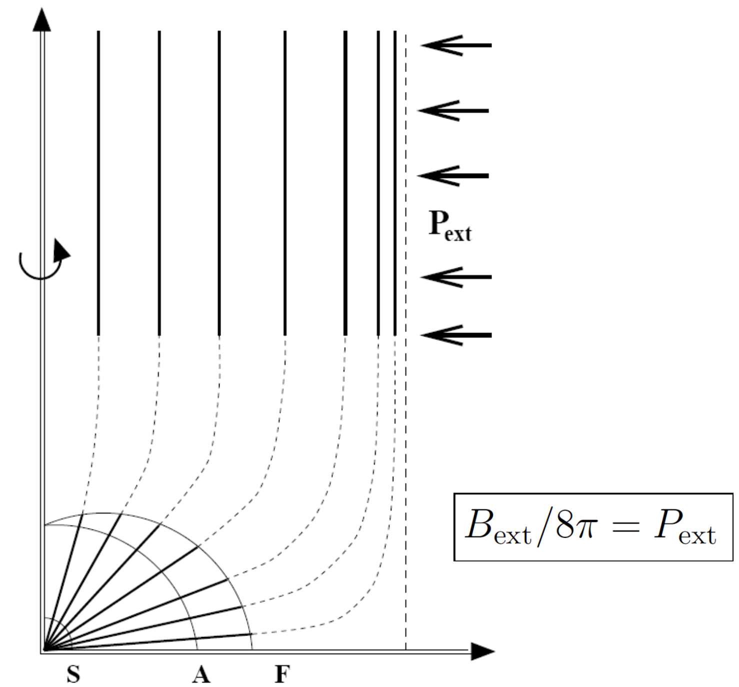
$$W_{\mathrm{A}} = m_{\mathrm{e}}^2 c^5 / e^2 \approx 10^{17}\,\mathrm{erg\,s^{-1}}$$

Jets – theory

- It is necessary to include the external media into consideration. It is the ambient pressure that determines the jet transverse scale and particle energy.

1D approach for cylindrical jets

$$
\begin{cases}
\dfrac{\mathrm{d}\mathcal{M}^2}{\mathrm{d}r_\perp} = F_1(\mathcal{M}^2, \Psi, r_\perp) \\
\dfrac{\mathrm{d}\Psi}{\mathrm{d}r_\perp} = F_2(\mathcal{M}^2, \Psi, r_\perp)
\end{cases}
$$



VB, L.M.Malyshkin, Astron. Lett., **26**, 208 (2000)
VB, Phys. Uspekhi, **40**, 659 (1997)

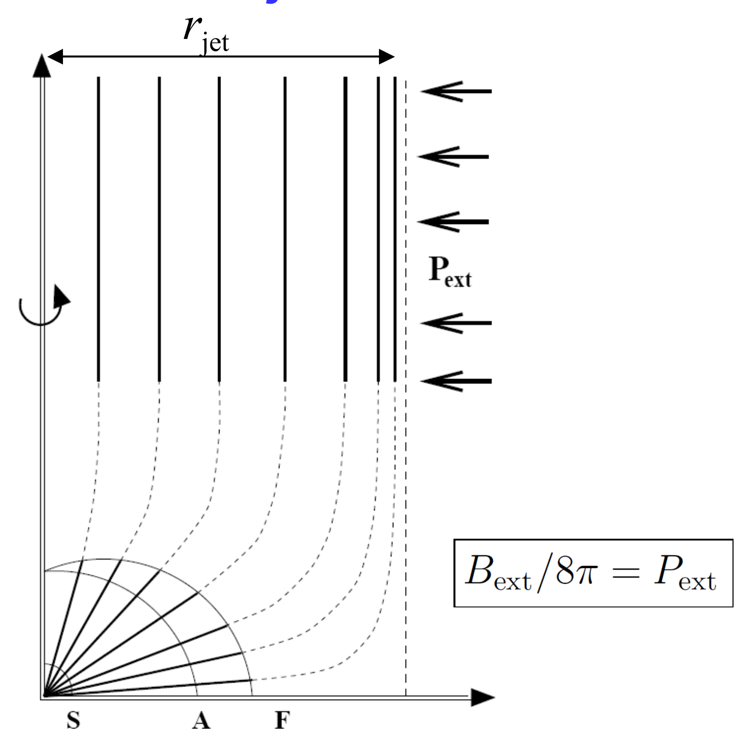
T.Lery, J.Heyvaerts, S.Appl, C.A.Norman, A&A, **347**, 1055 (1999)

Jets – theory

- It is necessary to include the external media into consideration. It is the ambient pressure that determines the jet transverse scale and particle energy.

$$r_{\rm jet} \sim R \left(\frac{B_{\rm in}^2}{8\pi P_{\rm ext}} \right)^{1/4}$$

$$\boxed{\frac{W_{\rm part}}{W_{\rm tot}} \sim \frac{1}{\sigma_{\rm M}} \left[\frac{B^2(R_{\rm L})}{8\pi P_{\rm ext}} \right]^{1/4}}$$



VB, L.M.Malyshkin, Astron. Lett., **26**, 208 (2000)
VB, Phys. Uspekhi, **40**, 659 (1997)

T.Lery, J.Heyvaerts, S.Appl, C.A.Norman, A&A, **347**, 1055 (1999)

Jets – theory

Simple asymptotic solutions for Lorentz-factor

Quasi-cylindrical flows ($\Gamma < \sigma_{\mathrm{M}}$)

$$\boxed{\Gamma = x_r} \qquad x_r = \Omega_{\mathrm{F}} r_{\perp}/c$$

Quasi-radial flows

$$\boxed{\Gamma = C\sqrt{\frac{R_{\mathrm{c}}}{r_{\perp}}}}$$

Jets – theory

Simple asymptotic solutions for Lorentz-factor

Quasi-cylindrical flows $(\Gamma < \sigma_{\mathrm{M}})$

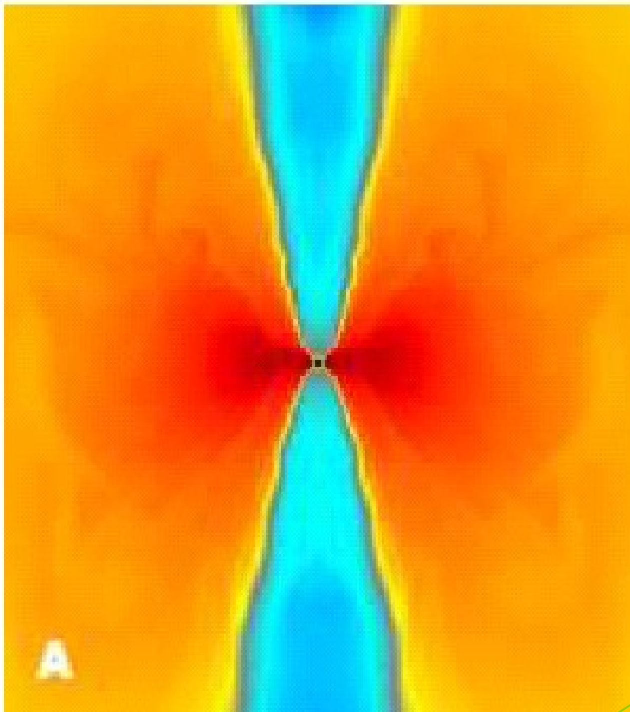
$$\boxed{\Gamma = x_r} \qquad x_r = \Omega_{\mathrm{F}} r_{\perp}/c$$

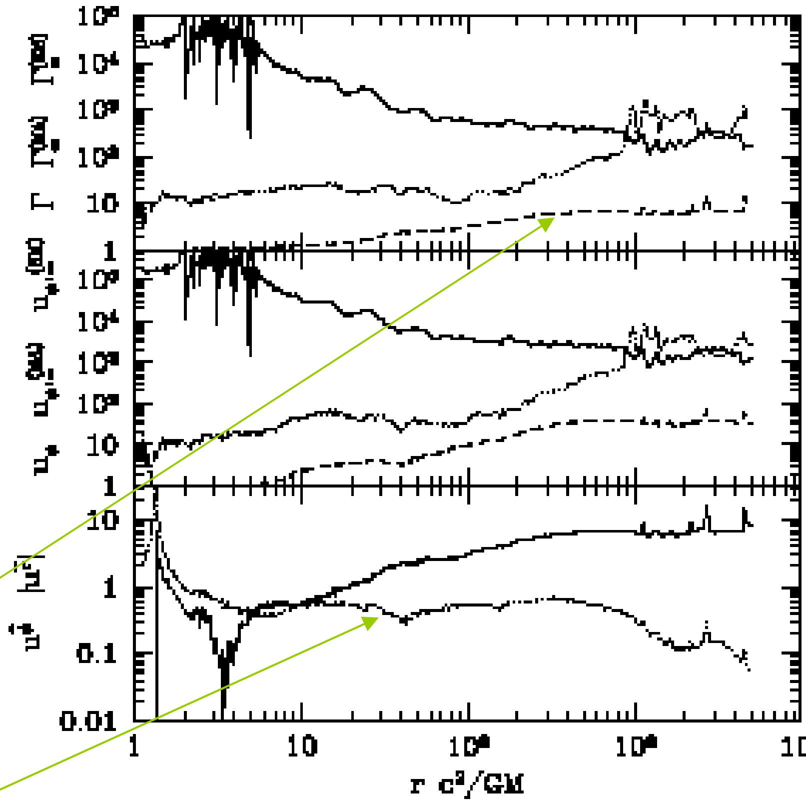
- Asymptotic solution (corresponds to drift velocity only)
- In general case

$$\boxed{\Gamma^2 = \Gamma_0^2 + x^2}$$

Jets – theory

J.McKinney. MNRAS, **367**, 1797 (2006)





$$\Gamma(z) = (z/\, R_{\mathrm{L}})^{1/2},\; u_{\varphi} = 1$$

Jets – theory

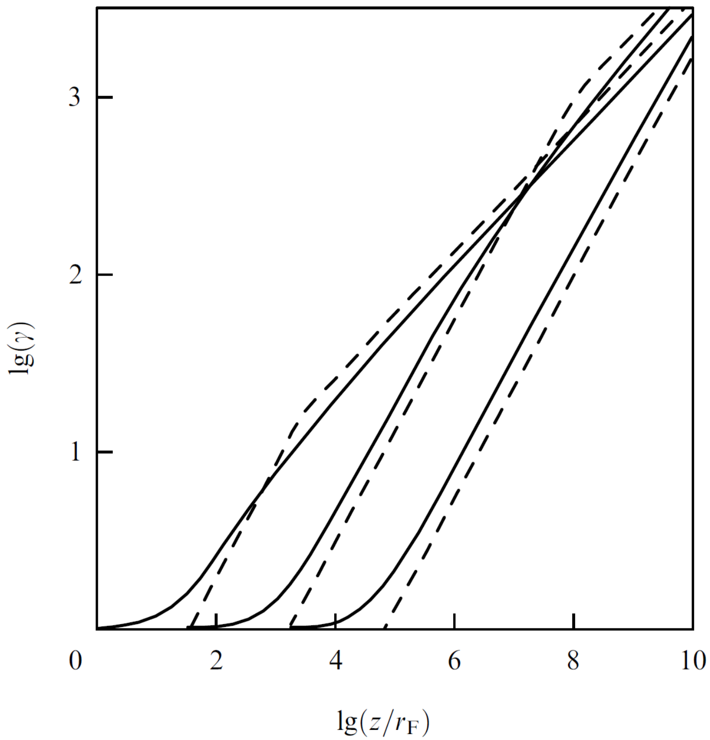
Parabolic structure terminates the efficiency of acceleration

- Self-similar solution $z \sim r_{\perp}{}^{k}$
- For $k > 2$

  $\Gamma = x_r \sim z^{1/k}$
- For $k < 2$

  $\Gamma = (R_c r_{\perp})^{1/2}$

  $\sim z^{(k-1)/k}$
- *Parabolic* $k = 2$

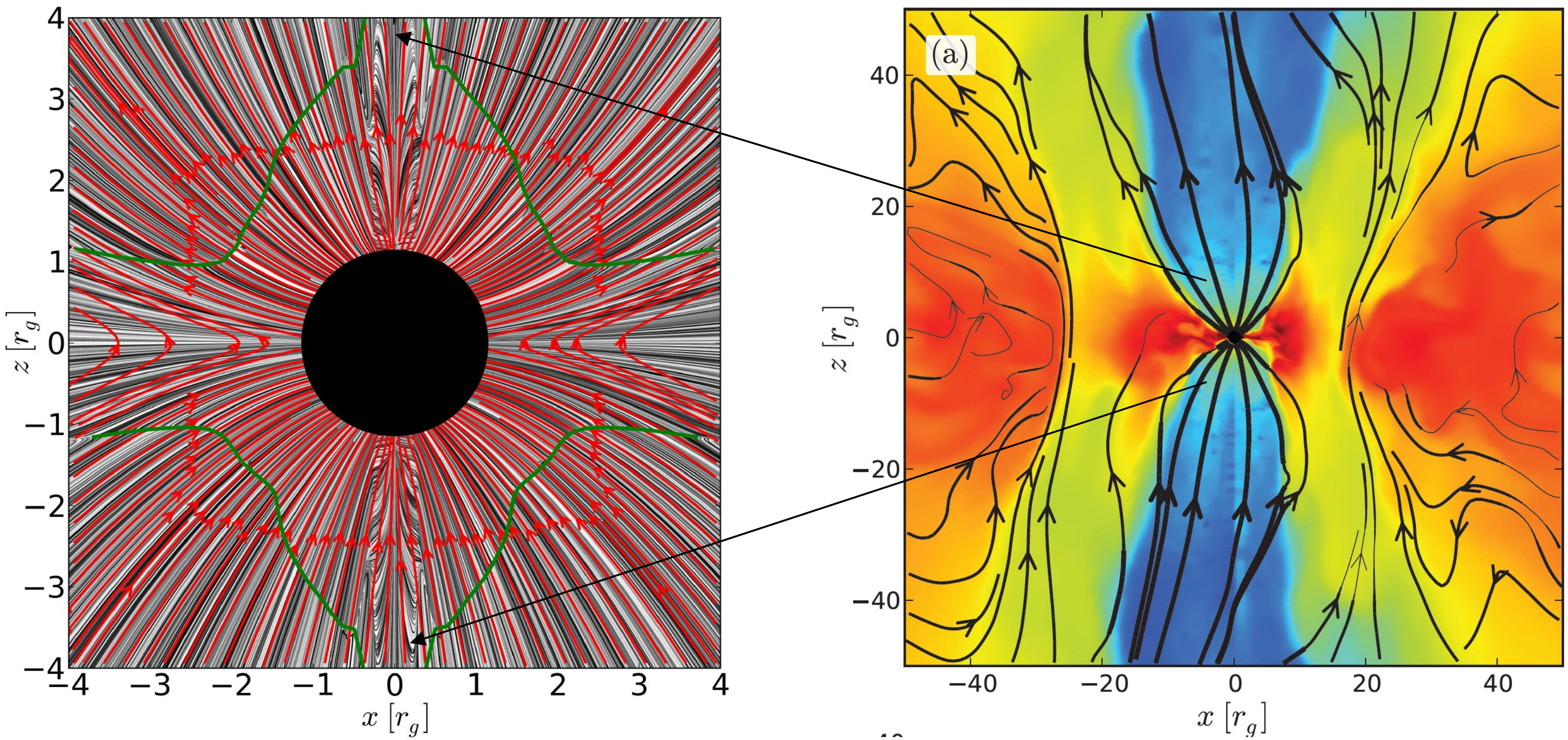


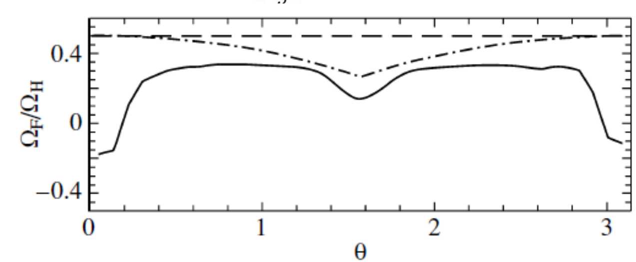
R. Narayan, J.McKinney, A.F.Farmer, MNRAS, **375**, 548, 2006

In all cases $\Gamma\theta \sim 1$

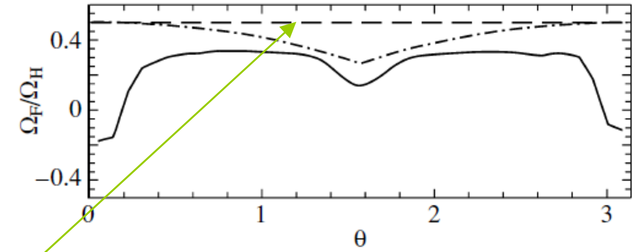
J.C.McKinney, A.Tchekhovskoy, R.D.Blandford. MNRAS, **423**, 3083 (2012)

Parabolic?





R.Blandford & R.Znajek. MNRAS, **179**, 433 (1977)

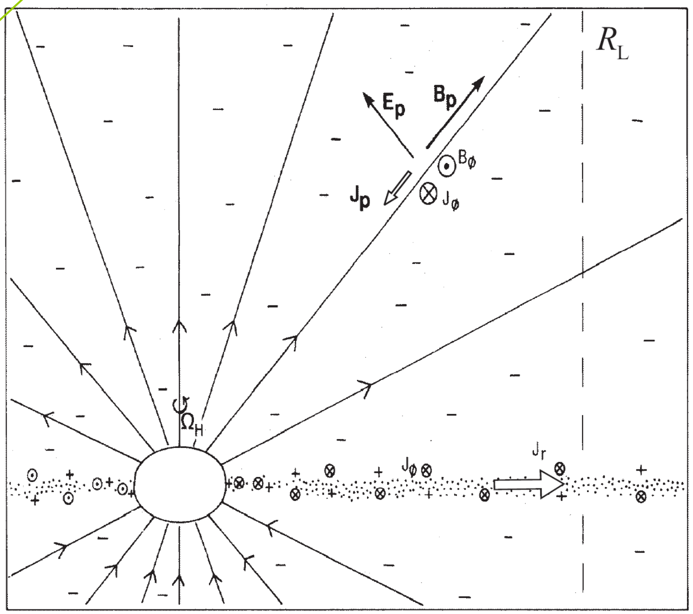


Monopole + Monopole

$$\Psi_0^{(2)} = \Psi_0(1-\cos\theta).$$

Horizon 'boundary condition'

$$4\pi I(\theta) = [\Omega_H - \Omega_F(\theta)]\Psi_0 \sin^2\theta.$$



At large distances

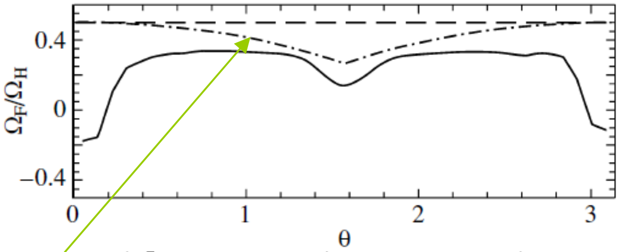
$$4\pi I(\theta) = \Omega_F(\theta)\Psi_0 \sin^2\theta.$$

Then

$$\Omega_F = \frac{\Omega_H}{2}, \qquad I(\Psi) = I_M = \frac{\Omega_F}{4\pi}\left(2\Psi - \frac{\Psi^2}{\Psi_0}\right).$$

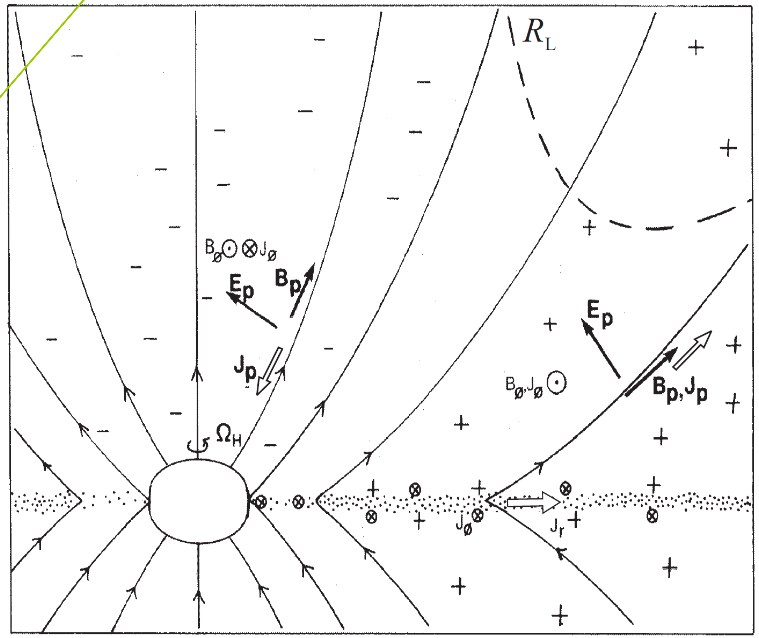
$$E_{\hat\theta} = -B_{\hat\varphi}$$

R.Blandford & R.Znajek. MNRAS, **179**, 433 (1977)



Parabolic + Parabolic

$$\Psi_0^{(1)}(r,\theta) = r(1-\cos\theta) + r_g(1+\cos\theta)[1-\ln(1+\cos\theta)] - 2r_g(1-\ln 2)$$



Horizon ‘boundary condition’

$$4\pi I(\Psi) = [\Omega_H - \Omega_F(\Psi)]\sin\theta\frac{d\Psi}{d\theta}$$

At large distances

$$4\pi I(\theta) = \Omega_F(\theta)\Psi_0\sin^2\theta.$$

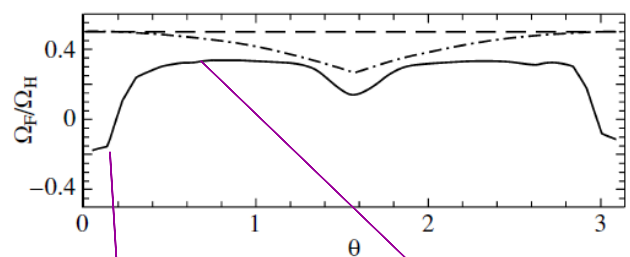
Then

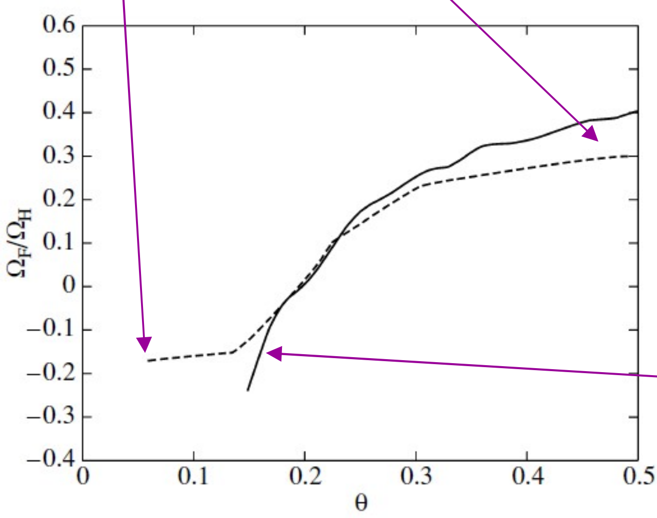
$$\Omega_F(r_g,\theta) = \frac{\Omega_H\sin^2\theta[1+\ln(1+\cos\theta)]}{4\ln 2+\sin^2\theta+[\sin^2\theta-2(1+\cos\theta)]\ln(1+\cos\theta)}$$

Excellent agreement with analytical force-free behaveour

VB, A.A.Zheltoukov. Astron. Lett., **39**, 215 (2013)

Monopole + Cylinder





$$\begin{cases} A_1(\Psi) = \varpi^2 B_z \\ A_2(\Psi) = c^2 \displaystyle\int_0^{\varpi} x^2 \frac{d}{dx} (B_z)^2 \, dx \\ A_3(\Psi) = \dfrac{1}{2\pi} \sin\theta \dfrac{r_g^2 + a^2}{r_g^2 + a^2 \cos^2\theta} \left(\dfrac{d\Psi}{d\theta} \right) \end{cases}$$

$$\boxed{\Omega_F = \Omega_H \left[\frac{A_3}{A_3 + A_1} + \frac{A_2}{\Omega_H^2 A_1 A_3 \left(1 + \sqrt{1 - \frac{A_2 (A_3^2 - A_1^2)}{\Omega_H^2 A_1^2 A_3^2}} \right)} \right]}$$

Jets – theory

Transverse profile of the poloidal magnetic field

T.Chiueh, Zh.-Yu.Li, M.C.Begelman. ApJ, **377**, 462 (1991)

D.Eichler. ApJ, **419**, 111 (1993)

S.V.Bogovalov. Astron. Lett., **21**,565 (1995)

M.Camenzind. In Herbig-Haro Flows and the Birth of Low Mass Stars. Eds. Reipurth B., Bertout C. (1997)

$$\boxed{B_{\mathrm{p}} = \frac{B_0}{1 + (r_\perp / r_{\mathrm{core}})^2}}$$

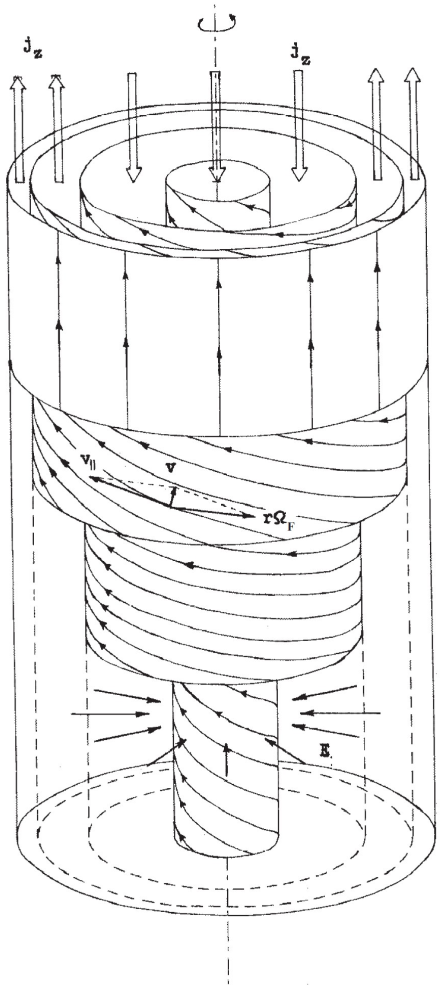
$$r_{\mathrm{core}} = \gamma_{\mathrm{in}} R_{\mathrm{L}}$$

Jets – theory

Transverse profile of the poloidal magnetic field

And this was odd, because…

homogeneneous
poloidal magnetic
field is the solution
for force-free (and,
hence, for magnetically
dominated) flow.



Jets – theory

Transverse profile of the poloidal magnetic field

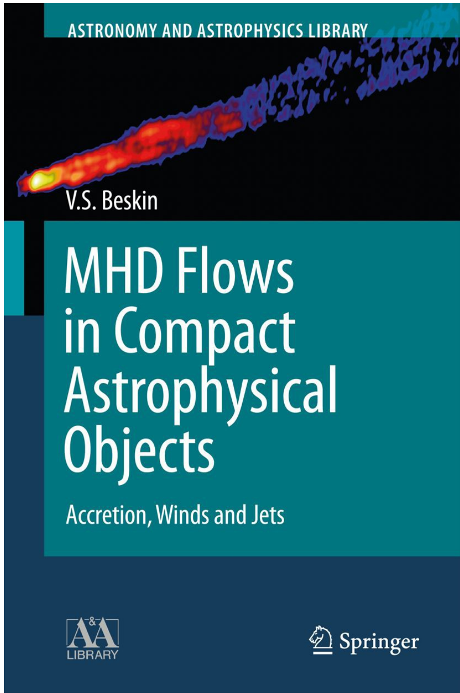
Theorem 5.2. *In the relativistic case, in the presence of the environment with rather high pressure ($B_{\rm ext} > B_{\rm min}$) the poloidal magnetic field inside the jet remains practically constant: $B_{\rm p} \approx B_{\rm ext}$. For small external pressure ($B_{\rm ext} < B_{\rm min}$) in the vicinity of the rotation axis there must form a core region $r_\perp < \varpi_{\rm c} = \gamma_{\rm in} R_{\rm L}$ with the magnetic field $B_{\rm p} \approx B_{\rm min}$ (5.69) containing only a small part of the total magnetic flux Ψ_0:*

$$\frac{\Psi_{\rm core}}{\Psi_0} \approx \frac{\gamma_{\rm in}}{\sigma}.$$

For $r_\perp < \varpi_{\rm c}$, the poloidal magnetic field $B_{\rm p}$ decreases as

$$B_{\rm p} \propto r_\perp^{2-\alpha},$$

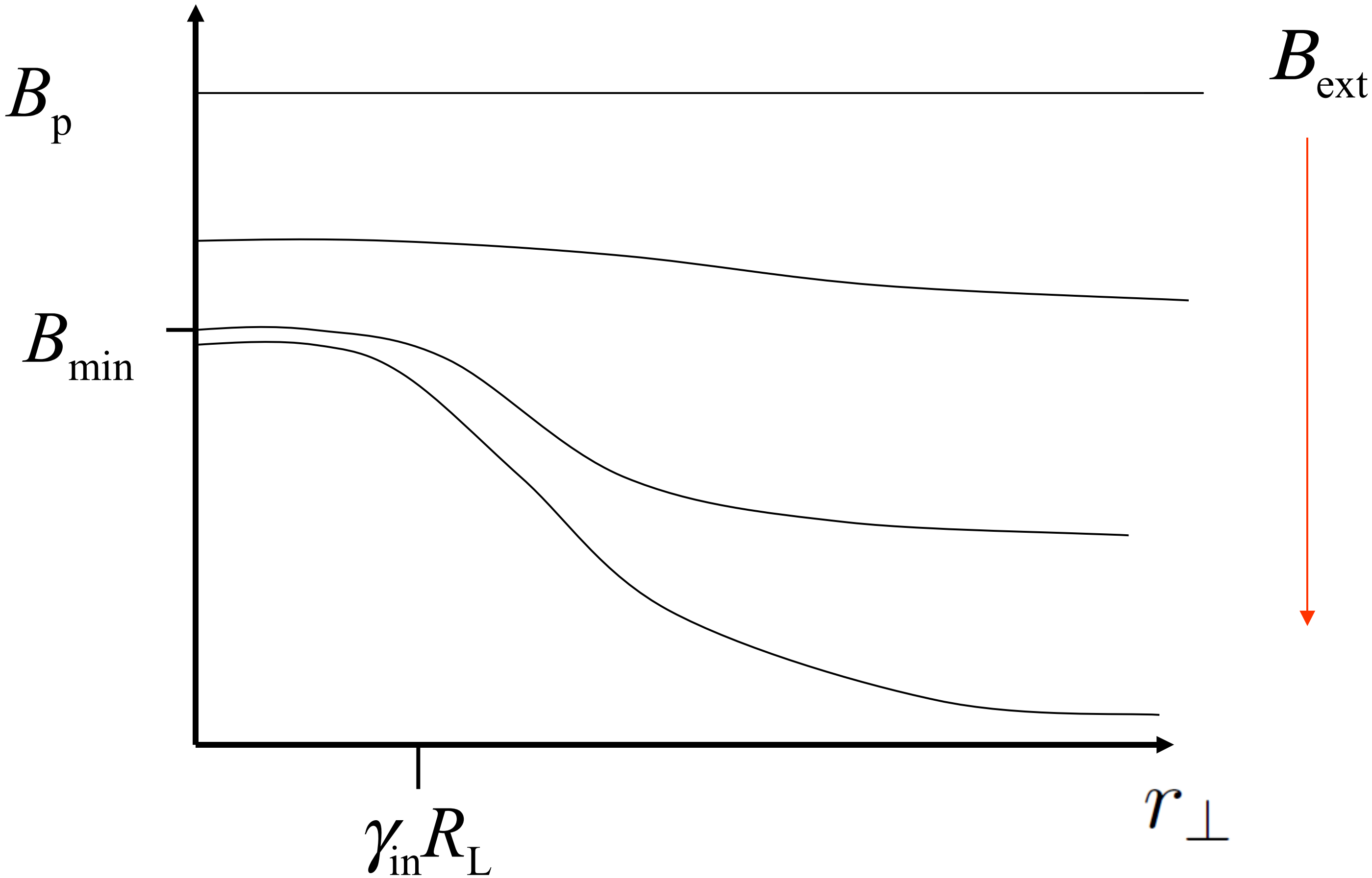
where $\alpha < 2$.



$$B_{\rm min} = \frac{1}{\sigma \gamma_{\rm in}} B(R_{\rm L}) \qquad B(R_{\rm L}) = \Omega^2 \Psi_{\rm tot}/\pi c^2 \qquad B_{\rm p}^2/8\pi \approx P_{\rm ext}$$

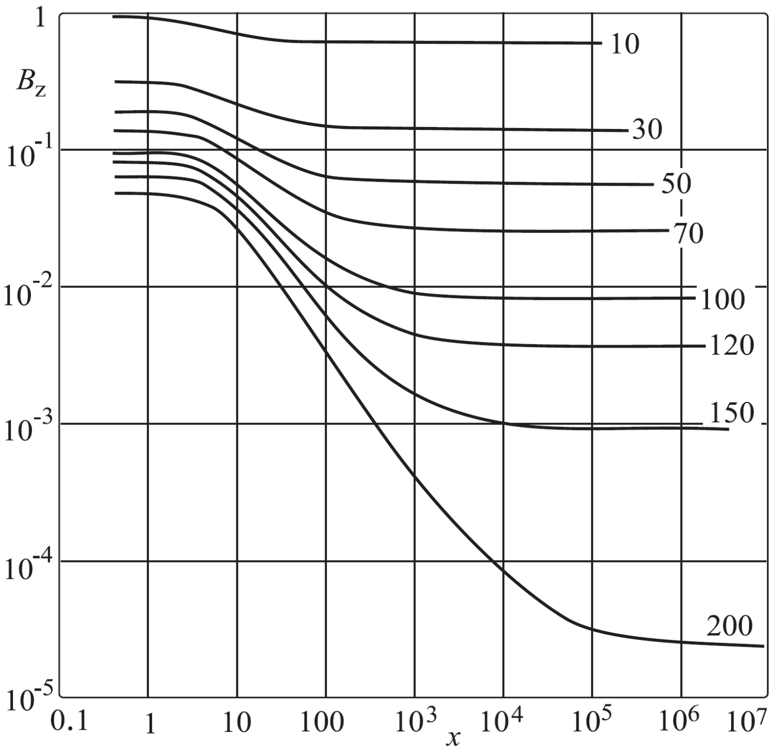
Central core

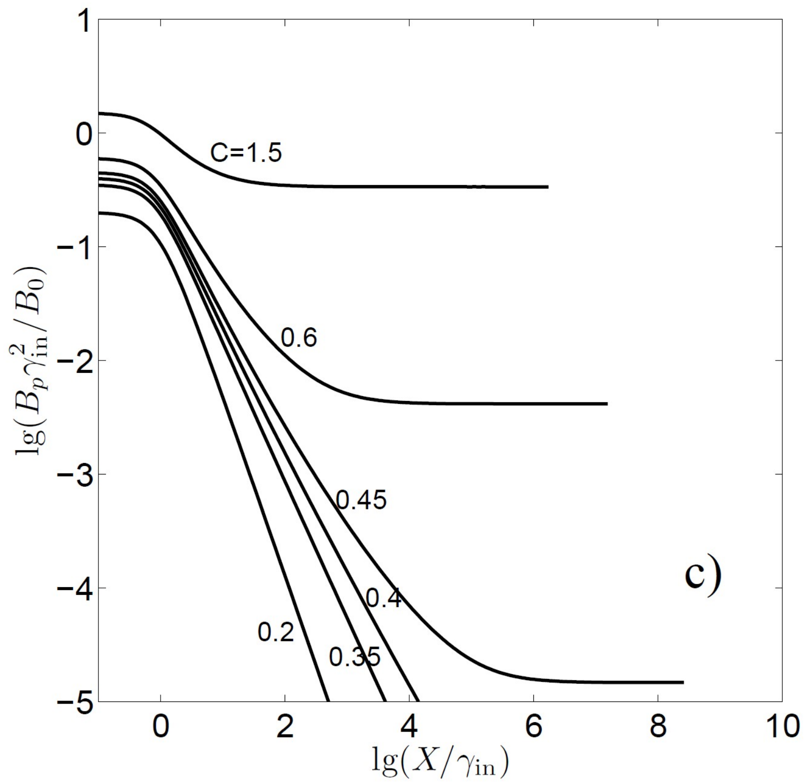
$$B_{\min} = \frac{1}{\sigma_{\mathrm{M}} \gamma_{\mathrm{in}}} B(R_{\mathrm{L}}) \qquad r_{\mathrm{core}} = \gamma_{\mathrm{in}} R_{\mathrm{L}}$$



Central core

$$\begin{cases} \dfrac{\mathrm{d}\mathcal{M}^2}{\mathrm{d}r_\perp} & = & F_1(\mathcal{M}^2, \Psi, r_\perp) \\ \dfrac{\mathrm{d}\Psi}{\mathrm{d}r_\perp} & = & F_2(\mathcal{M}^2, \Psi, r_\perp) \end{cases}$$

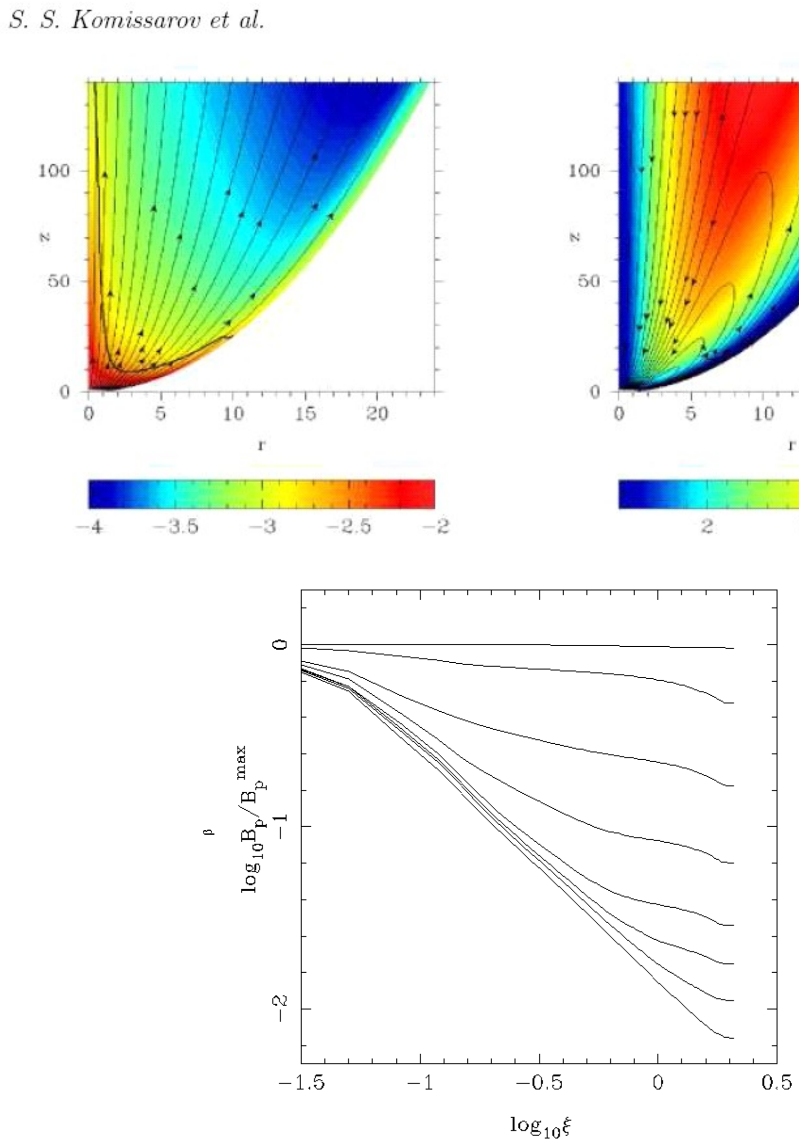




VB, E.E.Nokhrina.
MNRAS, **389**, 335 (2007)
MNRAS, **397**, 1486 (2009)

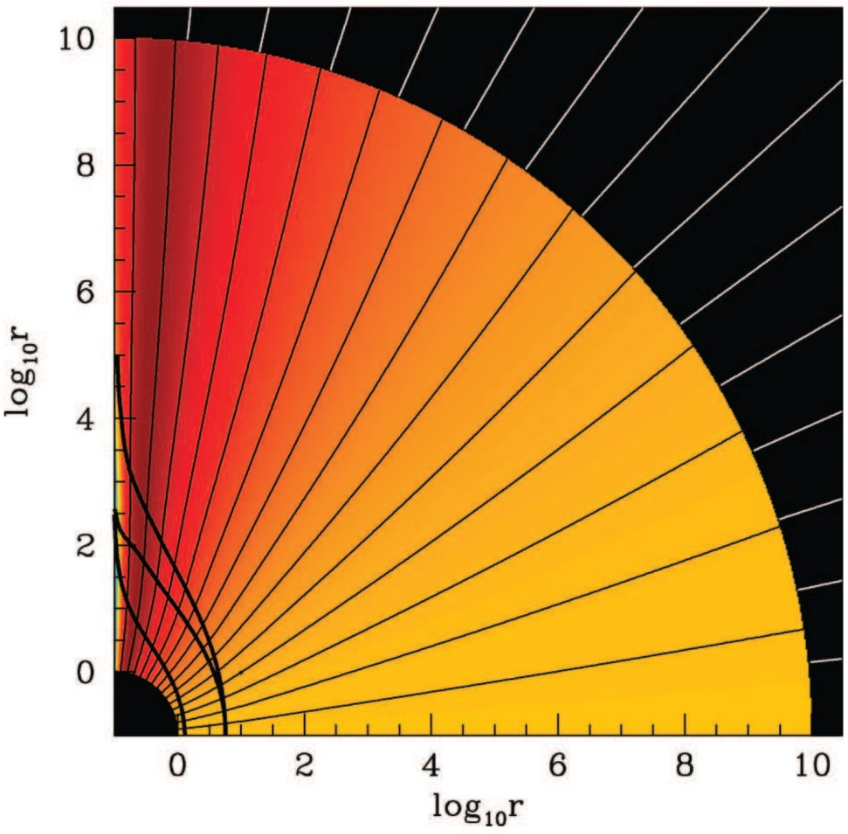
Yu.Lyubarsky. ApJ,
698, 1570 (2009)

Central core

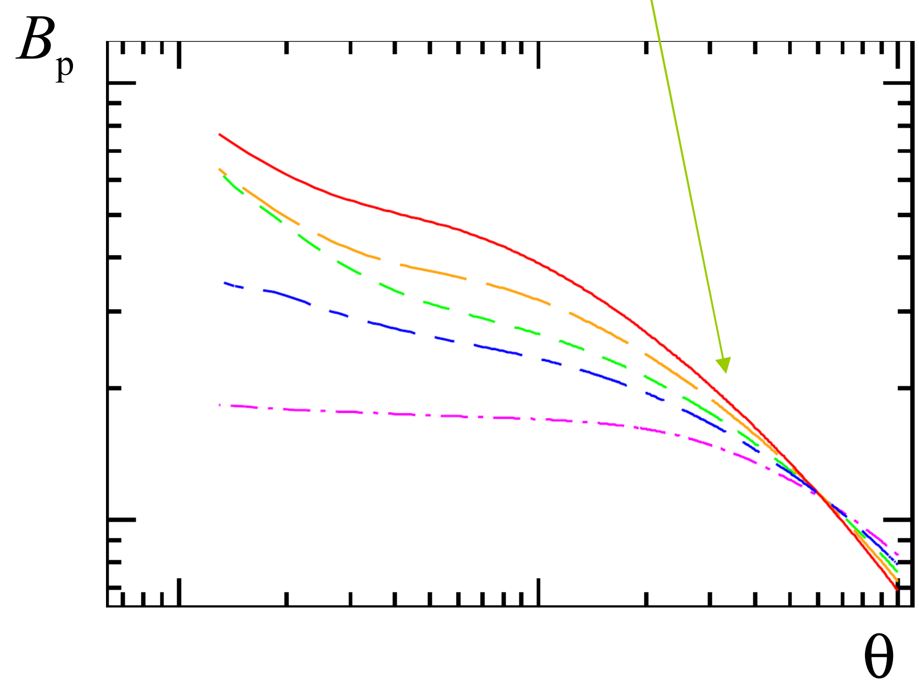


S.Komissarov, M.Barkov, N.Vlahakis, A.Königl. MNRAS, **380**, 51 (2006)

Central core

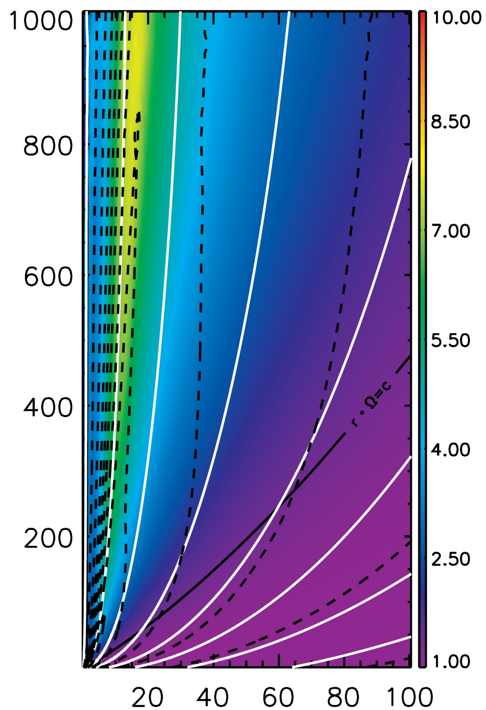


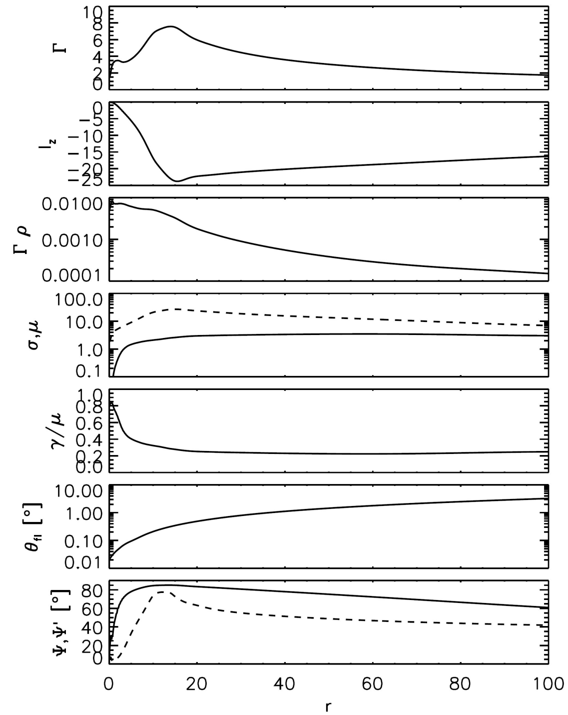
$$\boxed{B_{\min} = \frac{1}{\sigma_M \gamma_{\rm in}} B(R_L)}$$

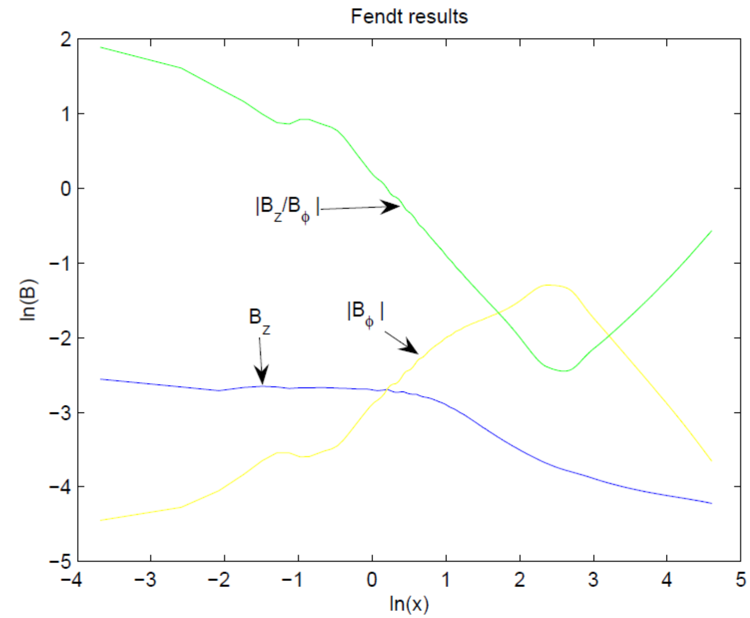


A.Tchekhovskoy, J.McKinney, R.Narayan. ApJ, **699**, 1789 (2009)

Central core







O.Porth, Ch.Fendt, Z.Meliani, B.Vaidya. ApJ, **737**, 42 (2011)

Jets – theory

- Real parameters

$$\begin{cases} \sigma_{\mathrm{M}} \sim \dfrac{1}{\lambda}\left(\dfrac{W_{\mathrm{tot}}}{W_{\mathrm{A}}}\right)^{1/2} \\ W_{\mathrm{A}} = m_{\mathrm{e}}^2 c^5/e^2 \approx 10^{17}\ \mathrm{erg\ s^{-1}} \end{cases}$$

$$\sigma_{\mathrm{M}}\lambda \sim 10^{14}$$

- As $\Gamma = r_{\mathrm{jet}} / R_{\mathrm{L}} \sim 10^4 - 10^5$, there are two possibilities:

1. Magnetically dominated flow

$$\sigma_{\mathrm{M}} > 10^5 \qquad \Gamma \sim 10^4 - 10^5$$

2. Saturation regime

$$\sigma_{\mathrm{M}} < 10^5 \qquad \Gamma \sim \sigma_{\mathrm{M}}$$

Core shift and jet parameters

E.E.Nokhrina, VB, Y.Y.Kovalev, A.A.Zheltoukhov. MNRAS, **447,** 2726 (2015)

- No assumption about equipartition (in both cases we know the bulk particle energy $\Gamma m c^2$).

$$\Gamma \sim \sigma_{\mathrm{M}}$$

- The only free parameter is the fraction of synchrotron radiating particles $n_{\mathrm{syn}} = \xi n_{\mathrm{e}}$.

$$\xi \approx 0.01$$

$$\lambda = 7.3 \times 10^{13} \left(\frac{\eta}{\mathrm{mas\,GHz}}\right)^{3/4} \left(\frac{D_{\mathrm{L}}}{\mathrm{Gpc}}\right)^{3/4} \times \left(\frac{\chi}{1+z}\right)^{3/4} \frac{1}{(\delta \sin\varphi)^{1/2}} \frac{1}{(\xi \gamma_{\min})^{1/4}}$$

$$\sigma_{\mathrm{M}} = 1.4 \left[\left(\frac{\eta}{\mathrm{mas\,GHz}}\right) \left(\frac{D_{\mathrm{L}}}{\mathrm{Gpc}}\right) \frac{\chi}{1+z}\right]^{-3/4} \times \sqrt{\delta \sin\varphi}\, (\xi \gamma_{\min})^{1/4} \sqrt{\frac{P_{\mathrm{jet}}}{10^{45}\ \mathrm{erg\ s^{-1}}}}$$

Core shift and jet parameters

E.E.Nokhrina, VB, Y.Y.Kovalev, A.A.Zheltoukhov. MNRAS, **447,** 2726 (2015)

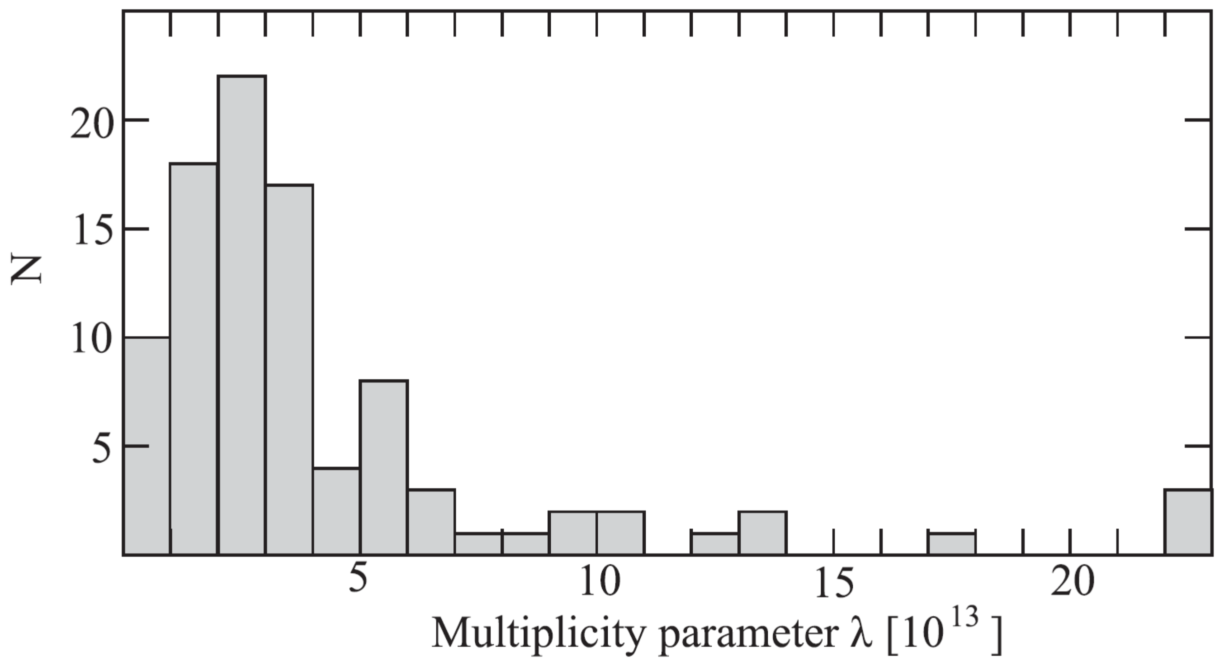


Figure 1. Distributions of the multiplicity parameter λ for the sample of 97 sources. Two objects with $\lambda = 2.8 \times 10^{14}$ and 3.6×10^{14} lie out of the shown range of values.

Core shift and jet parameters

E.E.Nokhrina, VB, Y.Y.Kovalev, A.A.Zheltoukhov. MNRAS, **447,** 2726 (2015)

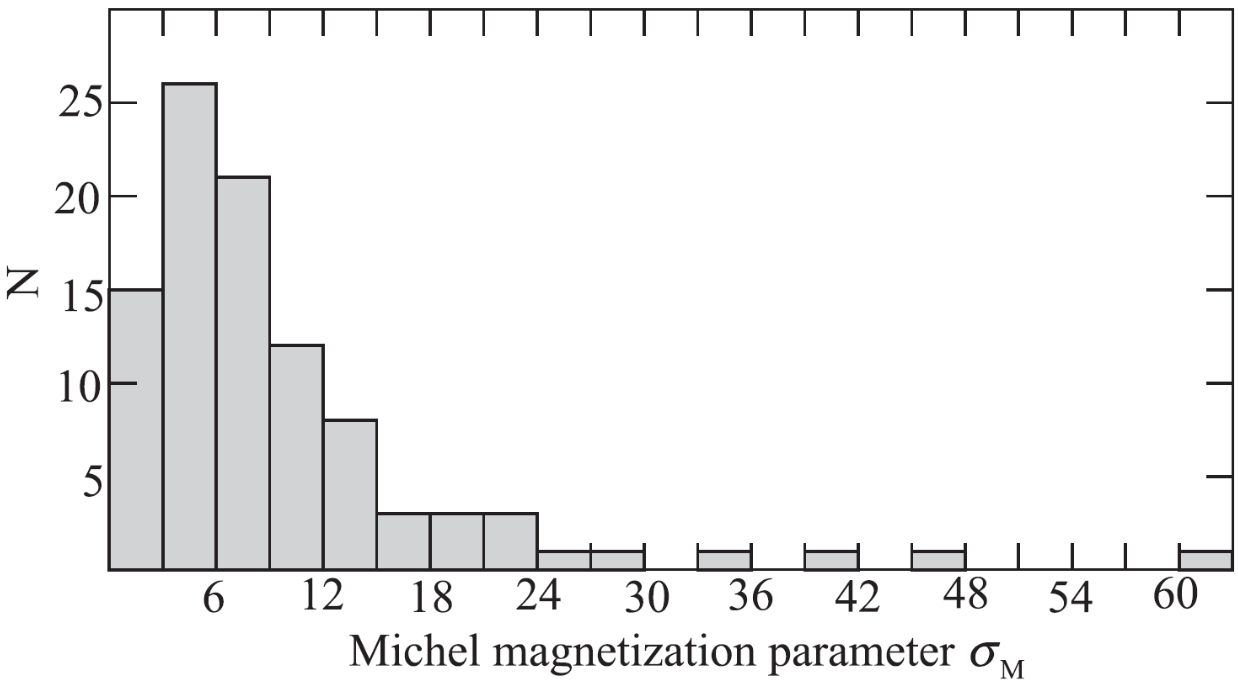


Figure 2. Distributions of the Michel magnetization parameter σ_M for the sample of 97 sources.

Core shift and jet parameters

E.E.Nokhrina, VB, Y.Y.Kovalev, A.A.Zheltoukhov. MNRAS, **447,** 2726 (2015)

$$\begin{cases} \dfrac{\mathrm{d}\mathcal{M}^2}{\mathrm{d}r_\perp} & = & F_1(\mathcal{M}^2, \Psi, r_\perp) \\ \dfrac{\mathrm{d}\Psi}{\mathrm{d}r_\perp} & = & F_2(\mathcal{M}^2, \Psi, r_\perp) \end{cases}$$

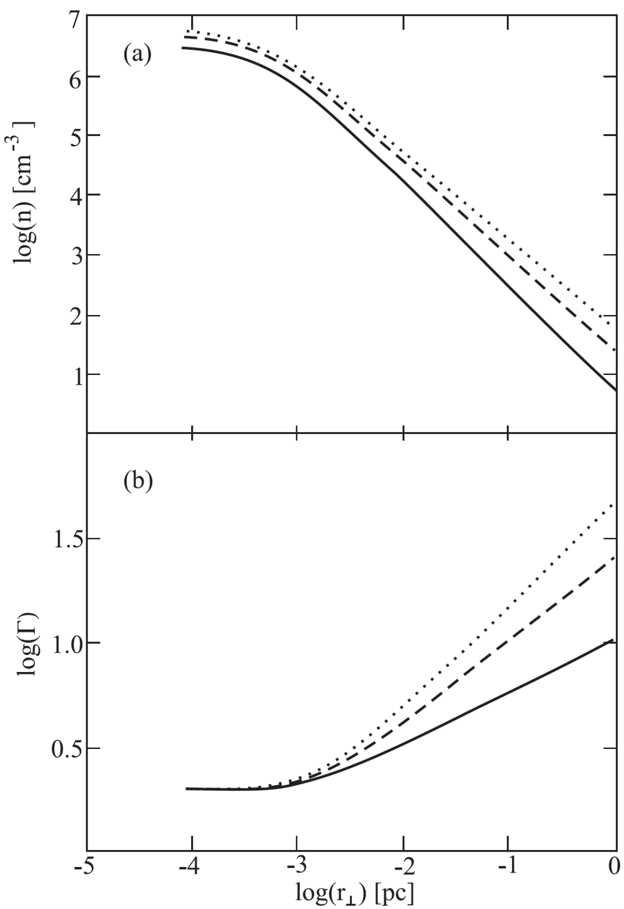


Figure 3. Transversal profile of the number density n_{e} (a) and Lorentz factor Γ (b) in logarithmical scale for $\lambda = 10^{13}$, jet radius $R_{\mathrm{jet}} = 1$ pc and three different values of σ: 5 (solid line), 15 (dashed line) and 30 (dotted line).

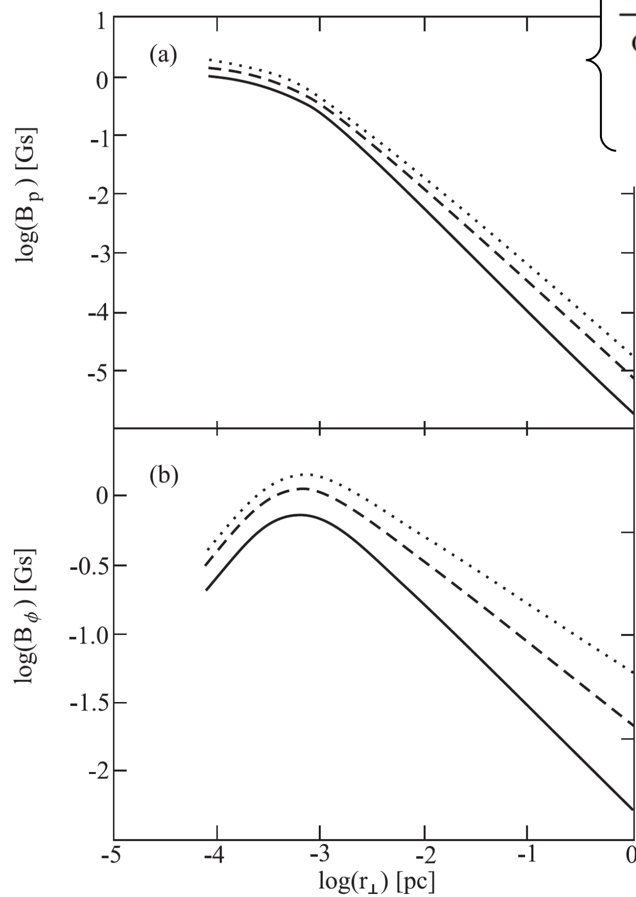


Figure 4. Transversal profile of poloidal (a) and toroidal (b) components of magnetic field in logarithmical scale for the same parameters and line types as in Fig. 3.

Core shift and jet parameters

E.E.Nokhrina, VB, Y.Y.Kovalev, A.A.Zheltoukhov. MNRAS, **447,** 2726 (2015)

Slow acceleration
along the jet

$$\dot{\Gamma}/\,\Gamma = 10^{-3}\ \mathrm{yr}^{-1}$$

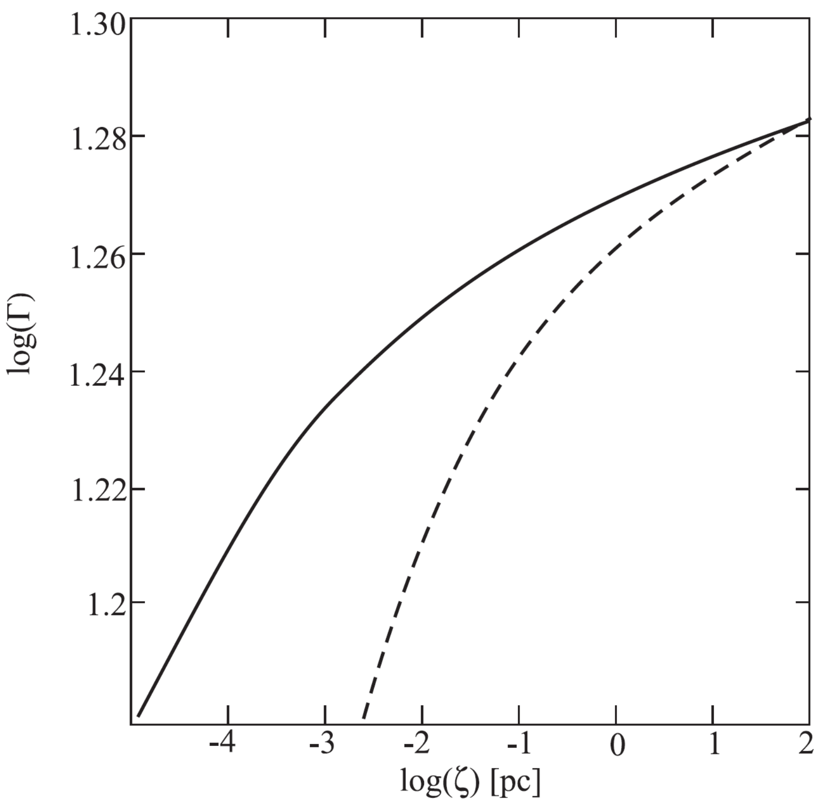


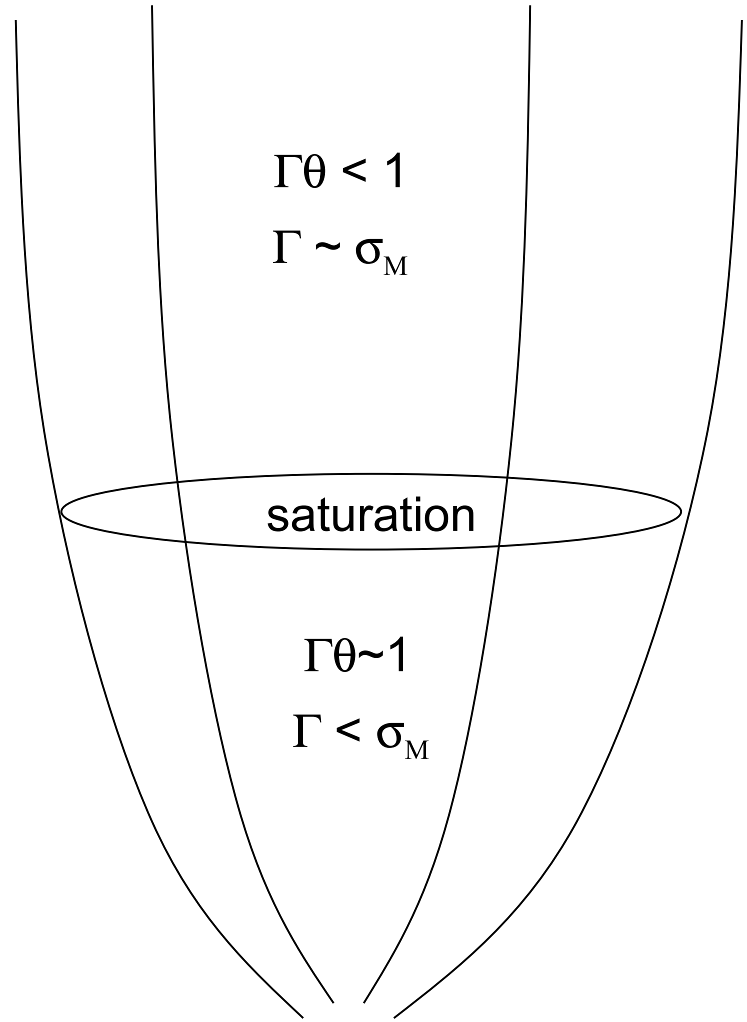
Figure 5. Dependence of Lorentz factor on coordinate along the jet in assumption of $\zeta \propto r_{\perp}^{3}$ (solid line) and $\zeta \propto r_{\perp}^{2}$ (dashed line) form of the jet.

Collimation parameter

For magnetically dominated flow
the theory prediction is

$$\Gamma\theta \sim 1$$

But in the saturation regime
$(\Gamma \sim \mathrm{const})$ $\quad \Gamma\theta \sim 0.1$
becomes possible.



Main conclusions

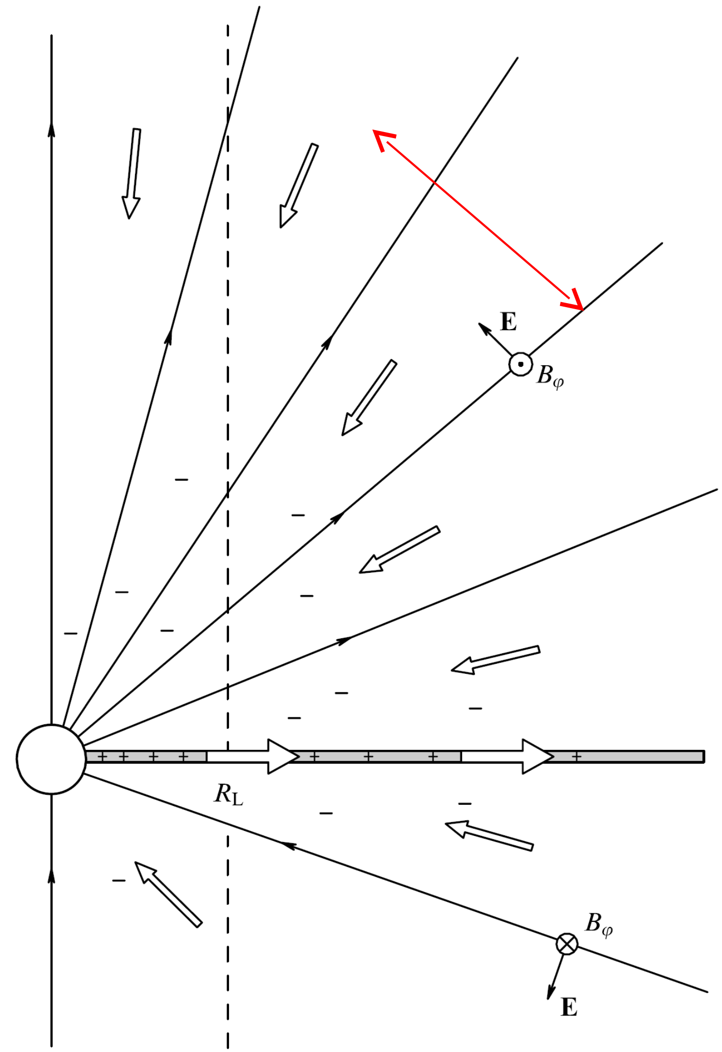
- Saturation
- Central core

Theoretical reason – a problem

F.C.Michel (1973)

What to do with (enormous) potential difference?

Ferraro isorotation law implies constant electric potential (Ω_{F}) along magnetic field lines.



A problem

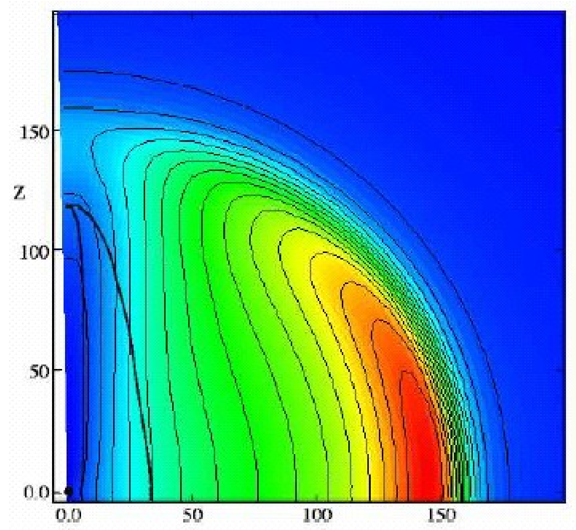
Longitudinal electric field?

Magnetically dominated wind → Any disturbance → Longitudinal electric field

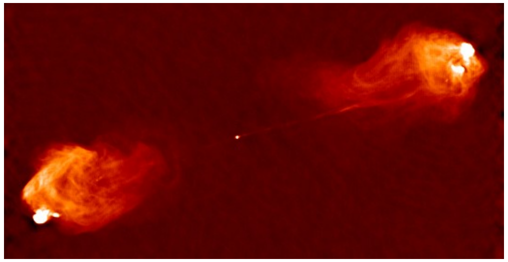
$$\mathrm{E}_{\perp} \longrightarrow E_{\parallel}$$

A problem

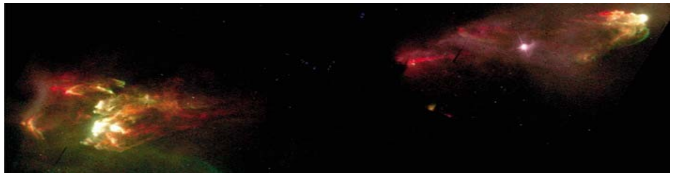
Switch-on wave, if there is no ambient pressure



S.Komissarov, MNRAS, **350**, 1431 (2004)



Lobes in AGN



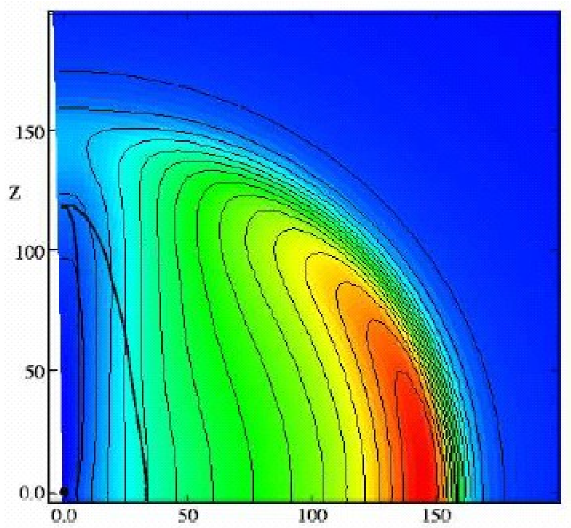
HH objects in YSO

Stellar wind in close binaries

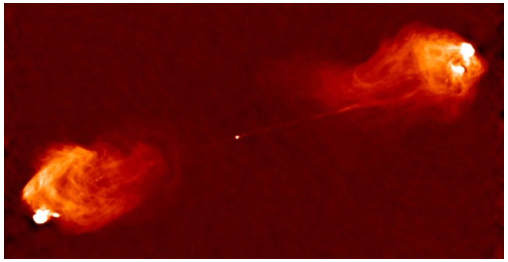
But what to do if we have it?

A problem

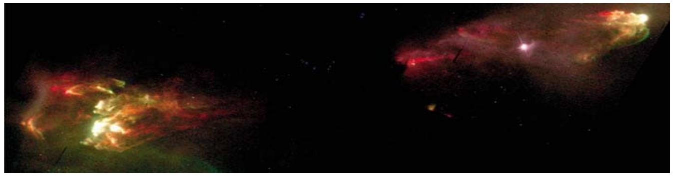
If there is no external environment, one can prolong the solution up to infinity.



S.Komissarov, MNRAS,**350**, 1431 (2004)



Lobes in AGN



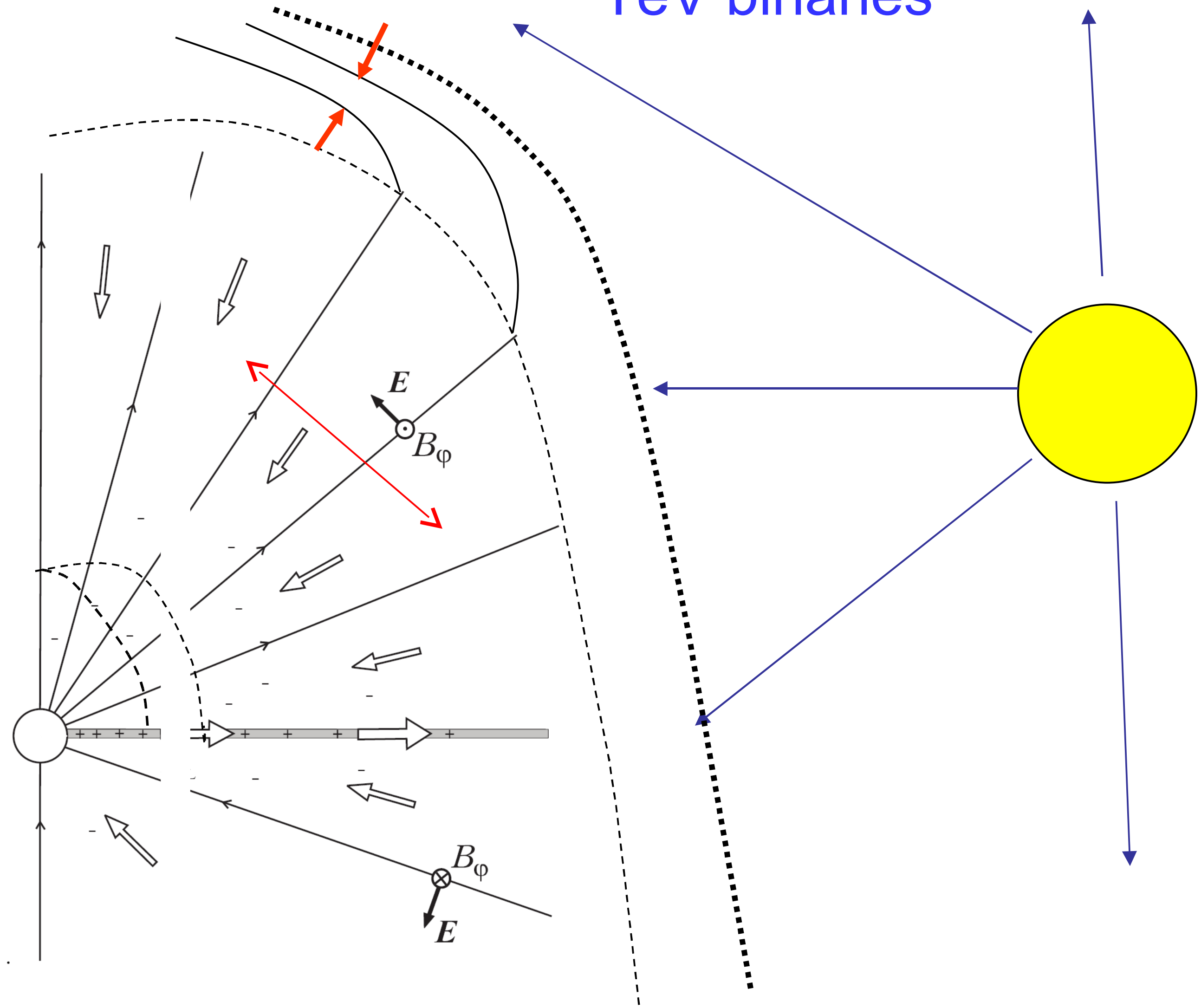
HH objects in YSOs

Stellar wind in binaries

But what to do if the wind meets the ambient?

A problem

TeV binaries



A problem

A question

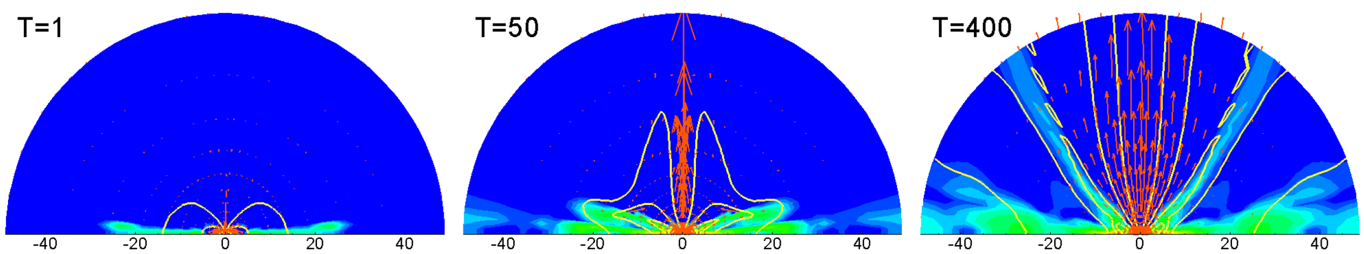
How to diminish the potential drop
(How to diminish the angular velocity Ω_{F})

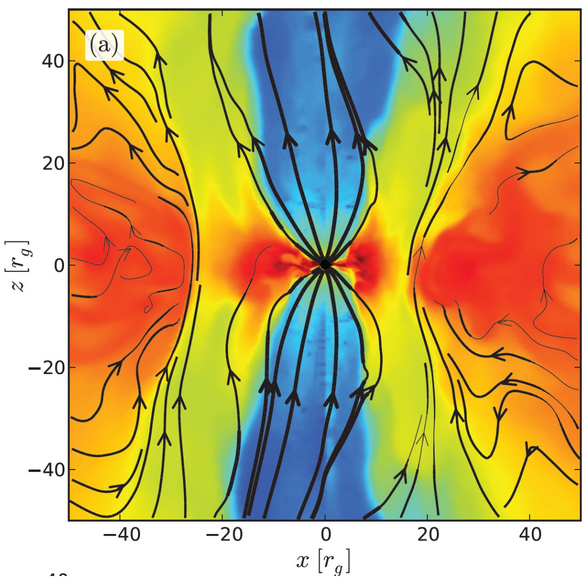
Two hints

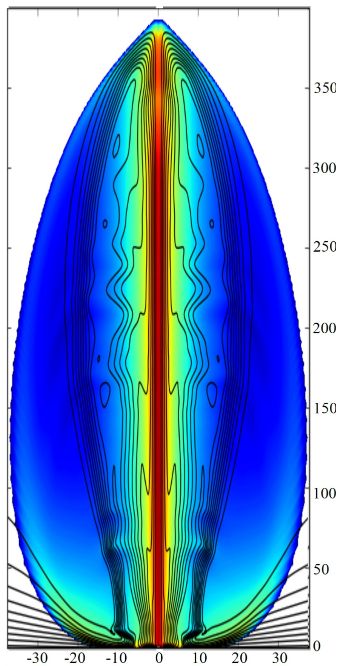
Dipole magnetic field topology
Not a vacuum electric field topology

A problem

[Magnetic field]







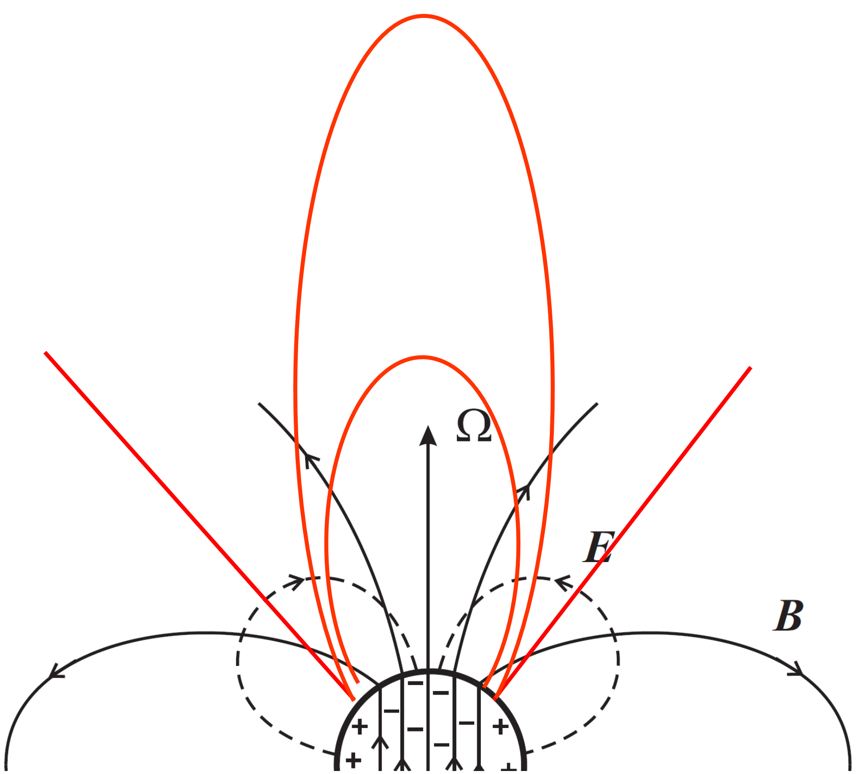
J.McKinney, A.Tchekhovskoy, R.Blandford

O. Bromberg, A. Tchekhovskoy

A problem

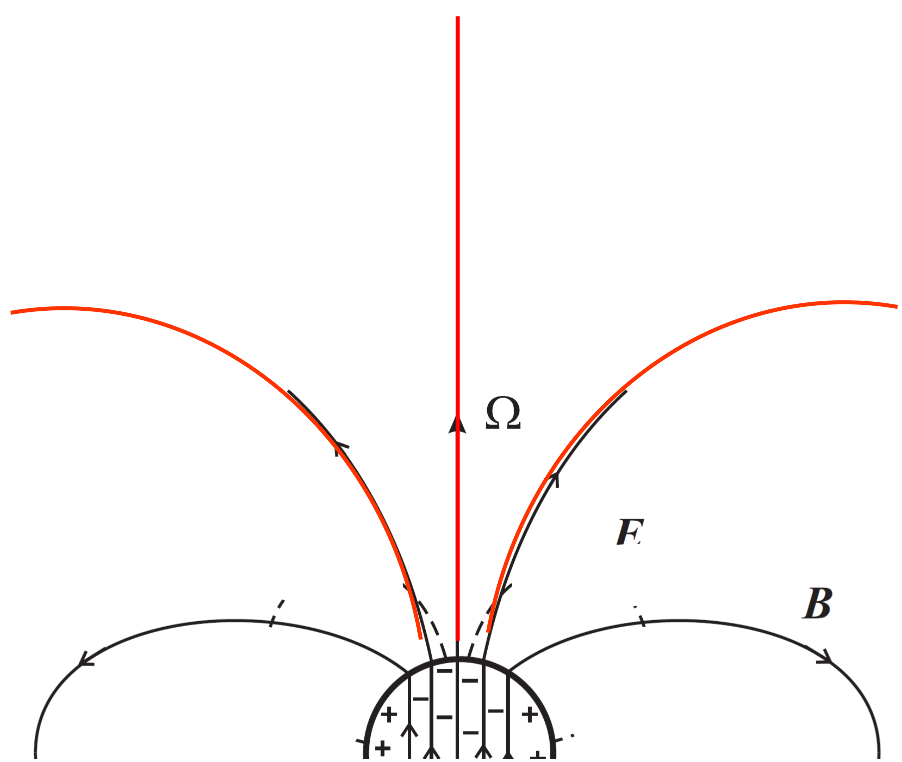
Electric potential

Vacuum



$Q = 0$

Full screening

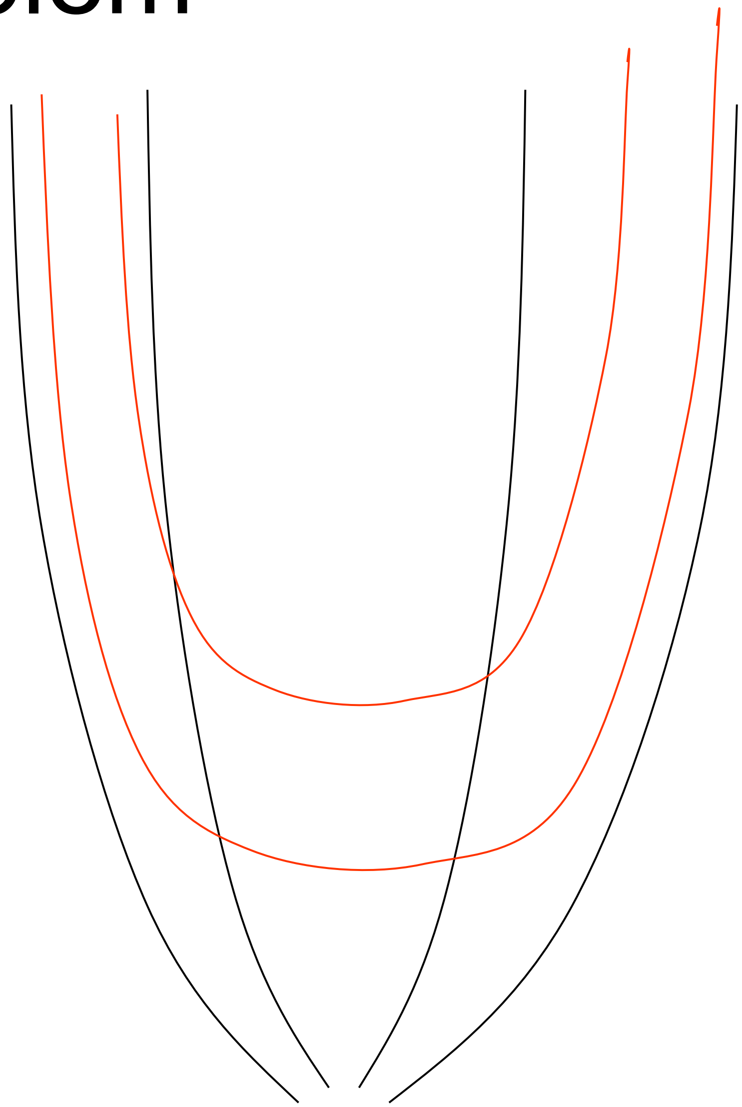


$Q \neq 0$

A problem

[Electric potential]

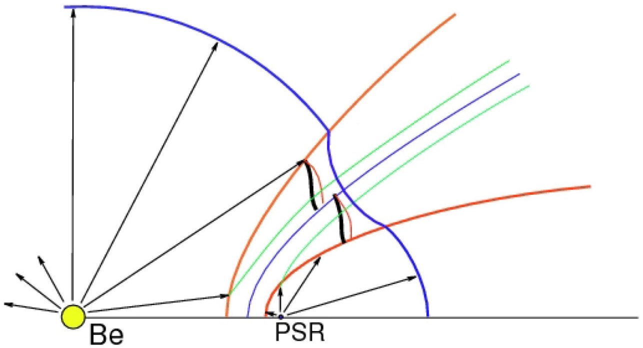
It is impossible to switch on the disturbance without generating the longitudinal electric field.



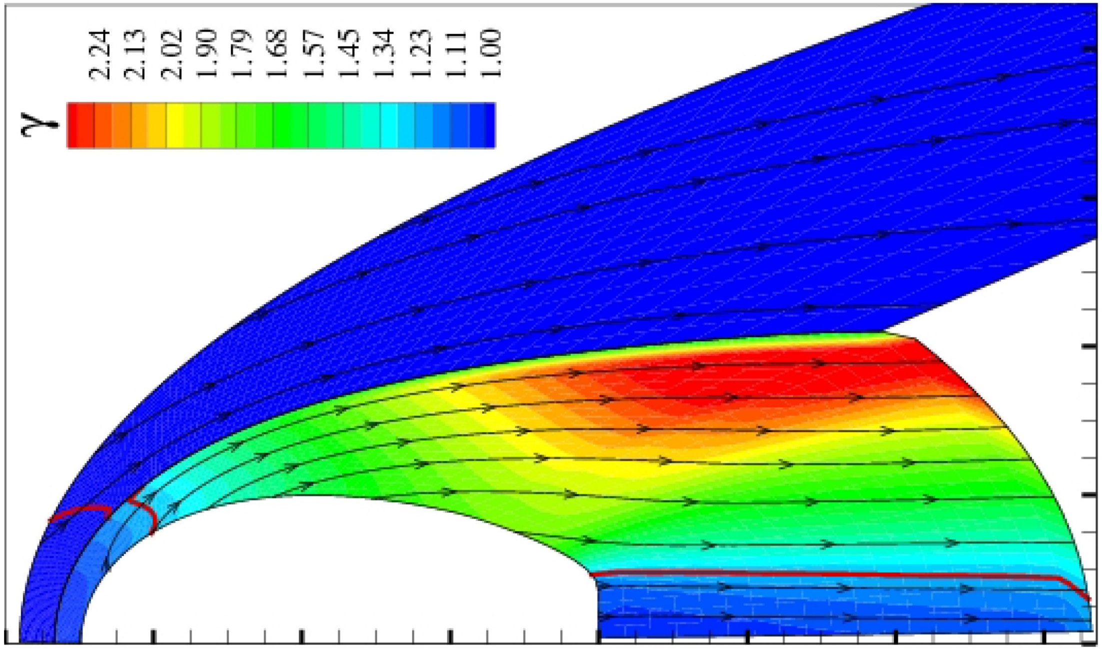
Statement #1

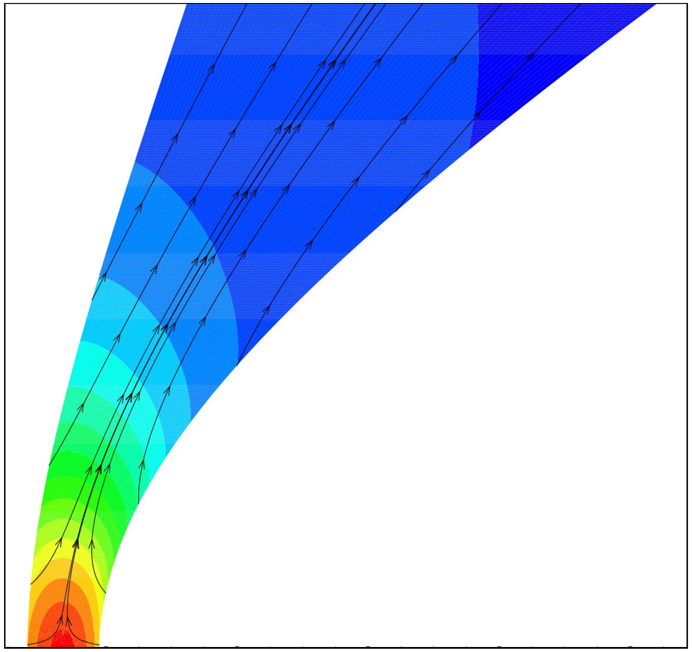
NOONE HAS ANALYZED CARFULLY ENOUGH THE PRESENCE OF THE TRANSVERSE POTENTIAL DROP WHEN THE HIGHLY MAGNETIZED WIND MEETS THE TARGET.

An example



!

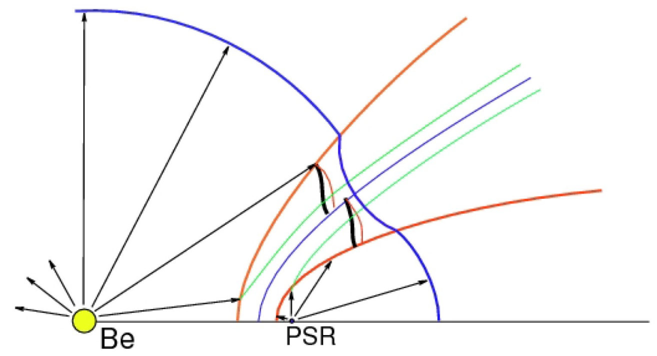


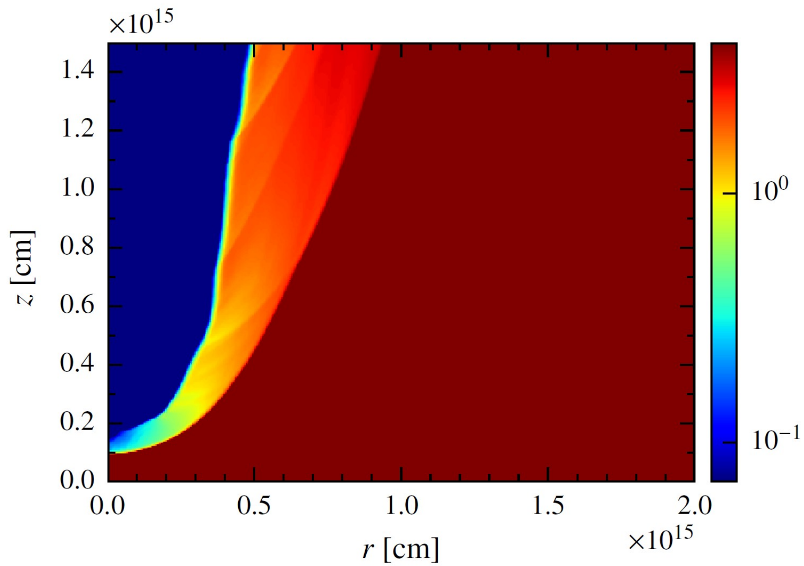


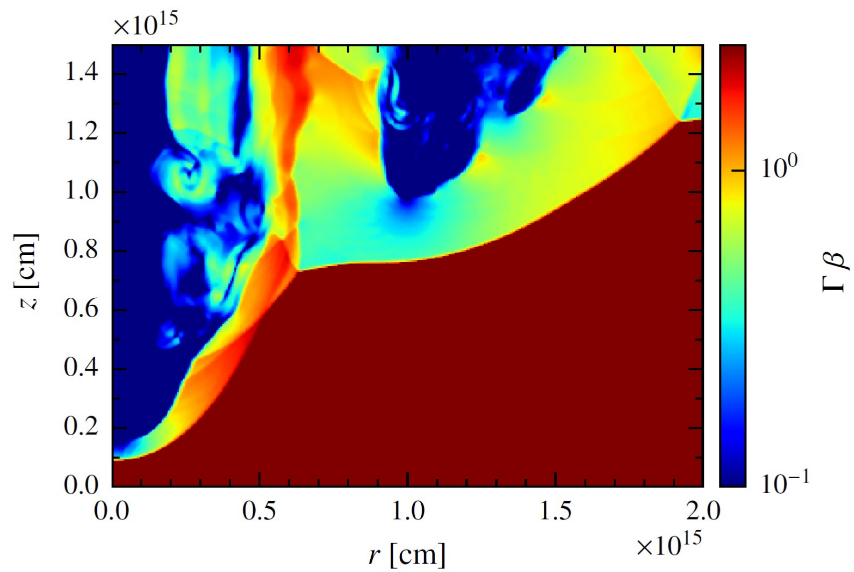
S.V.Bogovalov, D.Khangulyan, A.V.Koldoba, G.V.Ustyugova, F.Aharonian,
MNRAS, **387**, 63 (2008) MNRAS **419**, 3426 (2012)

An example

!



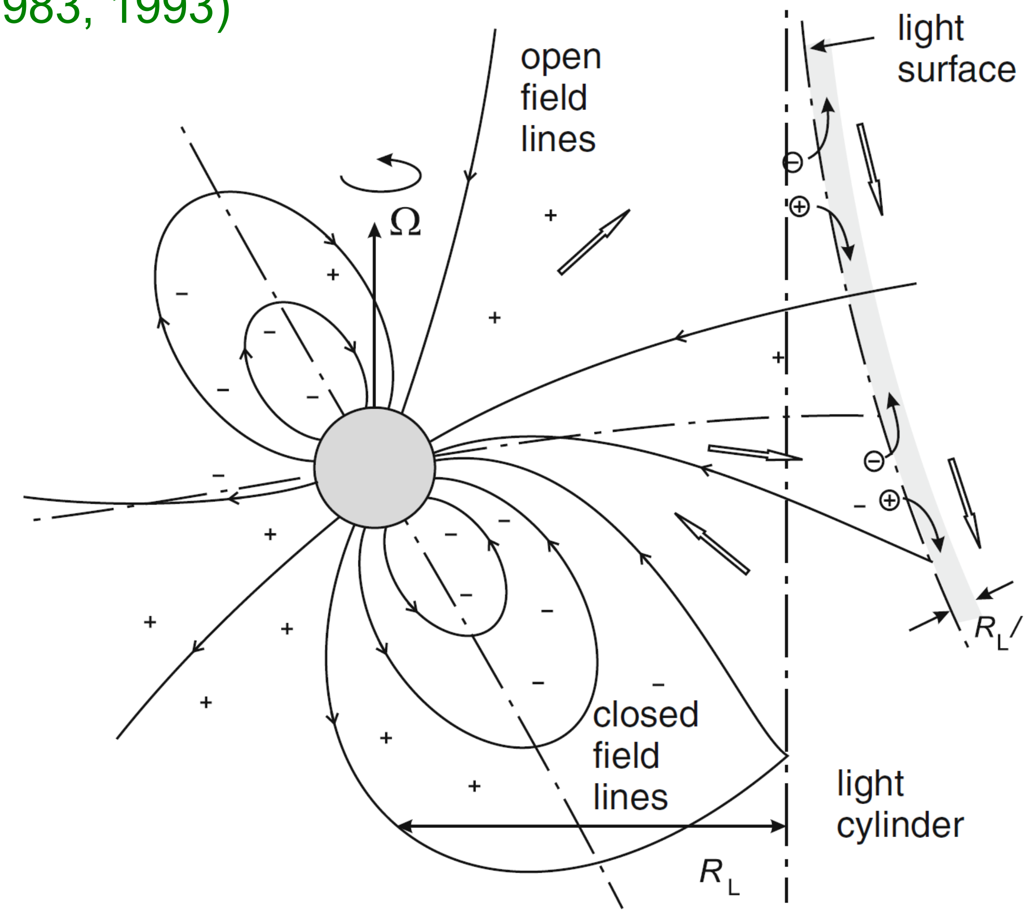




de la Cita et al, (2016) http://arxiv.org/abs/1604.02070

Our predictions

VB, Ya.N.Istomin, A.V.Gurevich (1983, 1993)
VB, R.R.Rafikov (2000)



- narrow sheet $\Delta r \sim R_L/\lambda$
- effective particle acceleration

up to $\Gamma \sim \sigma_M$ (10^6 for Crab)

- transverse displacement

$\Delta r \sim R_L/\lambda$

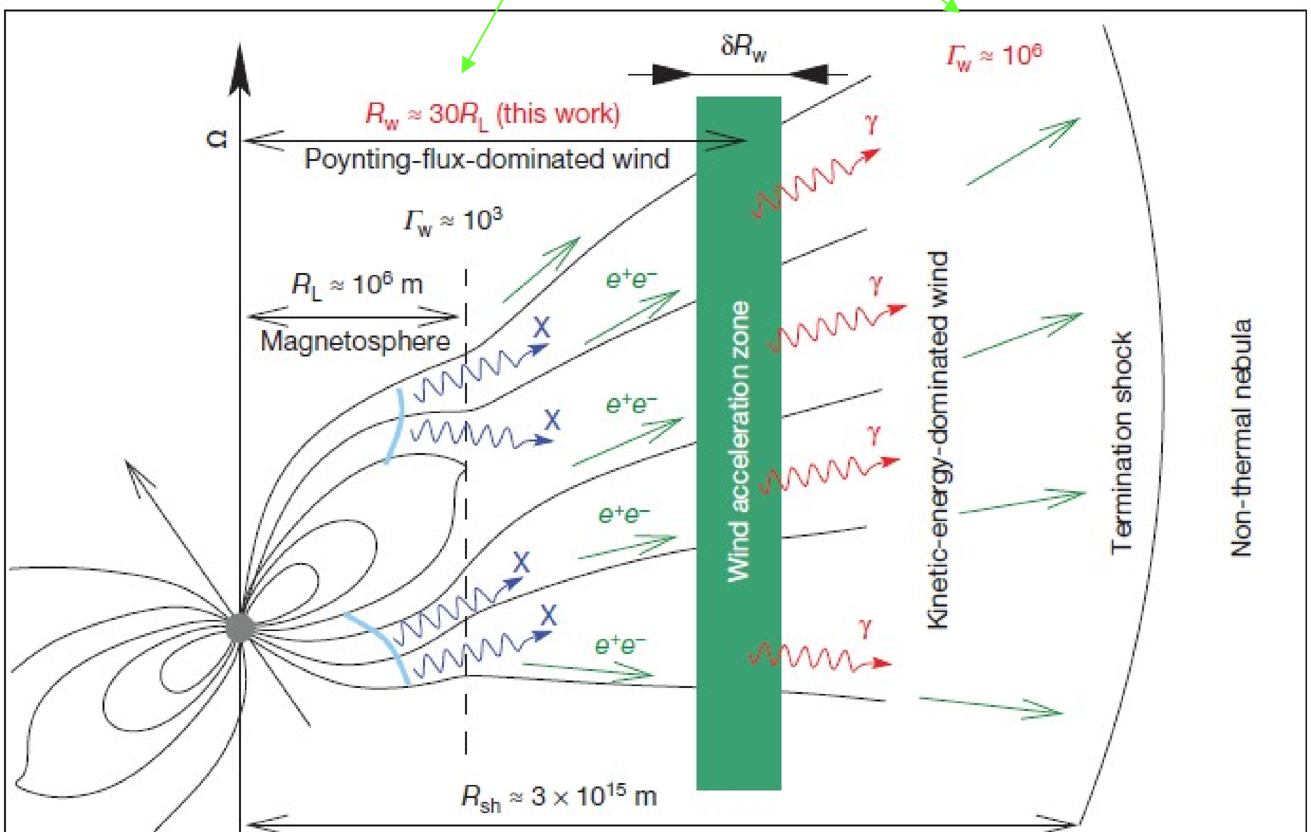
- Stop point!

LETTER

doi:10.1038/nature10793

Abrupt acceleration of a 'cold' ultrarelativistic wind from the Crab pulsar

F. A. Aharonian[1,2], S. V. Bogovalov[3] & D. Khangulyan[4]



Deceleration

Reconnection

G.Drenkhahn, A&A, **387**, 714 (2002)

G.Drenkhahn, H.C.Spruit, A&A, **391**, 1141 (2002)

…

A.Levinson, N.Globus, MNRAS, **458,** 2269 (2016)

Deceleration

Photon drag

Zhi-Yun Li, M.Begelman, T.Chiueh, ApJ, **384**, 567 (1992)

VB, N.Zakamska, H.Sol, MNRAS, **347**, 587 (2004)

M.Russo, Ch.Thompson, ApJ, **773**, 24 (2013)

Particle loading

R.Svensson, MNRAS, **227**, 403 (1987)

M.Lyutikov, MNRAS, **339**, 632 (2003)

E.V.Derishev, F.Aharonian, V.V.Kocharovsky, Vl.V.Kocharovsky, Phys.Rev.D, **68**, 043003 (2003)

B.Stern, J.Poutanen, MNRAS, **372**,1217 (2006)

M.Barkov et al., 2014 Fermi Symposium proceedings, arXiv:1502.02383

I: Photon drag

V.S.Beskin, A.V.Chernoglazov, MNRAS, **463**, 3398 (2016)

- Zero approximation – force-free
- MHD
- MHD with photon drag – qualitative
- MHD with photon drag – quantitative

Photon drag

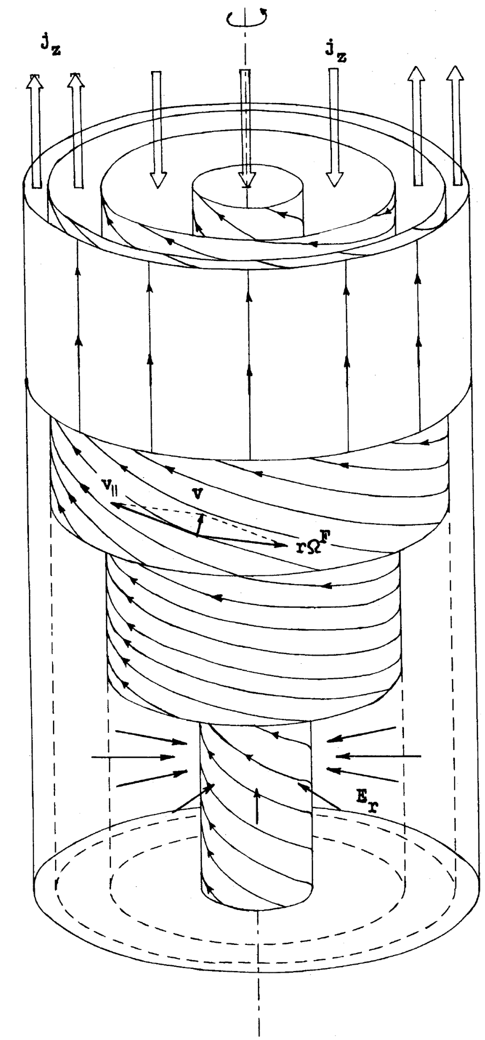
Zero force-free approximation

$$v_z^0 = c, \quad v_\varpi^0 = 0, \quad v_\varphi^0 = 0$$

$$\begin{cases} \mathbf{B} &= \dfrac{\nabla\Psi \times \mathbf{e}_\varphi}{2\pi r_\perp} - \dfrac{2I}{cr_\perp}\mathbf{e}_\varphi, \\ \mathbf{E} &= -\dfrac{\Omega_{\mathrm{F}}(\Psi)}{2\pi c}\nabla\Psi. \end{cases}$$

$$\begin{cases} B_z^0 = B_0 \\ B_\varphi^{(0)} = -\dfrac{2I}{cr_\perp}, \\ E_r^{(0)} = B_\varphi^0, \end{cases}$$

$$\boxed{4\pi I(\Psi) = 2\Omega_F(\Psi)\Psi}$$



Photon drag

Zero force-free approximation

$$v_z^0 = c, \quad v_\varpi^0 = 0, \quad v_\varphi^0 = 0$$

$$4\pi I(\Psi) = 2\Omega_F(\Psi)\Psi$$

$$\boxed{\Omega_{\mathrm{F}}(r_\perp) = \Omega_0 \left(1 - \frac{r_\perp^2}{r_{\mathrm{jet}}^2}\right)}$$

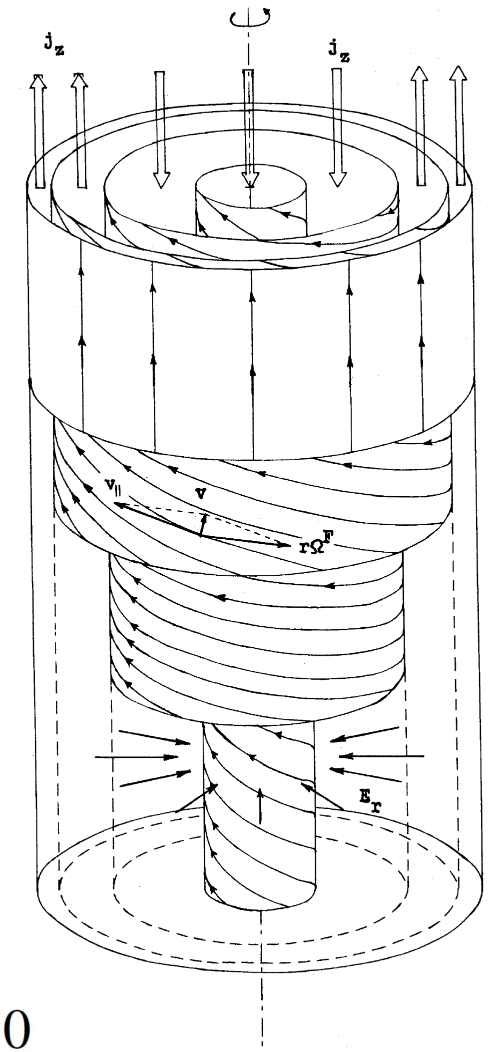
$$\left\{\begin{array}{lcl} B_z^0 = B_0 & & \\ B_\varphi^{(0)} & = & -\dfrac{2I}{cr_\perp}, \\ E_r^{(0)} & = & B_\varphi^0, \end{array}\right.$$

$$K = \frac{1}{4r_\perp}\frac{\mathrm{d}}{\mathrm{d}r_\perp}\left(r_\perp^2 \frac{\Omega_{\mathrm{F}}}{\Omega_0}\right)$$

$$K(0) = 1/2$$

$$\rho_{\mathrm{e}}^0(r_\perp) = -\frac{\Omega_0 B_0}{\pi c} K(r_\perp)$$

$$\pi \int_0^{r_{\mathrm{jet}}} K(r')r'\,\mathrm{d}r' = 0$$

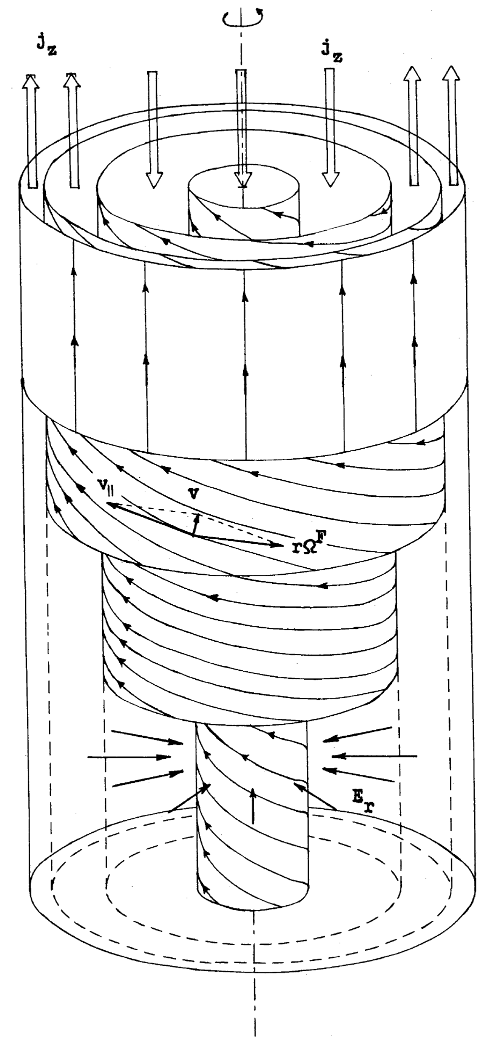


Photon drag

MHD flow without drag

- MHD cylindrical jet
- electron-positron plasma

$$(\mathbf{v}^{\pm}\nabla)\mathbf{p}^{\pm} \quad = \quad e\left(\mathbf{E}+\frac{\mathbf{v}^{\pm}}{c}\times\mathbf{B}\right)$$



Photon drag

MHD flow without drag

small disturbances

$$n^+ = \frac{\Omega_0 B_0}{2\pi ce}\left[\lambda - \frac{1}{4r_\perp}\frac{\mathrm{d}}{\mathrm{d}r_\perp}\left(r_\perp^2\frac{\Omega_\mathrm{F}}{\Omega_0}\right) + \eta^+(r_\perp, z)\right],$$

$$n^- = \frac{\Omega_0 B_0}{2\pi ce}\left[\lambda + \frac{1}{4r_\perp}\frac{\mathrm{d}}{\mathrm{d}r_\perp}\left(r_\perp^2\frac{\Omega_\mathrm{F}}{\Omega_0}\right) + \eta^-(r_\perp, z)\right],$$

$$v_z^\pm = c\left[1 - \xi_z^\pm(r_\perp, z)\right],$$

$$v_r^\pm = c\xi_r^\pm(r_\perp, z),$$

$$v_\varphi^\pm = c\xi_\varphi^\pm(r_\perp, z).$$

$$\Phi(r_\perp, z) = \frac{B_0}{c}\left[\int_0^{r_\perp}\Omega_\mathrm{F}(r')r'\mathrm{d}r' + \Omega_0 r_\perp^2\delta(r_\perp, z)\right],$$

$\delta \sim 1$

$$\Psi(r_\perp, z) = \pi B_0 r_\perp^2\left[1 + \varepsilon f(r_\perp, z)\right].$$

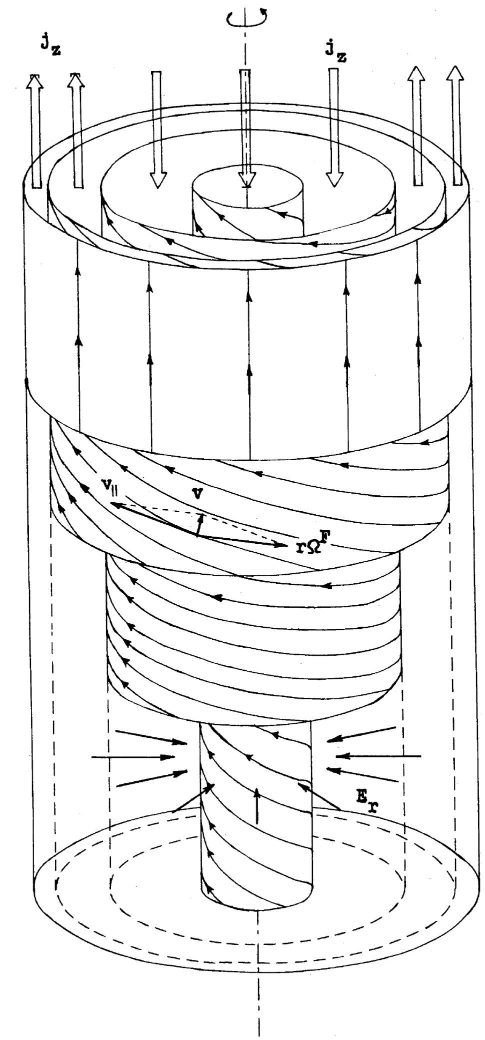
$$B_r = -\frac{\varepsilon}{2}r_\perp B_0\frac{\partial f}{\partial z},$$

$$B_\varphi = \frac{\Omega_0 r_\perp}{c}B_0\left[-\frac{\Omega_\mathrm{F}}{\Omega_0} - \zeta(r_\perp, z)\right],$$

$$B_z = B_0\left[1 + \frac{\varepsilon}{2r_\perp}\frac{\partial}{\partial r_\perp}\left(r_\perp^2 f\right)\right],$$

$$E_r = \frac{\Omega_0 r_\perp}{c}B_0\left[-\frac{\Omega_\mathrm{F}}{\Omega_0} - \frac{1}{r_\perp}\frac{\partial}{\partial r_\perp}(r_\perp^2\delta)\right],$$

$$E_z = -\frac{\Omega_0 r_\perp^2}{c}B_0\frac{\partial\delta}{\partial z},$$



Photon drag

MHD flow without drag

with N.Zakamska

$$
\left\{
\begin{aligned}
&-\frac{1}{r_\perp}\frac{\partial}{\partial r_\perp}(r_\perp^2\zeta) = \\
&2(\eta^+-\eta^-) - 2\,\underline{\left[(\lambda-K)\,\xi_z^+ - (\lambda+K)\,\xi_z^-\right]}\,, \\
&2(\eta^+-\eta^-) + \frac{1}{r_\perp}\frac{\partial}{\partial r_\perp}\left[r_\perp\frac{\partial}{\partial r_\perp}\left(r_\perp^2\delta\right)\right] + r_\perp^2\frac{\partial^2\delta}{\partial z^2} = 0, \\
&r_\perp\frac{\partial\zeta}{\partial z} = 2\left[(\lambda-K)\,\xi_r^+ - (\lambda+K)\,\xi_r^-\right], \\
&-\varepsilon r_\perp^2\frac{\partial^2 f}{\partial z^2} - \varepsilon\frac{\partial^2}{\partial r_\perp^2}\left(r_\perp^2 f\right) = \\
&4\,\frac{\Omega_0 r_\perp}{c}\,\underline{\left[(\lambda-K)\,\xi_\varphi^+ - (\lambda+K)\,\xi_\varphi^-\right]}\,, \\
&\frac{\partial}{\partial z}\left(\xi_r^+\gamma^+\right) = -\xi_r^+ F_{\mathrm{d}}(\gamma^+)^2 \\
&+4\,\frac{\lambda\sigma_{\mathrm{M}}}{r_{\mathrm{jet}}^2}\left[-\frac{\partial}{\partial r_\perp}(r_\perp^2\delta) + r_\perp\zeta - \underline{r_\perp\frac{\Omega_{\mathrm{F}}}{\Omega_0}\xi_z^+ + \frac{c}{\Omega_0}\xi_\varphi^+}\right], \\
&\frac{\partial}{\partial z}\left(\xi_r^-\gamma^-\right) = -\xi_r^- F_{\mathrm{d}}(\gamma^-)^2 \\
&-4\,\frac{\lambda\sigma_{\mathrm{M}}}{r_{\mathrm{jet}}^2}\left[-\frac{\partial}{\partial r_\perp}(r_\perp^2\delta) + r_\perp\zeta - \underline{r_\perp\frac{\Omega_{\mathrm{F}}}{\Omega_0}\xi_z^- + \frac{c}{\Omega_0}\xi_\varphi^-}\right], \\
&\frac{\partial}{\partial z}\left(\gamma^+\right) = -F_{\mathrm{d}}(\gamma^+)^2 + 4\,\frac{\lambda\sigma_{\mathrm{M}}}{r_{\mathrm{jet}}^2}\left(-r_\perp^2\frac{\partial\delta}{\partial z} - r_\perp\frac{\Omega_{\mathrm{F}}}{\Omega_0}\xi_r^+\right), \\
&\frac{\partial}{\partial z}\left(\gamma^-\right) = -F_{\mathrm{d}}(\gamma^-)^2 - 4\,\frac{\lambda\sigma_{\mathrm{M}}}{r_{\mathrm{jet}}^2}\left(-r_\perp^2\frac{\partial\delta}{\partial z} - r_\perp\frac{\Omega_{\mathrm{F}}}{\Omega_0}\xi_r^-\right), \\
&\frac{\partial}{\partial z}\left(\xi_\varphi^+\gamma^+\right) = -\xi_\varphi^+ F_{\mathrm{d}}(\gamma^+)^2 \\
&+4\,\frac{\lambda\sigma_{\mathrm{M}}}{r_{\mathrm{jet}}^2}\left(-\frac{\varepsilon}{2}\frac{c r_\perp}{\Omega_0}\frac{\partial f}{\partial z} - \frac{c}{\Omega_0}\xi_r^+\right), \\
&\frac{\partial}{\partial z}\left(\xi_\varphi^-\gamma^-\right) = -\xi_\varphi^- F_{\mathrm{d}}(\gamma^-)^2 \\
&-4\,\frac{\lambda\sigma_{\mathrm{M}}}{r_{\mathrm{jet}}^2}\left(-\frac{\varepsilon}{2}\frac{c r_\perp}{\Omega_0}\frac{\partial f}{\partial z} - \frac{c}{\Omega_0}\xi_r^-\right).
\end{aligned}
\right.
$$

Force-free structure remains the exact MHD solution, i.e.

$$\zeta = \delta = f = 0$$

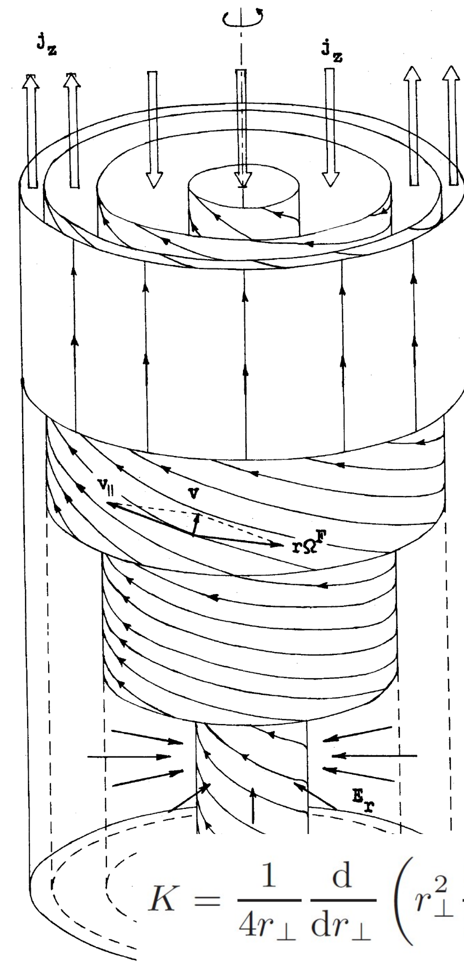
$$\xi_r^\pm = 0$$

if

$$(\lambda - K)\,\xi_z^+ = (\lambda + K)\,\xi_z^-$$

$$\xi_\varphi^\pm = x\xi_z^\pm$$

But now for finite particle energy



$$K = \frac{1}{4r_\perp}\frac{\mathrm{d}}{\mathrm{d}r_\perp}\left(r_\perp^2\frac{\Omega_{\mathrm{F}}}{\Omega_0}\right)$$

$$x = \Omega_{\mathrm{F}}(r_\perp)r_\perp/c$$

Photon drag

$$\Gamma = \frac{\gamma^+ + \gamma^-}{2}$$

$$G = \gamma^+ - \gamma^-$$

[MHD flow without drag](#)

with N.Zakamska

$$P_+ = \frac{\xi_z^+ + \xi_z^-}{2}$$

$$Q_+ = \frac{\xi_\varphi^+ + \xi_\varphi^-}{2}$$

$$P_- = \xi_z^+ - \xi_z^-$$

$$Q_- = \xi_\varphi^+ - \xi_\varphi^-$$

$$\left\{\begin{aligned}
&-\frac{1}{r_\perp}\frac{\partial}{\partial r_\perp}(r_\perp^2\zeta) = \\
&2(\eta^+ - \eta^-) - 2\left[(\lambda - K)\,\xi_z^+ - (\lambda + K)\,\xi_z^-\right], \\
&2(\eta^+ - \eta^-) + \frac{1}{r_\perp}\frac{\partial}{\partial r_\perp}\left[r_\perp \frac{\partial}{\partial r_\perp}\left(r_\perp^2\delta\right)\right] + r_\perp^2\frac{\partial^2\delta}{\partial z^2} = 0, \\
&r_\perp\frac{\partial\zeta}{\partial z} = 2\left[(\lambda - K)\,\xi_r^+ - (\lambda + K)\,\xi_r^-\right], \\
&-\varepsilon r_\perp^2\frac{\partial^2 f}{\partial z^2} - \varepsilon\frac{\partial^2}{\partial r_\perp^2}\left(r_\perp^2 f\right) = \\
&4\,\frac{\Omega_0 r_\perp}{c}\left[(\lambda - K)\,\xi_\varphi^+ - (\lambda + K)\,\xi_\varphi^-\right], \\
&\frac{\partial}{\partial z}\left(\xi_r^+\gamma^+\right) = -\xi_r^+ F_{\rm d}(\gamma^+)^2 \\
&+4\,\frac{\lambda\sigma_{\rm M}}{r_{\rm jet}^2}\left[-\frac{\partial}{\partial r_\perp}(r_\perp^2\delta) + r_\perp\zeta - r_\perp\frac{\Omega_{\rm F}}{\Omega_0}\xi_z^+ + \frac{c}{\Omega_0}\xi_\varphi^+\right], \\
&\frac{\partial}{\partial z}\left(\xi_r^-\gamma^-\right) = -\xi_r^- F_{\rm d}(\gamma^-)^2 \\
&-4\,\frac{\lambda\sigma_{\rm M}}{r_{\rm jet}^2}\left[-\frac{\partial}{\partial r_\perp}(r_\perp^2\delta) + r_\perp\zeta - r_\perp\frac{\Omega_{\rm F}}{\Omega_0}\xi_z^- + \frac{c}{\Omega_0}\xi_\varphi^-\right], \\
&\frac{\partial}{\partial z}\left(\gamma^+\right) = -F_{\rm d}(\gamma^+)^2 + 4\,\frac{\lambda\sigma_{\rm M}}{r_{\rm jet}^2}\left(-r_\perp^2\frac{\partial\delta}{\partial z} - r_\perp\frac{\Omega_{\rm F}}{\Omega_0}\xi_r^+\right), \\
&\frac{\partial}{\partial z}\left(\gamma^-\right) = -F_{\rm d}(\gamma^-)^2 - 4\,\frac{\lambda\sigma_{\rm M}}{r_{\rm jet}^2}\left(-r_\perp^2\frac{\partial\delta}{\partial z} - r_\perp\frac{\Omega_{\rm F}}{\Omega_0}\xi_r^-\right), \\
&\frac{\partial}{\partial z}\left(\xi_\varphi^+\gamma^+\right) = -\xi_\varphi^+ F_{\rm d}(\gamma^+)^2 \\
&+4\,\frac{\lambda\sigma_{\rm M}}{r_{\rm jet}^2}\left(-\frac{\varepsilon}{2}\frac{c r_\perp}{\Omega_0}\frac{\partial f}{\partial z} - \frac{c}{\Omega_0}\xi_r^+\right), \\
&\frac{\partial}{\partial z}\left(\xi_\varphi^-\gamma^-\right) = -\xi_\varphi^- F_{\rm d}(\gamma^-)^2 \\
&-4\,\frac{\lambda\sigma_{\rm M}}{r_{\rm jet}^2}\left(-\frac{\varepsilon}{2}\frac{c r_\perp}{\Omega_0}\frac{\partial f}{\partial z} - \frac{c}{\Omega_0}\xi_r^-\right).
\end{aligned}\right.$$

$$(\lambda - K)\,\xi_z^+ = (\lambda + K)\,\xi_z^-$$

$$\xi_\varphi^\pm = x\xi_z^\pm$$

$$\xi_r^\pm = 0$$

- Only one free function

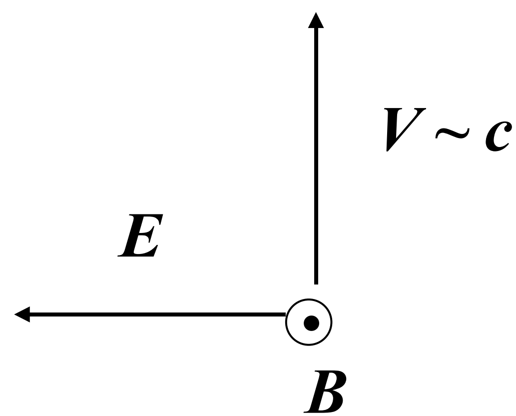
$$\boxed{\Gamma^2 = \Gamma_0^2 + x^2}$$

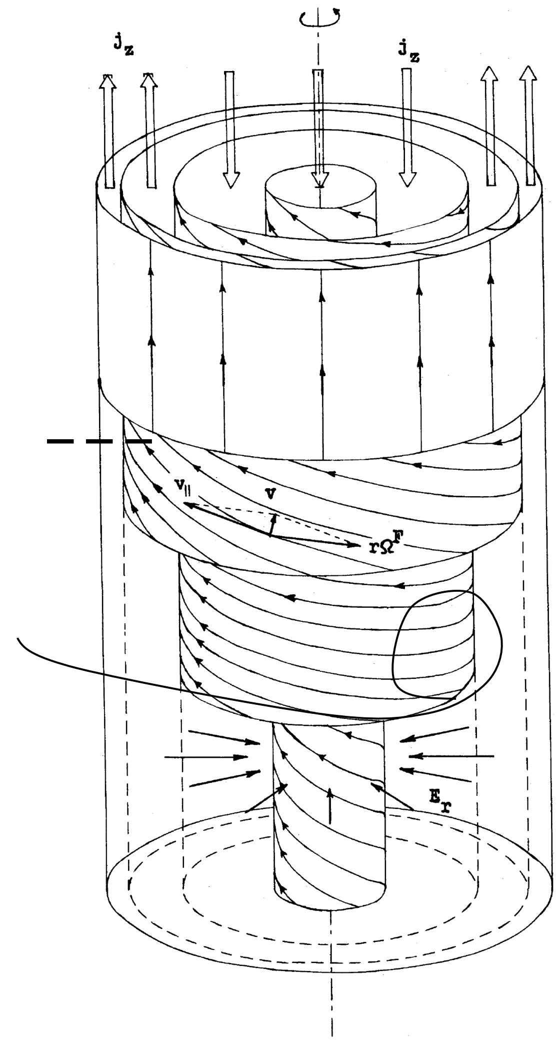
$$\begin{aligned}
Q_\pm &= xP_\pm, \\
P_- &= 2\frac{K}{\lambda}P_+, \\
Q_- &= 2\frac{K}{\lambda}Q_+, \\
G &= -\Gamma^3(1 - x^2P_+)P_-
\end{aligned}$$

$$P_+ = \frac{1}{\Gamma(\Gamma + \sqrt{\Gamma^2 - x^2})}$$

Photon drag

MHD flow + isotropic radiation field

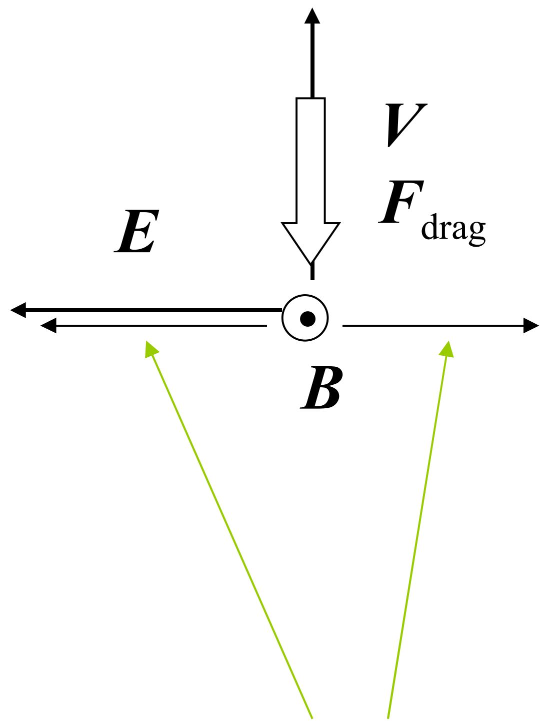


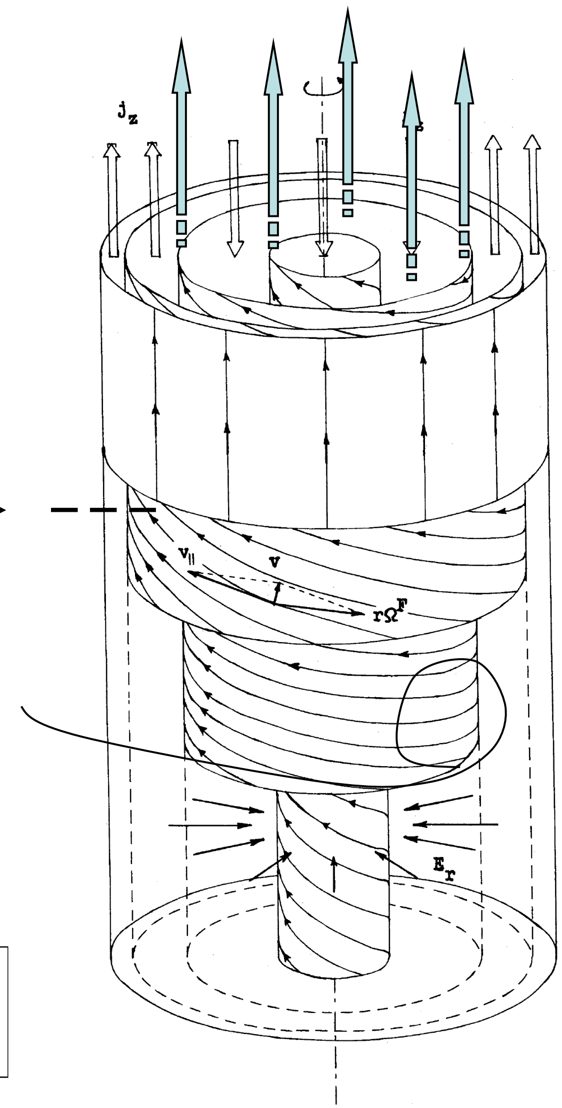


Photon drag

IC

MHD flow + isotropic radiation field





No work if the force is orthogonal to magnetic field

But IC photons take the energy away.

$$\mathbf{U}_{\mathrm{dr}} = c\frac{\mathbf{F}_{\mathrm{drag}} \times \mathbf{B}}{eB^2}$$

Photon drag

IC

MHD flow + isotropic radiation field

$$\nabla \mathbf{S} = -\mathbf{j}\,\mathbf{E}$$

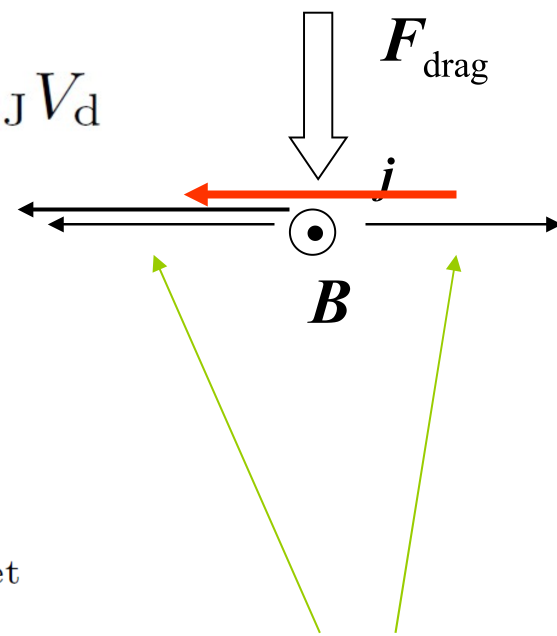
$$j_r \sim \lambda \rho_{\rm GJ} V_{\rm d}$$

$$\frac{c}{4\pi}\frac{{\rm d}B_\varphi^2}{{\rm d}z} \approx j_r B_\varphi$$

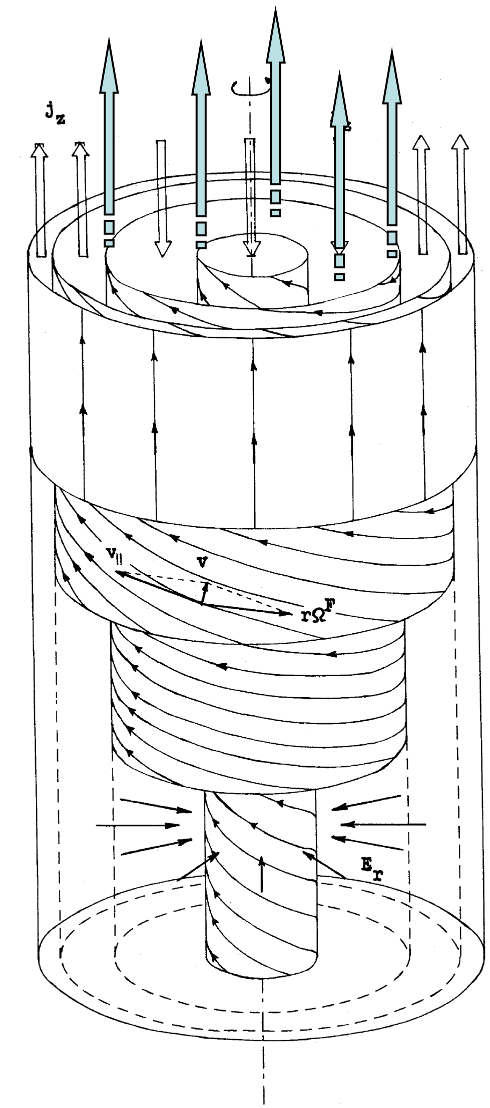
$$B_\varphi / B_z \sim r_{\rm jet}/R_{\rm L}$$

$$W_{\rm tot} \sim (c/4\pi) B_\varphi^2 r_{\rm jet}^2$$

$$L_{\rm dr} \sim \sigma_{\rm M}\frac{m_{\rm e}c^2}{F_{\rm drag}}$$



$$V_{\rm d} \sim c\frac{F_{\rm drag}}{eB_\varphi}$$



Photon drag

IC

MHD flow + isotropic radiation field

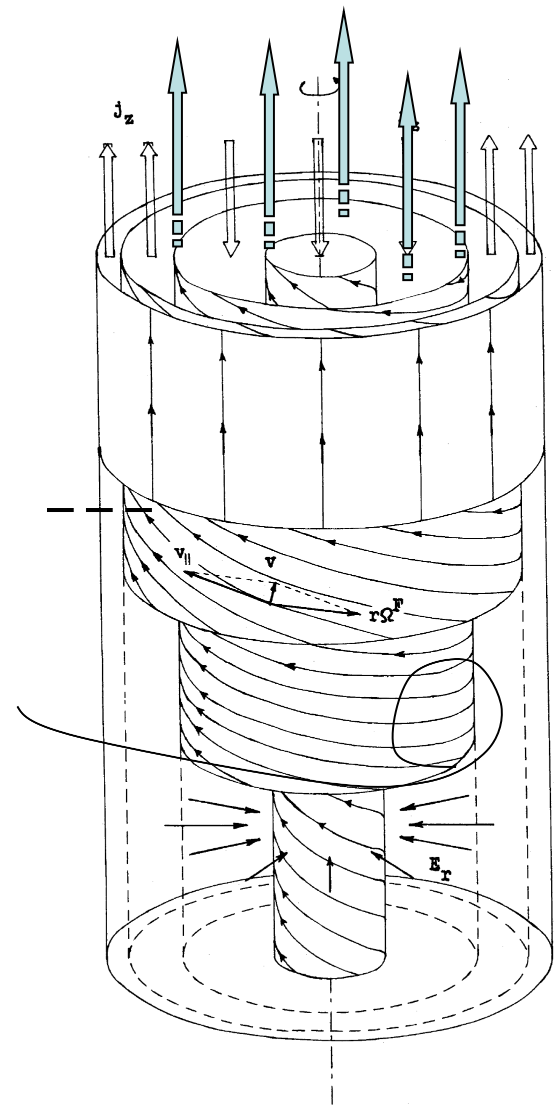
Damping length

$$L_{\rm dr} \sim \sigma_{\rm M} \frac{m_{\rm e} c^2}{F_{\rm drag}}$$

Appropriate work

$$A_{\rm dr} \sim \sigma_{\rm M} m_{\rm e} c^2$$

And IC photons take the energy away.

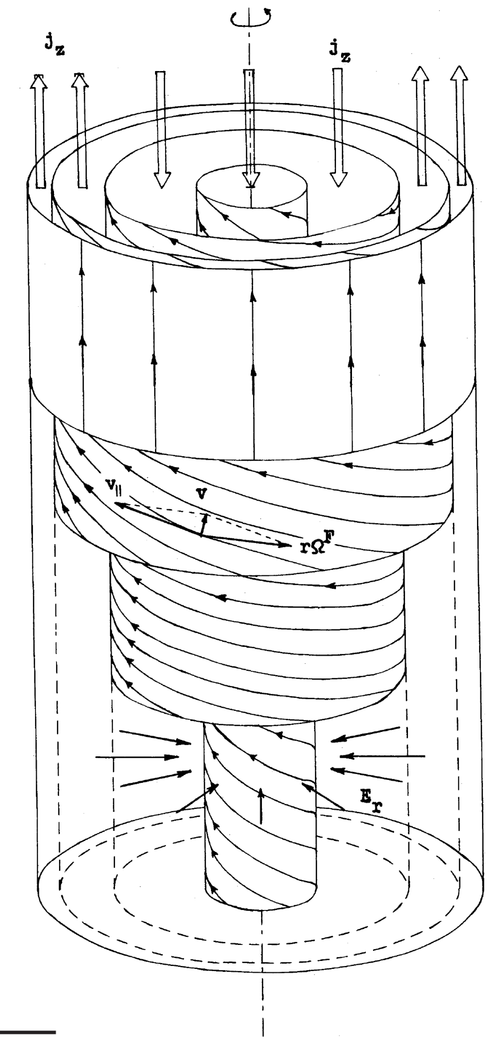


Photon drag – the task

MHD flow + radiation field

Step I: basic MHD equation

- MHD cylindrical jet
- electron-positron plasma
- isotropic photon field $U_{\rm iso}$



$$(\mathbf{v}^{\pm}\nabla)\mathbf{p}^{\pm} = e\left(\mathbf{E} + \frac{\mathbf{v}^{\pm}}{c}\times\mathbf{B}\right) + \mathbf{F}^{\pm}_{\rm drag}$$

$$\mathbf{F}^{\pm}_{\rm drag} = -\frac{4}{3}\frac{\mathbf{v}}{v}\sigma_T U_{\rm iso}(\gamma^{\pm})^2$$

$$U = U_{\rm iso} = \eta\frac{L_{\rm tot}}{4\pi r^2_{\rm cloud}c}$$

Photon drag

MHD flow + radiation field

Step I: basic MHD equations

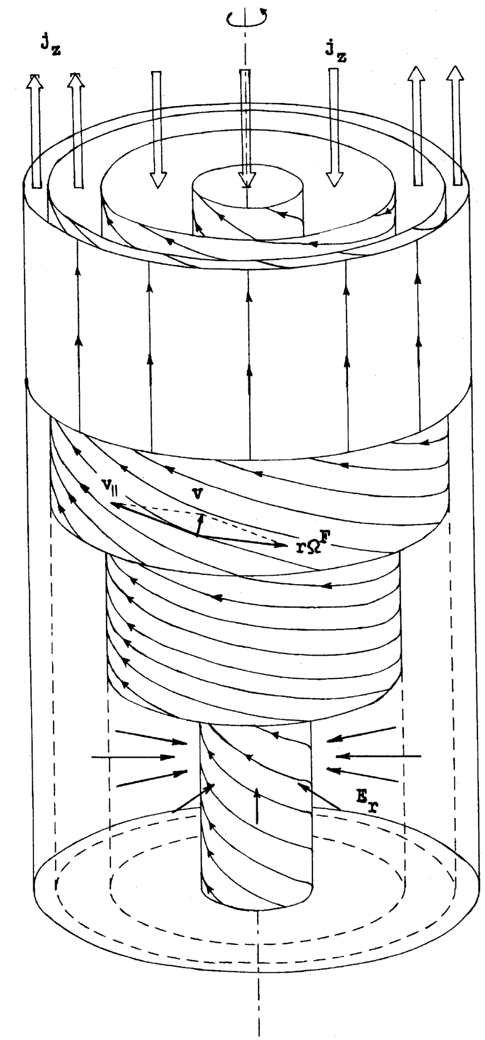
$$
\left\{
\begin{aligned}
&-\frac{1}{r_\perp}\frac{\partial}{\partial r_\perp}(r_\perp^2\zeta) = \\
&2(\eta^+ - \eta^-) - 2\left[(\lambda - K)\,\xi_z^+ - (\lambda + K)\,\xi_z^-\right], \\
&2(\eta^+ - \eta^-) + \frac{1}{r_\perp}\frac{\partial}{\partial r_\perp}\left[r_\perp\frac{\partial}{\partial r_\perp}\left(r_\perp^2\delta\right)\right] + r_\perp^2\frac{\partial^2\delta}{\partial z^2} = 0, \\
&r_\perp\frac{\partial\zeta}{\partial z} = 2\left[(\lambda - K)\,\xi_r^+ - (\lambda + K)\,\xi_r^-\right], \\
&-\varepsilon r_\perp^2\frac{\partial^2 f}{\partial z^2} - \varepsilon\frac{\partial^2}{\partial r_\perp^2}\left(r_\perp^2 f\right) = \\
&4\,\frac{\Omega_0 r_\perp}{c}\left[(\lambda - K)\,\xi_\varphi^+ - (\lambda + K)\,\xi_\varphi^-\right], \\
&\frac{\partial}{\partial z}\left(\xi_r^+\gamma^+\right) = -\xi_r^+ F_{\rm d}(\gamma^+)^2 \\
&+4\,\frac{\lambda\sigma_{\rm M}}{r_{\rm jet}^2}\left[-\frac{\partial}{\partial r_\perp}(r_\perp^2\delta) + r_\perp\zeta - r_\perp\frac{\Omega_{\rm F}}{\Omega_0}\xi_z^+ + \frac{c}{\Omega_0}\xi_\varphi^+\right], \\
&\frac{\partial}{\partial z}\left(\xi_r^-\gamma^-\right) = -\xi_r^- F_{\rm d}(\gamma^-)^2 \\
&-4\,\frac{\lambda\sigma_{\rm M}}{r_{\rm jet}^2}\left[-\frac{\partial}{\partial r_\perp}(r_\perp^2\delta) + r_\perp\zeta - r_\perp\frac{\Omega_{\rm F}}{\Omega_0}\xi_z^- + \frac{c}{\Omega_0}\xi_\varphi^-\right], \\
&\frac{\partial}{\partial z}\left(\gamma^+\right) = -F_{\rm d}(\gamma^+)^2 + 4\,\frac{\lambda\sigma_{\rm M}}{r_{\rm jet}^2}\left(-r_\perp^2\frac{\partial\delta}{\partial z} - r_\perp\frac{\Omega_{\rm F}}{\Omega_0}\xi_r^+\right), \\
&\frac{\partial}{\partial z}\left(\gamma^-\right) = -F_{\rm d}(\gamma^-)^2 - 4\,\frac{\lambda\sigma_{\rm M}}{r_{\rm jet}^2}\left(-r_\perp^2\frac{\partial\delta}{\partial z} - r_\perp\frac{\Omega_{\rm F}}{\Omega_0}\xi_r^-\right), \\
&\frac{\partial}{\partial z}\left(\xi_\varphi^+\gamma^+\right) = -\xi_\varphi^+ F_{\rm d}(\gamma^+)^2 \\
&+4\,\frac{\lambda\sigma_{\rm M}}{r_{\rm jet}^2}\left(-\frac{\varepsilon}{2}\frac{c r_\perp}{\Omega_0}\frac{\partial f}{\partial z} - \frac{c}{\Omega_0}\xi_r^+\right), \\
&\frac{\partial}{\partial z}\left(\xi_\varphi^-\gamma^-\right) = -\xi_\varphi^- F_{\rm d}(\gamma^-)^2 \\
&-4\,\frac{\lambda\sigma_{\rm M}}{r_{\rm jet}^2}\left(-\frac{\varepsilon}{2}\frac{c r_\perp}{\Omega_0}\frac{\partial f}{\partial z} - \frac{c}{\Omega_0}\xi_r^-\right).
\end{aligned}
\right.
$$

$$\sigma_{\rm M} = \frac{\Omega_0 e B_0 r_{\rm jet}^2}{4\lambda m c^3}$$

$$K = \frac{1}{4r_\perp}\frac{\rm d}{{\rm d}r_\perp}\left(r_\perp^2\frac{\Omega_{\rm F}}{\Omega_0}\right)$$

$$F_{\rm d} = \frac{4}{3}\,\frac{\sigma_{\rm T}U_{\rm iso}}{m_{\rm e}c^2}$$

with N.Zakamska



Photon drag

MHD flow + isotropic radiation field

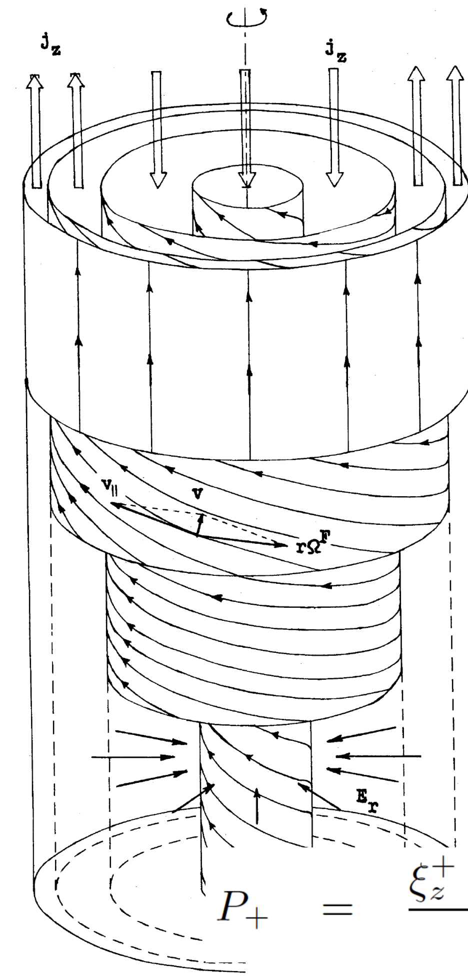
Step II: drift approximation

$$\mathbf{V}_{\mathrm{dr}} = c\frac{(e\mathbf{E} + \mathbf{F}_{\mathrm{drag}}) \times \mathbf{B}}{eB^2}$$

$$\frac{\mathrm{d}\mathcal{E}}{\mathrm{d}t} = (\mathbf{F}_{\mathrm{drag}} + e\mathbf{E})\mathbf{v}$$

$$\frac{\mathrm{d}\mathcal{E}}{\mathrm{d}t} = (F_{\|} + eE_{\|})v_{\|}$$

$$\boxed{\begin{aligned}\frac{\partial\gamma^{\pm}}{\partial z} &= -\frac{(1 - x^2 P_+)^2}{(1 + x^2)} F_{\mathrm{d}}(\gamma^{\pm})^2 \\ \mp\frac{4\lambda\sigma_{\mathrm{M}}}{r_{\mathrm{jet}}^2}\frac{(1 - x^2 P_+)}{(1 + x^2)} &\left(-r_{\perp}^2\frac{\partial\delta}{\partial z} + r_{\perp}^2\frac{\Omega_{\mathrm{F}}}{\Omega_0}\frac{\varepsilon}{2}\frac{\partial f}{\partial z}\right).\end{aligned}}$$



$$P_+ = \frac{\xi_z^+ + \xi_z^-}{2}$$

$$F_{\mathrm{d}} = \frac{4}{3}\frac{\sigma_{\mathrm{T}}U_{\mathrm{iso}}}{m_{\mathrm{e}}c^2}$$

Photon drag

MHD flow + radiation field

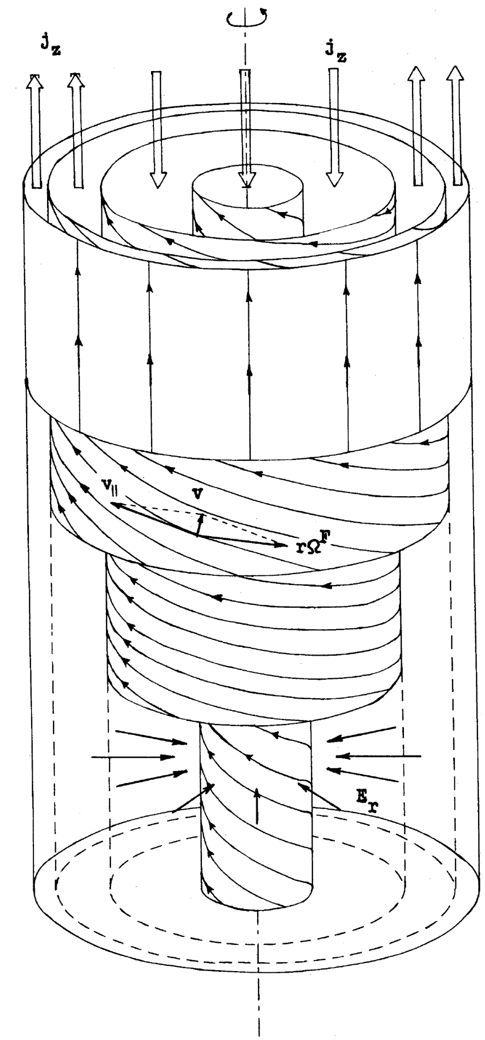
Decollimation

$$\boxed{\frac{\mathrm{d}^2}{\mathrm{d}x_0^2}\left(D - \frac{F}{2}\right) - \frac{16\lambda^2\sigma_{\mathrm{M}}}{\Gamma^3 x_{\mathrm{jet}}^2}\left(D - \frac{F}{2}\right) + \ldots = 0}$$

$$D = x_0^2\delta \text{ and } F = xx_0 f,$$

$$\delta = \frac{\varepsilon}{2}\frac{\Omega_{\mathrm{F}}}{\Omega_0} f$$

$$\Psi(r_\perp, z) \quad = \quad \pi B_0 r_\perp^2 \left[1 + \varepsilon f(r_\perp, z)\right]$$



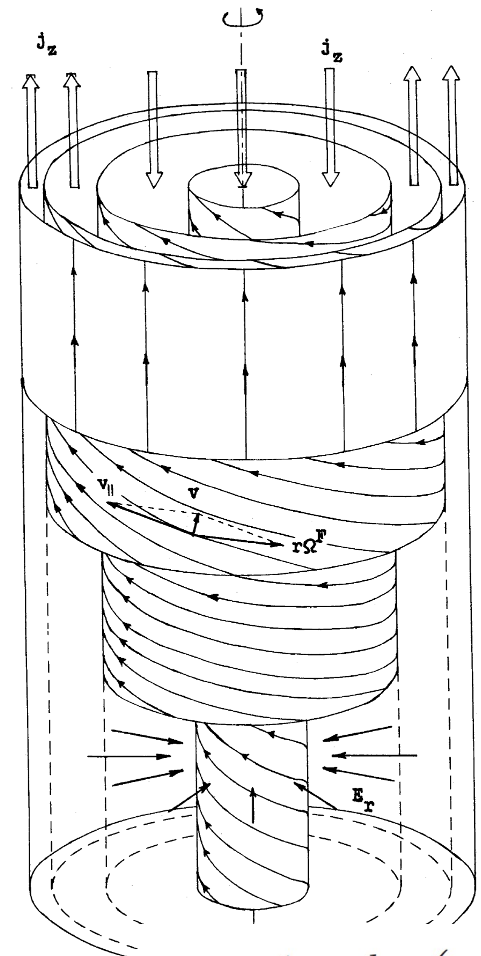
Photon drag

MHD flow + isotropic radiation field

Step III: Disturbances of electric field and magnetic surfaces (MHD approximation)

$$\delta = \frac{\varepsilon}{2}\frac{\Omega_{\rm F}}{\Omega_0} f$$

$$\boxed{\begin{aligned} & 2x\frac{\mathrm{d}}{\mathrm{d}x_0}\left[x_0\frac{\mathrm{d}}{\mathrm{d}x_0}D\right] - 2x_0\frac{\mathrm{d}}{\mathrm{d}x_0}\left[\frac{1}{x_0}\frac{\mathrm{d}}{\mathrm{d}x_0}\left(\frac{\Omega_0}{\Omega_{\rm F}}D\right)\right] + \\ & -8x\frac{\mathrm{d}}{\mathrm{d}x_0}\left[K\frac{(x_0 x + \Omega_0/\Omega_{\rm F} - x^2 P_+ \Omega_0/\Omega_{\rm F})}{(1+x^2)}D\right] + \\ & 8Kx_0\frac{\mathrm{d}}{\mathrm{d}x_0}D - \frac{32K^2 x_0(x^2+1-x^2P_+)}{x(1+x^2)}D \\ & = -2x\frac{\mathrm{d}}{\mathrm{d}x_0}\left[x_0^2\mathcal{G}\right] - 8Kx_0^2\mathcal{G}, \end{aligned}}$$



$$K = \frac{1}{4r_\perp}\frac{\mathrm{d}}{\mathrm{d}r_\perp}\left(r_\perp^2\frac{\Omega_{\rm F}}{\Omega_0}\right)$$

$$D = x_0^2\delta$$

$$\mathcal{G} = A\Gamma^2(F_{\rm d}z)/\sigma_{\rm M}$$

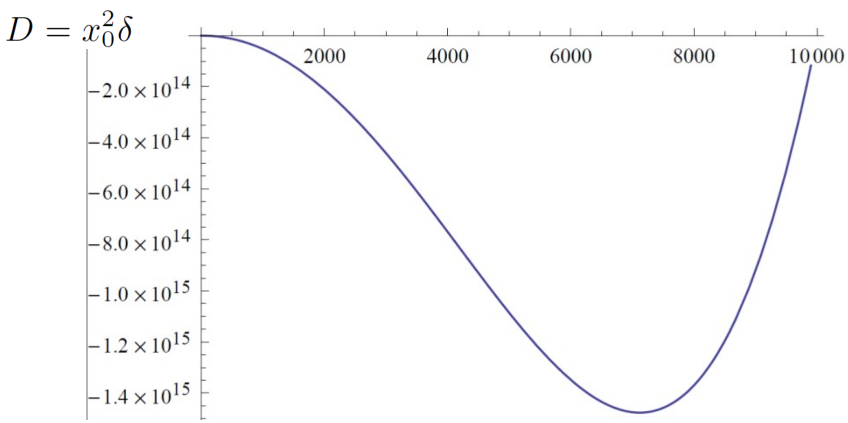
$$x_0 = \Omega_0 r_\perp/c$$

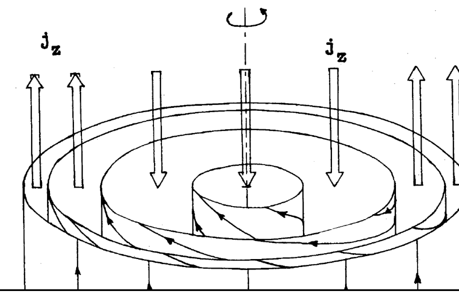
$$x = \Omega(r_\perp)r_\perp/c$$

Photon drag

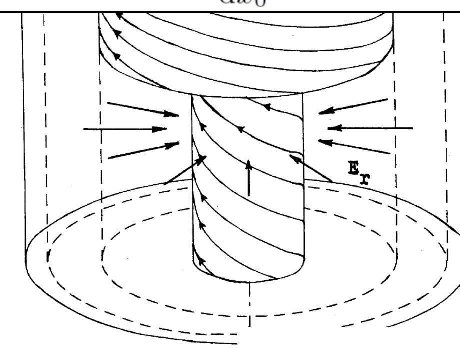
MHD flow + isotropic radiation field

Step III: Disturbances of electric field and magnetic surfaces (MHD approximation)





$$
\begin{aligned}
&2x\frac{\mathrm{d}}{\mathrm{d}x_0}\left[x_0\frac{\mathrm{d}}{\mathrm{d}x_0}D\right]-2x_0\frac{\mathrm{d}}{\mathrm{d}x_0}\left[\frac{1}{x_0}\frac{\mathrm{d}}{\mathrm{d}x_0}\left(\frac{\Omega_0}{\Omega_\mathrm{F}}D\right)\right]+\\
&-8x\frac{\mathrm{d}}{\mathrm{d}x_0}\left[K\frac{(x_0x+\Omega_0/\Omega_\mathrm{F}-x^2P_+\Omega_0/\Omega_\mathrm{F})}{(1+x^2)}D\right]+\\
&8Kx_0\frac{\mathrm{d}}{\mathrm{d}x_0}D-\frac{32K^2x_0(x^2+1-x^2P_+)}{x(1+x^2)}D\\
&=-2x\frac{\mathrm{d}}{\mathrm{d}x_0}\left[x_0^2\mathcal{G}\right]-8Kx_0^2\mathcal{G},
\end{aligned}
$$



$$\delta \sim \frac{A}{\sigma_\mathrm{M}}\Gamma^2(F_\mathrm{d}z) \qquad L_\mathrm{dr} \sim \frac{\sigma_\mathrm{M}}{\Gamma^2 F_\mathrm{d}}$$

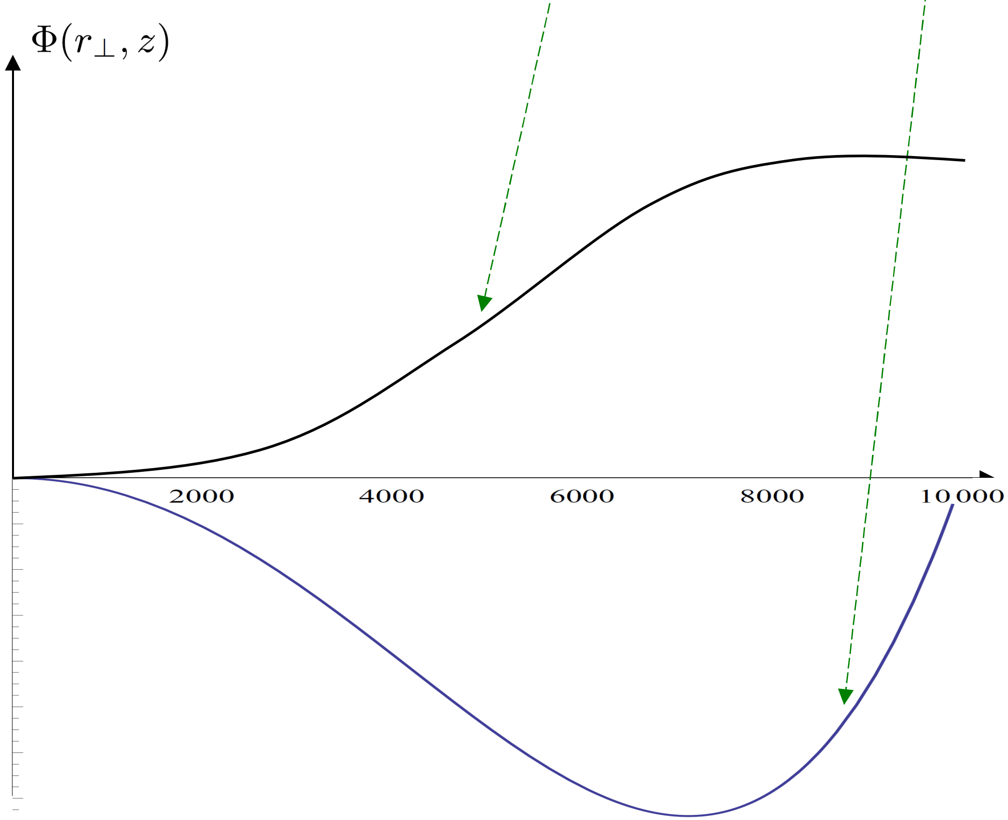
$$L_\mathrm{dr} \sim 300\left(\frac{\sigma_\mathrm{M}}{10}\right)\left(\frac{\Gamma}{10}\right)^{-2}\left(\frac{U_\mathrm{iso}}{10^{-4}\,\mathrm{erg/cm^3}}\right)^{-1}\mathrm{pc}$$

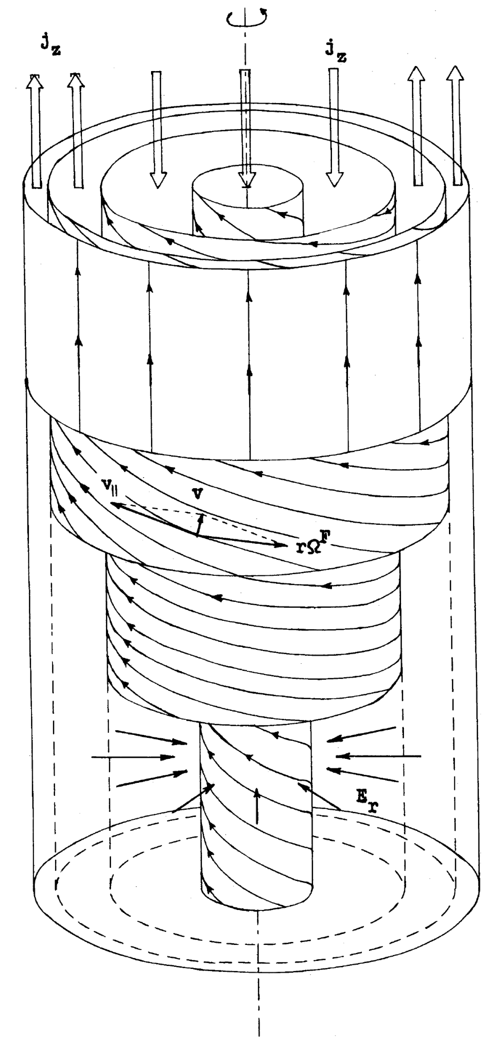
$$F_\mathrm{d} = \frac{4}{3}\frac{\sigma_\mathrm{T}U_\mathrm{iso}}{m_\mathrm{e}c^2}$$

Photon drag

MHD flow + radiation field

$$\Phi(r_\perp, z) \;=\; \frac{B_0}{c}\left[\int_0^{r_\perp} \Omega_{\rm F}(r')r'\,{\rm d}r' + \Omega_0 r_\perp^2\,\delta(r_\perp, z)\right]$$





Photon drag

MHD flow + radiation field

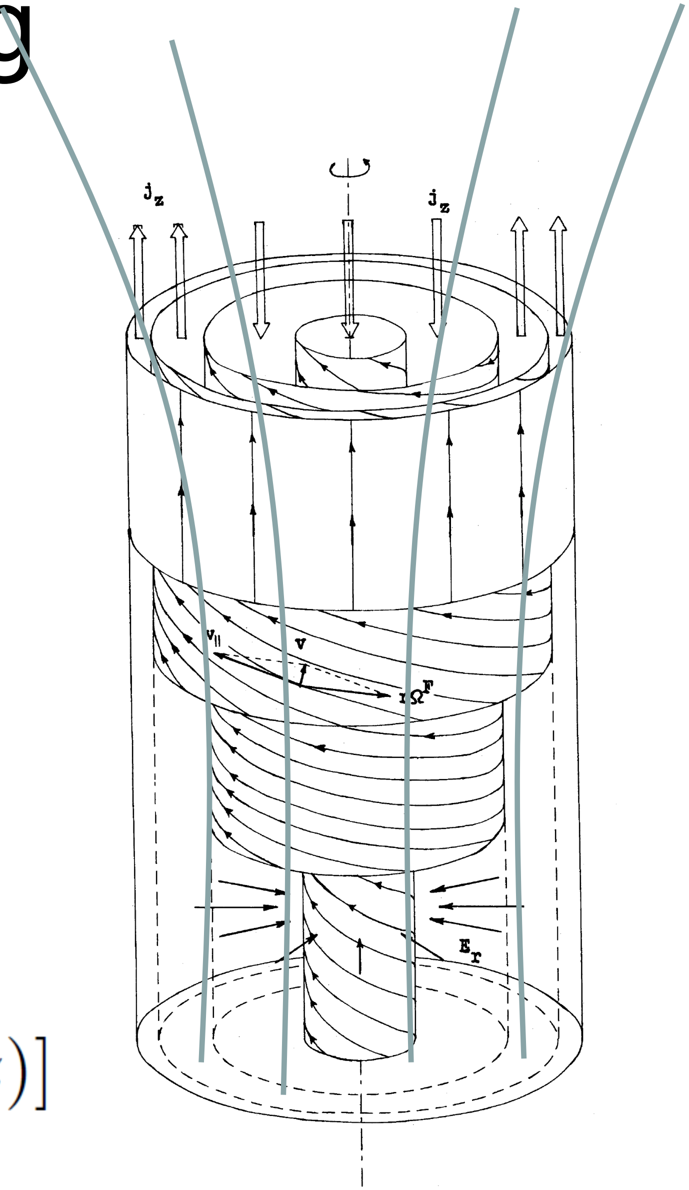
Decollimation

$$\boxed{\frac{\mathrm{d}^2}{\mathrm{d}x_0^2}\left(D-\frac{F}{2}\right)-\frac{16\lambda^2\sigma_{\mathrm{M}}}{\Gamma^3 x_{\mathrm{jet}}^2}\left(D-\frac{F}{2}\right)+\ldots=0}$$

$$D = x_0^2\delta \text{ and } F = xx_0 f,$$

$$\delta = \frac{\varepsilon}{2}\frac{\Omega_{\mathrm{F}}}{\Omega_0} f$$

$$\Psi(r_\perp, z) \quad = \quad \pi B_0 r_\perp^2 \left[1+\varepsilon f(r_\perp, z)\right]$$



Photon drag

MHD flow + radiation field

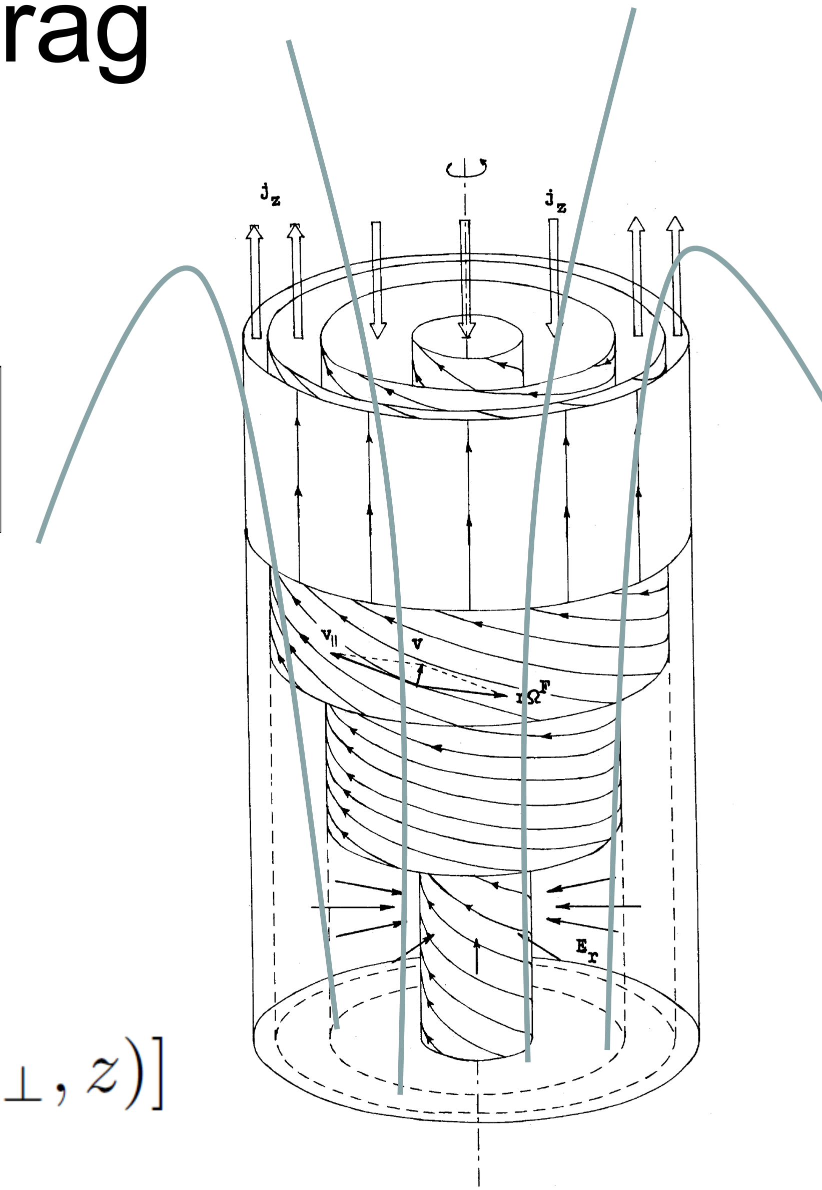
Decollimation –> reconnection

$$\boxed{\frac{\mathrm{d}^2}{\mathrm{d}x_0^2}\left(D-\frac{F}{2}\right)-\frac{16\lambda^2\sigma_{\mathrm{M}}}{\Gamma^3 x_{\mathrm{jet}}^2}\left(D-\frac{F}{2}\right)+\ldots=0}$$

$$D = x_0^2\delta \text{ and } F = xx_0 f,$$

$$\delta = \frac{\varepsilon}{2}\frac{\Omega_{\mathrm{F}}}{\Omega_0} f$$

$$\Psi(r_\perp, z) \quad = \quad \pi B_0 r_\perp^2\left[1+\varepsilon f(r_\perp, z)\right]$$



Photon drag

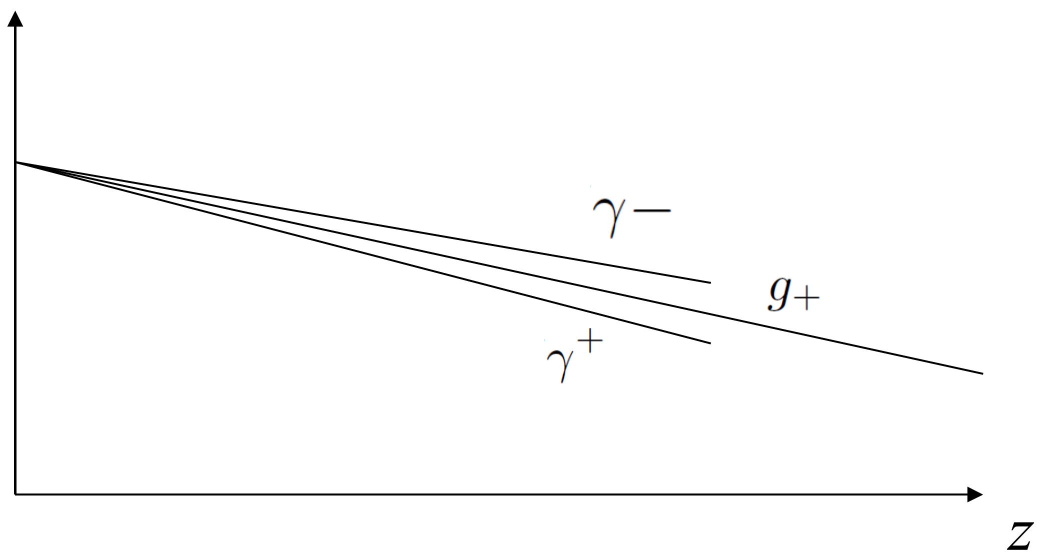
MHD flow + radiation field

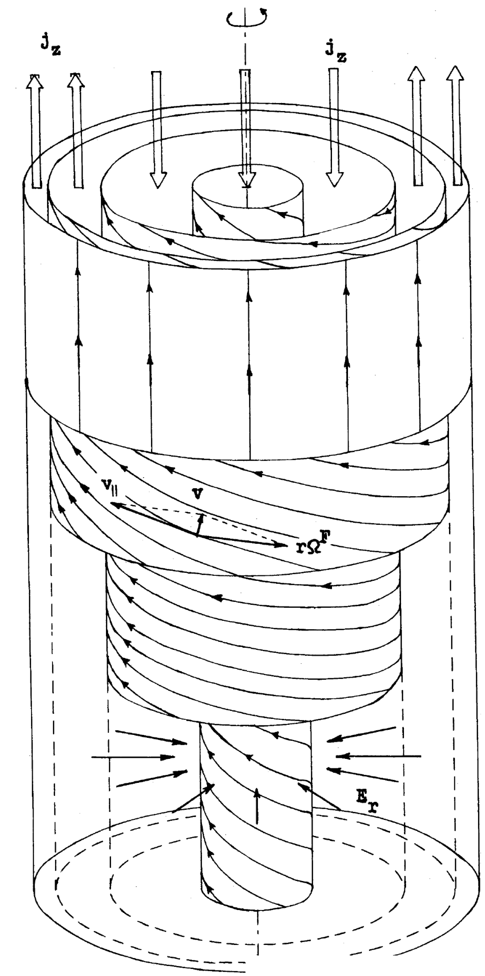
Step IV: Kinetic effects

$$g_+ = \frac{\delta\gamma^+ + \delta\gamma-}{2},$$

$$g_- = \delta\gamma^+ - \delta\gamma-$$

$$\frac{g_-}{g_+} \sim \frac{1}{\lambda\sigma_{\mathrm{M}}} \frac{(1+x^2)A}{(1-x^2P_+)^2}\Gamma^3 < 1$$





$$F_{\mathrm{d}} = \frac{4}{3}\frac{\sigma_{\mathrm{T}}U_{\mathrm{iso}}}{m_{\mathrm{e}}c^2}$$

Photon drag

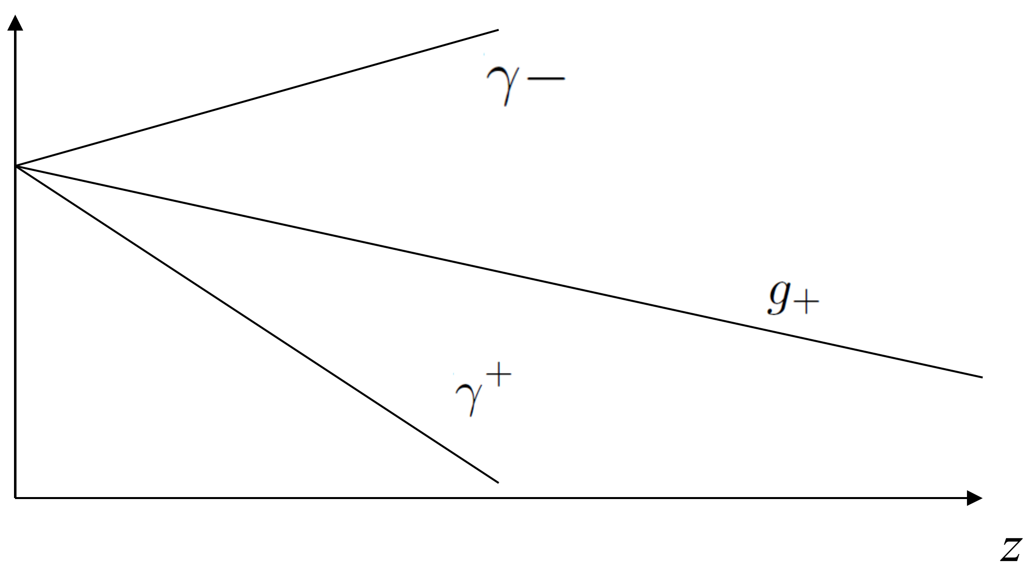
MHD flow + radiation field

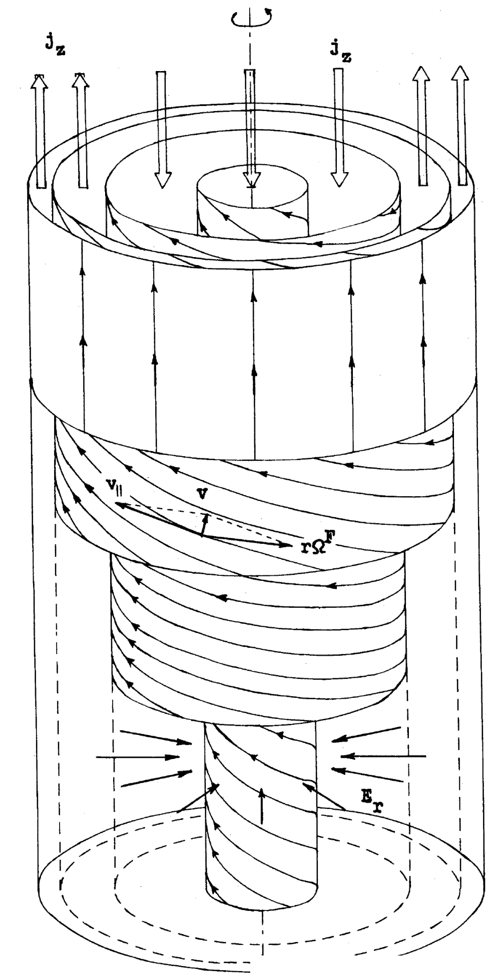
Step IV: Kinetic effects

$$g_+ = \frac{\delta\gamma^+ + \delta\gamma-}{2},$$

$$g_- = \delta\gamma^+ - \delta\gamma-$$

$$\frac{g_-}{g_+} \sim \frac{1}{\lambda\sigma_{\mathrm{M}}} \frac{(1+x^2)A}{(1-x^2P_+)^2}\Gamma^3 > 1$$





$$F_{\mathrm{d}} = \frac{4}{3}\frac{\sigma_{\mathrm{T}} U_{\mathrm{iso}}}{m_{\mathrm{e}} c^2}$$

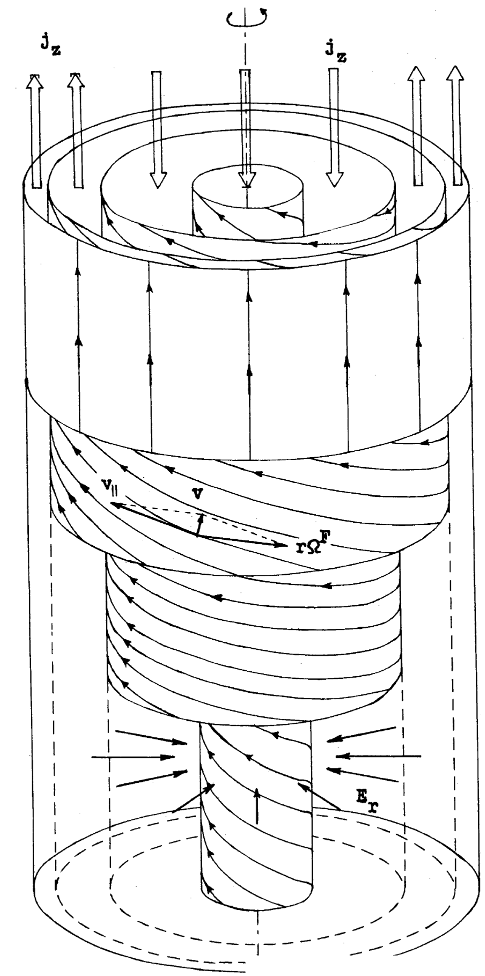
Photon drag

MHD flow + radiation field

Step IV: Kinetic effects

$$g_+ = \frac{\delta\gamma^+ + \delta\gamma-}{2},$$

$$g_- = \delta\gamma^+ - \delta\gamma-$$



$$\frac{g_-}{g_+} \sim \frac{1}{\lambda\sigma_{\rm M}} \frac{(1+x^2)A}{(1-x^2P_+)^2}\Gamma^3$$

Critical values

$$\lambda\sigma_{\rm M} = \frac{x_{\rm jet}^2\Gamma^5}{\Gamma_0^2}$$

$$W_\star = \frac{x_{\rm jet}^4\Gamma^{10}}{\Gamma_0^4}W_{\rm A}$$

$$W_{\rm A} = m_{\rm e}^2c^5/e^2 \approx 10^{17}\,{\rm erg\,s^{-1}}$$

$$F_{\rm d} = \frac{4}{3}\frac{\sigma_{\rm T}U_{\rm iso}}{m_{\rm e}c^2}$$

I: Photon drag – conclusion

V.S.Beskin, A.V.Chernoglazov, MNRAS, **463**, 3398 (2016)

Photon drag results in

- Decollimation
- Recconection

II: Particle Loading

E.E.Nokhrina, VB, MNRAS, **469**, 3850 (2017)

Loading

MHD flow + $e^{-}e^{+}$ pair creation

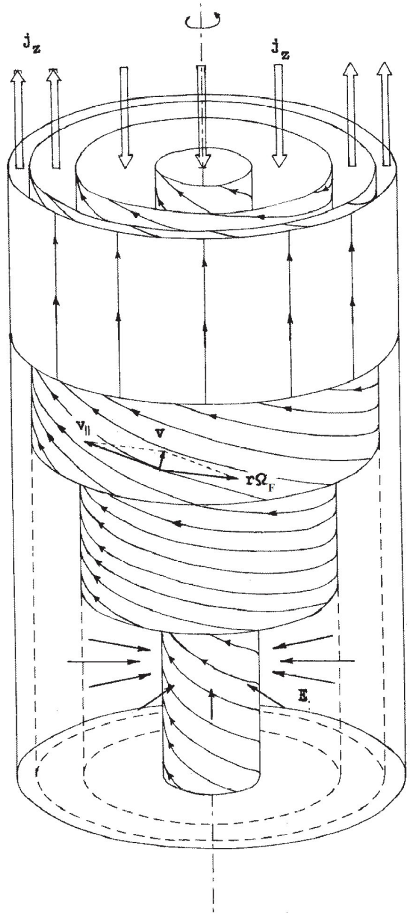
How the particle loading affects the MHD flow

- MHD cylindrical jet
- electron-positron plasma
- creation at rest

    no extra energy flux

    no extra angular momentum flux

$$\frac{1}{\Gamma^2} = \frac{1}{x_r^2} + \frac{B_\varphi^2 - \mathbf{E}^2}{B_\varphi^2}$$



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Anisotropic pressure

z – moving reference frame

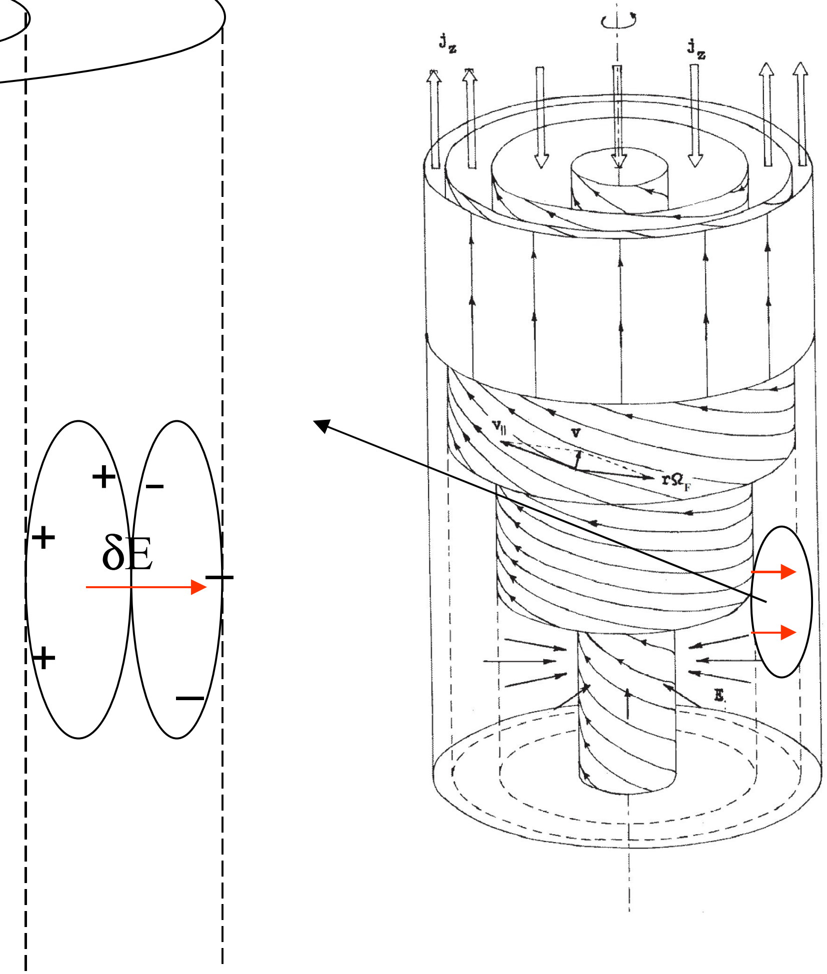
$$V = E_\theta / B_\varphi$$

$$\Gamma = 1/(1 - V^2/c^2)^{1/2}$$

In this frame

$$\begin{aligned} B_\phi &= B_\varphi/\Gamma, \\ B_z &= B_\mathrm{p}. \end{aligned}$$

$$\tan\alpha = B_z/B_\phi$$



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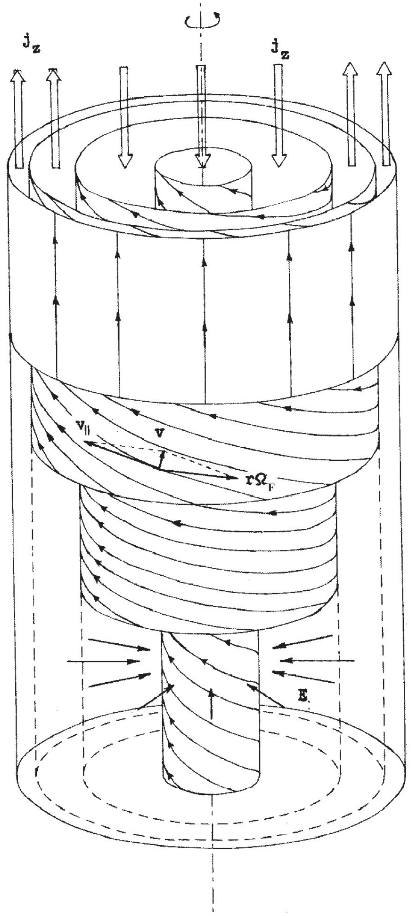
MHD flow + e^-e^+ pair creation (at rest)

M.Lyutikov (2005) – quasi-spherical

$$T^{ij} = (w + b^2)u^i u^j + \left(p + \frac{1}{2}b^2\right) g^{ij} - b^i b^j$$

$$\begin{cases} \frac{1}{r^2}\partial_r[r^2(w+b^2)\beta\gamma^2] = R \\ \frac{1}{r^2}\partial_r\{r^2[(w+b^2)\beta^2\gamma^2 + (p+b^2/2)]\} - \frac{2p}{r} = 0 \\ \frac{1}{r}\partial_r[rb\beta\gamma] = 0 \\ \frac{1}{r^2}\partial_r[r^2\rho\beta\gamma] = R \end{cases}$$



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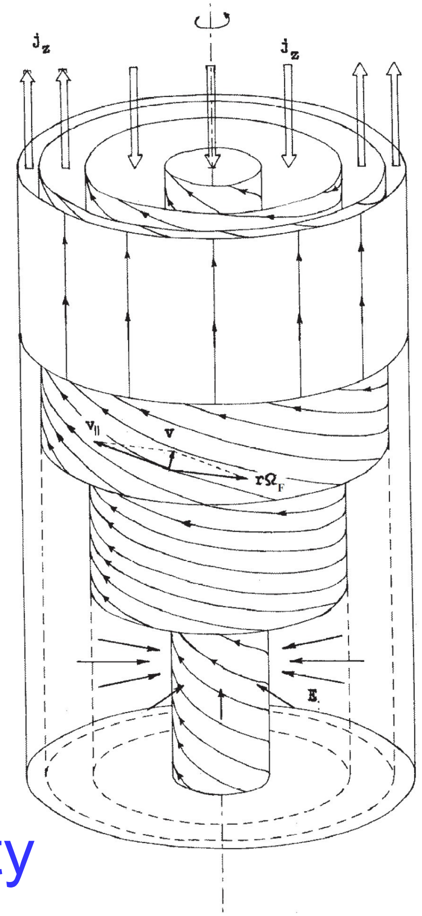
MHD flow + e⁻e⁺ pair creation (at rest)

$$T^{ik} = \left(e + P_s + \frac{\mathbf{b}^2}{4\pi}\right)u^i u^k + \left(P_s + \frac{\mathbf{b}^2}{8\pi}\right)g^{ik} + \left[\frac{(\cancel{P_n} - P_s)}{\mathbf{b}^2} - \frac{1}{4\pi}\right]b^i b^k$$

$$\begin{cases} \frac{1}{r^2}\partial_r[r^2(w+b^2)\beta\gamma^2] = R \\ \frac{1}{r^2}\partial_r\{r^2[(w+b^2)\beta^2\gamma^2 + (p + b^2/2)]\} - \frac{2p}{r} = 0 \\ \cancel{\frac{1}{r}\partial_r[rb\beta\gamma] = 0} \\ \frac{1}{r^2}\partial_r[r^2\rho\beta\gamma] = R \end{cases}$$

- Anisotropic pressure
- 2D – no equi-potentiality



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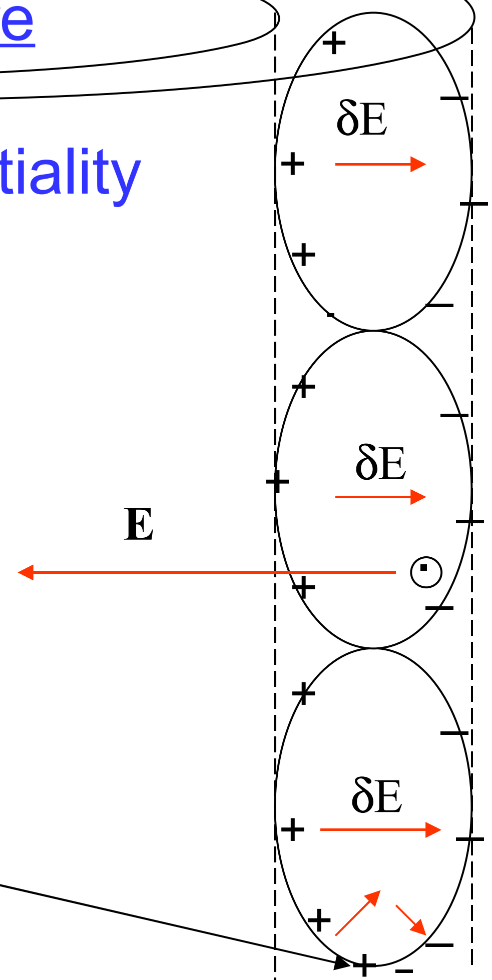
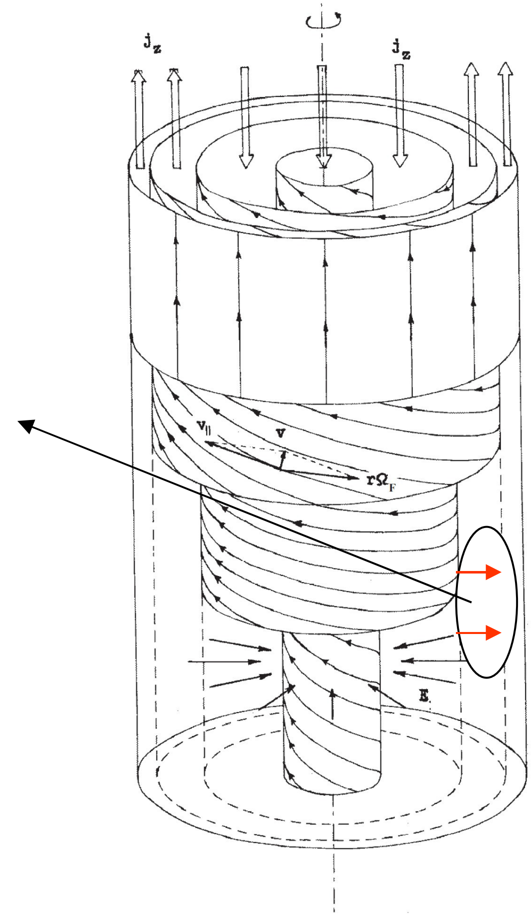
Anisotropic pressure

2D – no equi-potentiality



Pair creation at rest



Loading

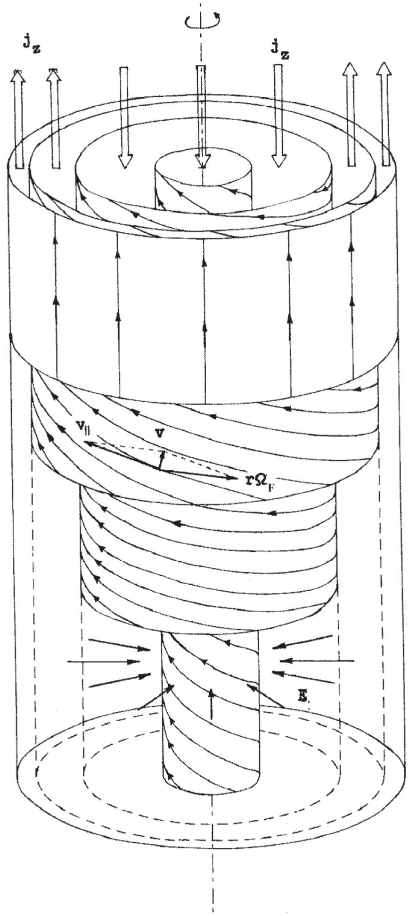
Anisotropic pressure

Rotation in the *rz*-plane

$$T^{ik} = \left(\varepsilon_{\rm ld} + P_s + \frac{\mathbf{b}^2}{4\pi}\right) U^i U^k + \left(P_s + \frac{\mathbf{b}^2}{8\pi}\right) g^{ik} - \left(\frac{P_s}{\mathbf{b}^2} + \frac{1}{4\pi}\right) b^i b^k.$$

$$\begin{aligned} \varepsilon_{\rm ld} &= n_{\rm ld}^{\rm com} m_{\rm e} c^2 \gamma_{\rm hd}, \\ P_s &= \frac{1}{2} n_{\rm ld}^{\rm com} m_{\rm e} c^2 \gamma_{\rm hd} \end{aligned}$$

$$P_n = 0$$



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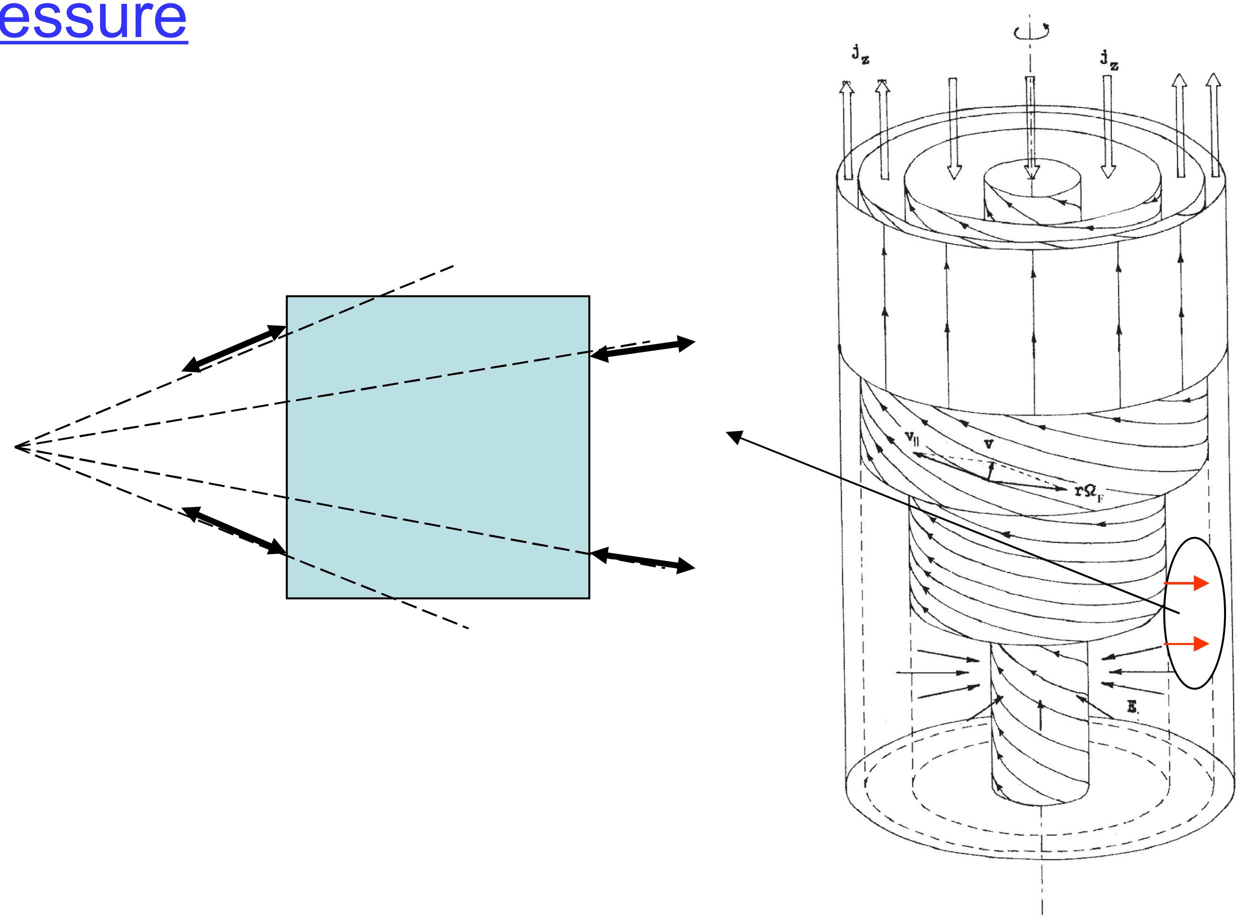
Anisotropic pressure

Radial force

$$\mathcal{F} = -\frac{P_s}{r}\mathbf{e}_r$$



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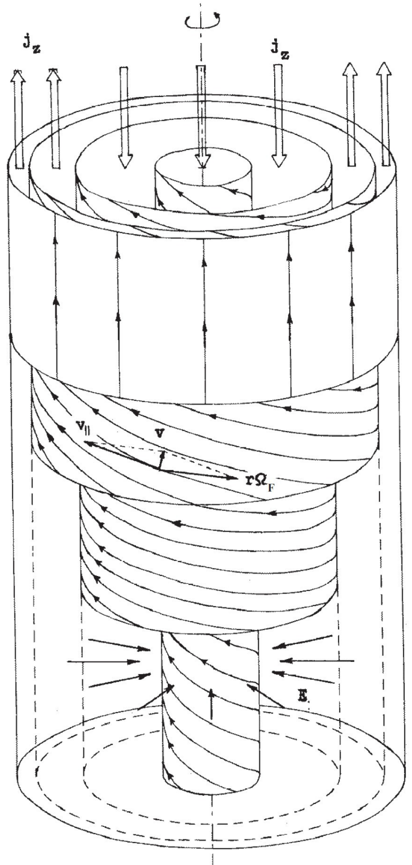
Anisotropic pressure

Full system of equation was known

E.Asseo & D.Beaufils. Ap&SS, **89**, 133 (1983)
R.Lovelace et al. ApJS, **62**, 1 (1986)
E.Tsikarisvili, A.Rogava, D.Tsikauri. ApJ, **439**, 822 (1992)
I.Kuznetsova, ApJ, **618**, 432 (2005)

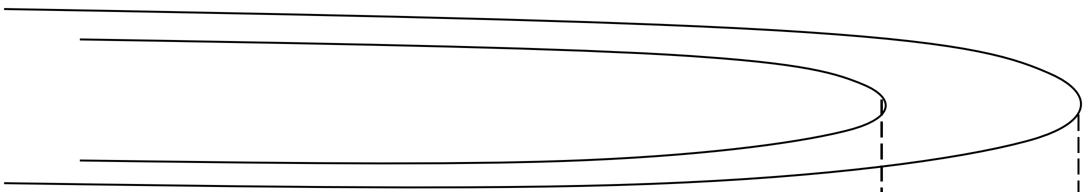


$$\begin{cases} E(\Psi) &= \dfrac{\Omega_{\rm F} I}{2\pi}(1+|\beta|) + \mu_{\rm ld}\eta_{\rm ld} < \gamma > + \mu\eta < \gamma >, \\ L(\Psi) &= \dfrac{I}{2\pi}(1+|\beta|) + \mu_{\rm ld}\eta_{\rm ld}\varpi u_\varphi + \mu\eta\varpi u_\varphi . \end{cases}$$

$$\begin{cases} \dfrac{I}{2\pi} = \dfrac{\alpha^2 L - (\Omega_F - \omega)\varpi^2 (E - \omega L)}{\left[\alpha^2 - (\Omega_F - \omega)^2\varpi^2\right](1-\beta) - M^2}, \\ \gamma = \dfrac{1}{\alpha\,\mu\eta}\,\dfrac{\alpha^2(E - \Omega_F L)(1-\beta) - M^2(E - \omega L)}{\alpha^2 - (\Omega_F - \omega)^2\varpi^2(1-\beta) - M^2}, \\ u_{\hat\varphi} = \dfrac{1}{\varpi\mu\eta}\,\dfrac{(E - \Omega_F L)(\Omega_F - \omega)\varpi^2(1-\beta) - LM^2}{\left[\alpha^2 - (\Omega_F - \omega)^2\varpi^2\right](1-\beta) - M^2} \end{cases}$$

$$\boxed{\beta = 4\pi\frac{P_n - P_s}{h^2}}$$

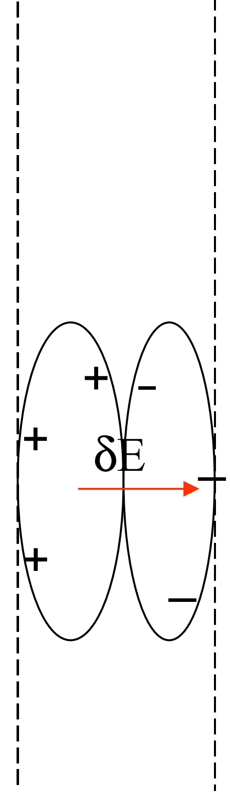
Loading

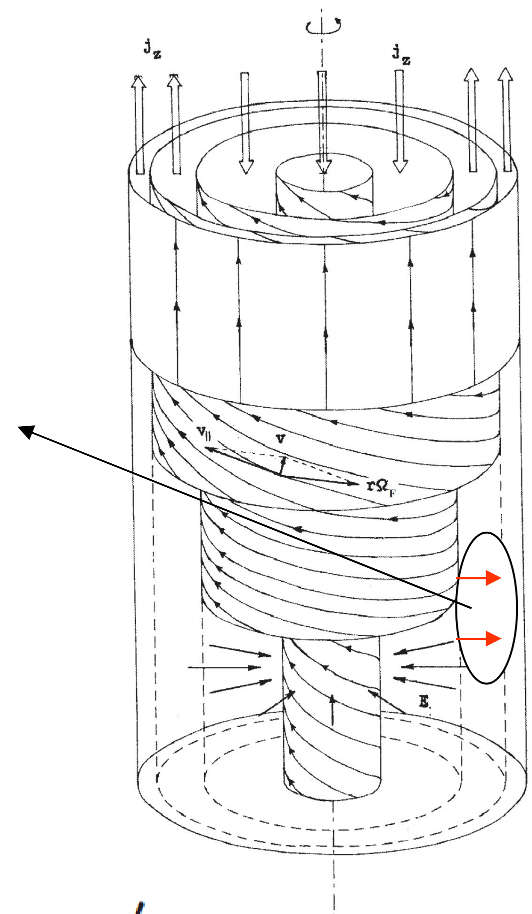


z –moving reference frame

$$V = E_\theta / B_\varphi$$

$$\Gamma = 1/(1 - V^2/c^2)^{1/2}$$





In this frame

$$\mathbf{p}' = p'_{||}\mathbf{b} + p'_{\perp}\cos\omega t'\,\mathbf{n}_1 + p'_{\perp}\sin\omega t'\,\mathbf{n}_2$$

Loading

Particle motion (laboratory frame)

$$\begin{aligned} p_r &= mcu_r = mV\Gamma \sin\alpha\cos\alpha(1-\cos\omega t'), \\ p_\phi &= mcu_\phi = mV\Gamma\cos\alpha\sin\omega t', \\ p_z &= mcu_z = mV\Gamma^2\cos^2\alpha(1-\cos\omega t'). \end{aligned}$$

$$\mathcal{E} = mc^2\Gamma^2\left[1 - V^2/c^2(\sin^2\alpha + \cos^2\alpha\cos\omega t')\right]$$

Averaging procedure

$$< A >_t = \frac{1}{T}\int_0^{T'} A(t')\frac{\mathrm{d}t}{\mathrm{d}t'}\mathrm{d}t' =< A(t')\frac{T'}{T}\frac{\mathrm{d}t}{\mathrm{d}t'} >_{t'}$$

Loading

Hydrodynamical motion

$$
\begin{aligned}
< v_r >_t &= \frac{V\Gamma^{-1} \sin\alpha \cos\alpha}{1 - V^2/c^2 \, \sin^2\alpha}, \\
< v_\phi >_t &= 0, \\
< v_z >_t &= \frac{V \cos^2\alpha}{1 - V^2/c^2 \, \sin^2\alpha}
\end{aligned}
$$

$$
\boxed{\gamma_{\rm hd} = \Gamma \sqrt{1 - V^2/c^2 \, \sin^2\alpha}}
$$

Mean energy

$$
< \gamma >_t = \Gamma^2 \left(1 - \frac{V^2}{c^2} \sin^2\alpha\right) \left[1 + \frac{1}{2} \frac{\cos^4\alpha}{(1 - V^2/c^2 \, \sin^2\alpha)^2}\right]
$$

$$
\boxed{< \gamma >_t \approx \frac{3}{2} \gamma_{\rm hd}^2}
$$

Damping via Loading

Hydrodynamical motion

$$
\begin{aligned}
E(\Psi) &= \frac{\Omega_{\mathrm{F}} I}{2\pi}(1+|\beta|) + \mu_{\mathrm{ld}}\eta_{\mathrm{ld}} < \gamma > + \mu\eta < \gamma >, \\
L(\Psi) &= \frac{I}{2\pi}(1+|\beta|) + \mu_{\mathrm{ld}}\eta_{\mathrm{ld}}\varpi u_{\varphi} + \mu\eta\varpi u_{\varphi}.
\end{aligned}
$$

$$\mu = \varepsilon/n = mc^2$$

$$\mu_{\mathrm{ld}} = \varepsilon_{\mathrm{ld}}/n_{\mathrm{ld}} = mc^2 < \gamma >$$

$$\boxed{\beta = 4\pi \frac{\cancel{P_n} - P_s}{h^2}}$$

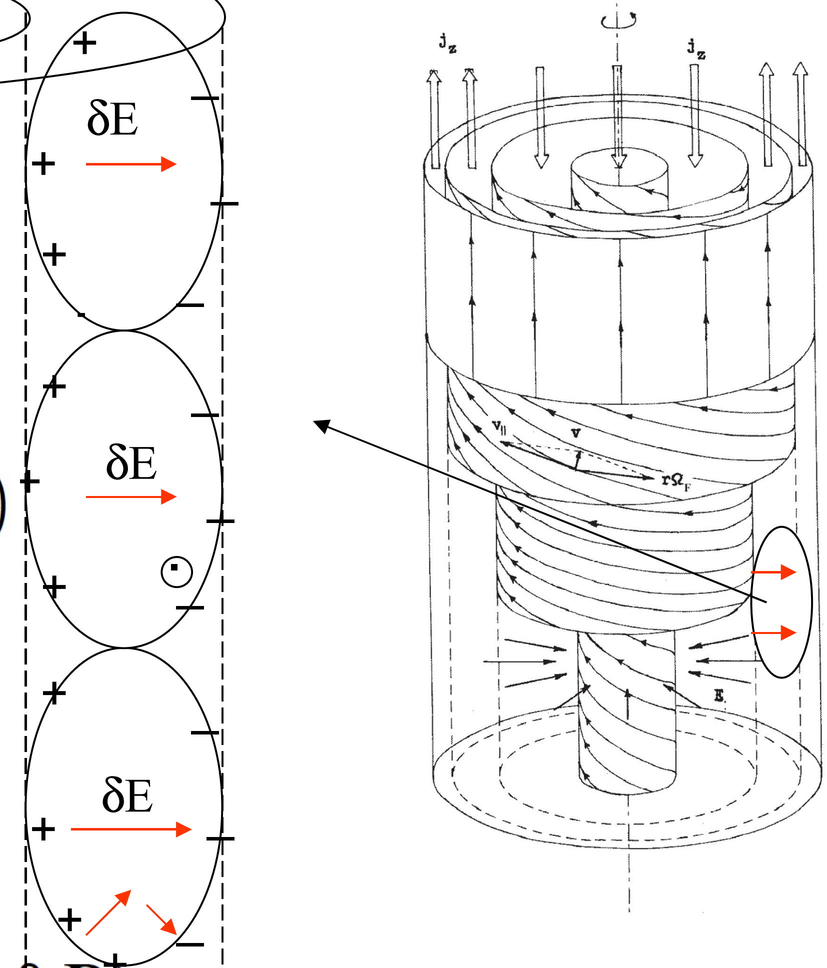
Damping via Poynting

Poynting damping

$$\sigma_{\rm e} = e n_{\rm ld}^{\rm lab} r_{\perp} = e\tilde{\Gamma} n_{\rm ld} r_{\perp}$$

$$\delta E = \frac{4\pi m_{\rm e} c^2 n_{\rm ld} \tilde{\beta}^2 \tilde{\Gamma}^3}{\tilde{E}} \left(2 + \tilde{\beta}^2\right)$$

$$\delta B = \tilde{\beta}\,\delta E$$

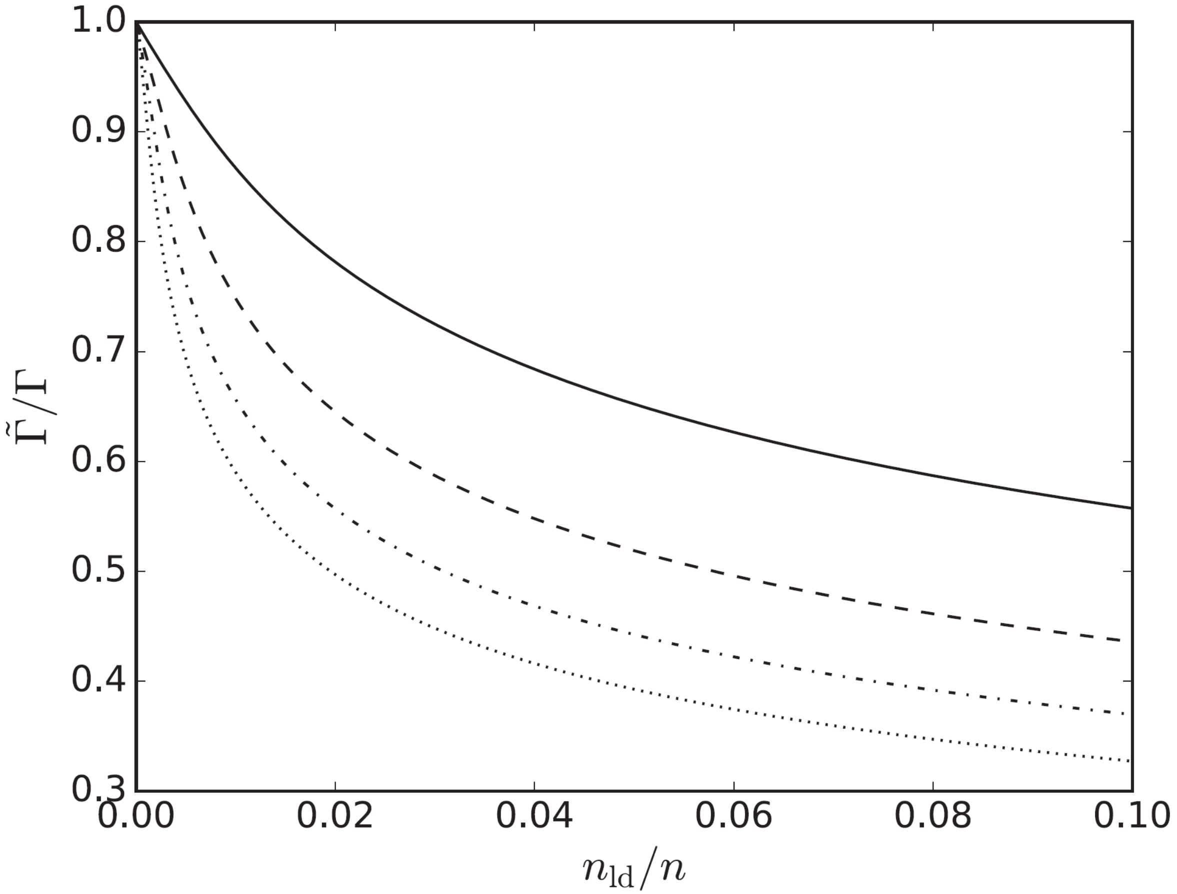


$$\mathrm{v/c} \;= (E - \delta E)/(B - \delta B)$$

Damping via Poynting

Poynting damping

$$\mathrm{v/c} = (E - \delta E)/(B - \delta B)$$



Loading

Critical number density

$$\frac{1}{\Gamma^2} = \frac{1}{x_r^2} + \frac{B_\varphi^2 - \mathbf{E}^2}{B_\varphi^2}$$

- Direct calculation of the field disturbances in
- Loading pressure $|\beta| \sim 1$
- Electric field disrurbance $\delta \boldsymbol{E} \sim \boldsymbol{E}$
- Anisotropic pressure force $\delta \boldsymbol{F} \sim \boldsymbol{F}$

$$\frac{1}{\alpha}\nabla_k\left[\frac{1}{\alpha\varpi^2}A\nabla^k\Psi\right] + \frac{(\Omega_F - \omega)}{\alpha^2}(1-\beta)\frac{d\Omega_F}{d\Psi}(\nabla\Psi)^2$$
$$+\frac{64\pi^4}{\alpha^2}\varpi^2\frac{1}{2M^2}\frac{\partial}{\partial\Psi}\left(\frac{G}{A}\right) - 8\pi^3\mu n\frac{1}{\eta}\frac{d\eta}{d\Psi}$$
$$\cancel{-8\pi^3 P_n\frac{1}{s_1}\frac{ds_1}{d\Psi}} - 16\pi^3 P_s\frac{1}{s_2}\frac{ds_2}{d\Psi} = 0.$$

I.Kuznetsova

Loading

Critical number density

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- Loading pressure $|\beta| \sim 1$
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- Anisotropic pressure force $\delta \boldsymbol{F} \sim \boldsymbol{F}$

$$\frac{1}{\Gamma^2} = \frac{1}{x_r^2} + \frac{B_\varphi^2 - \mathbf{E}^2}{B_\varphi^2}$$

$$\boxed{n_{\rm cr} = \frac{B_\varphi^2}{m_{\rm e} c^2 \Gamma^2}}$$

$$\frac{n_{\rm cr}}{\lambda n_{\rm GJ}} = \frac{\sigma_{\rm M}}{\Gamma^2} \sim \frac{1}{\Gamma}$$

Astrophysical applications?

Photon drag

$$R = 10 \text{ pc}$$

$$L_{\rm dr} \sim 300 \left(\frac{\sigma_{\rm M}}{10}\right) \left(\frac{\Gamma}{10}\right)^{-2} \left(\frac{U_{\rm iso}}{10^{-4}\,{\rm erg\,cm^{-3}}}\right)^{-1} {\rm pc}.$$

Loading

$$L_{\rm load} = \frac{c n_{\rm cr}}{\dot{n}} \sim 100 \left(\frac{\dot{n}}{10^{-4}\ {\rm cm}^{-3}}\right)^{-1} \ {\rm pc}$$

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Let us discuss…

Loading

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Conclusion

1. Radiation drag and particle loading might be a reason for deceleration for small enough magnetization.
2. Disturbances of electric potential AND magnetic surfaces are to be included into consideration.
3. Drag force acting on particles in highly magnetized flow diminishes mainly Poynting flux, not the particle energy.
4. Drag force results in decolimation (= reconnection?) of magnetic surfaces.

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THANKS AGAIN!