A Few Words About AGNs and PSRs

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Astrophysical jets: from observations to theory and laboratory experiment

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Laboratory of Fundamental and Applied Research of Relativistic Objects of the Universe in MIPT

Plan

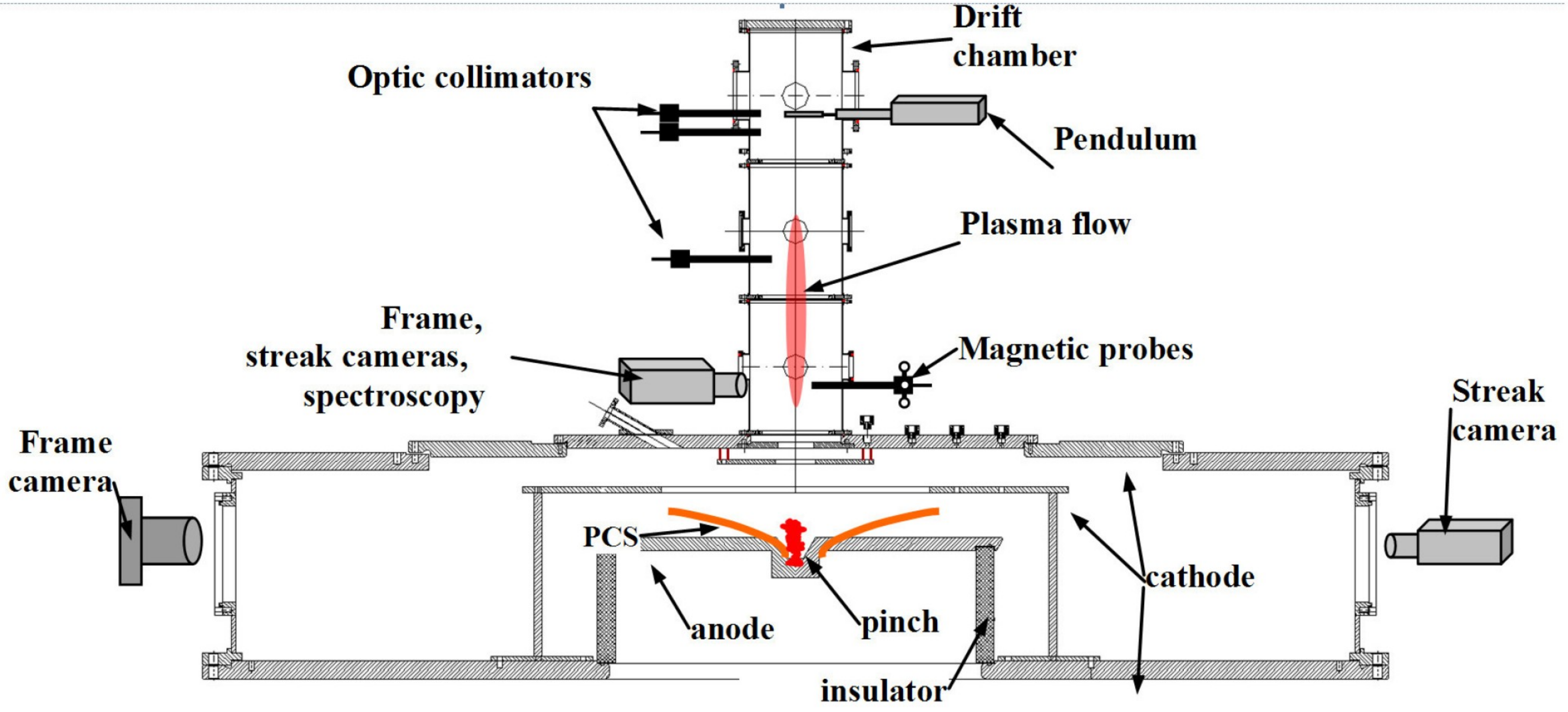
- AGNs (information only)
- PSRs (in more detail)

AGNs

- Laboratory experiment on plasma focus facility
- One (rather technical) theoretical result +

Laboratory experiment

Plasma focus facility in KI



Laboratory experiment

Plasma focus facility in KI

Laboratory experiment

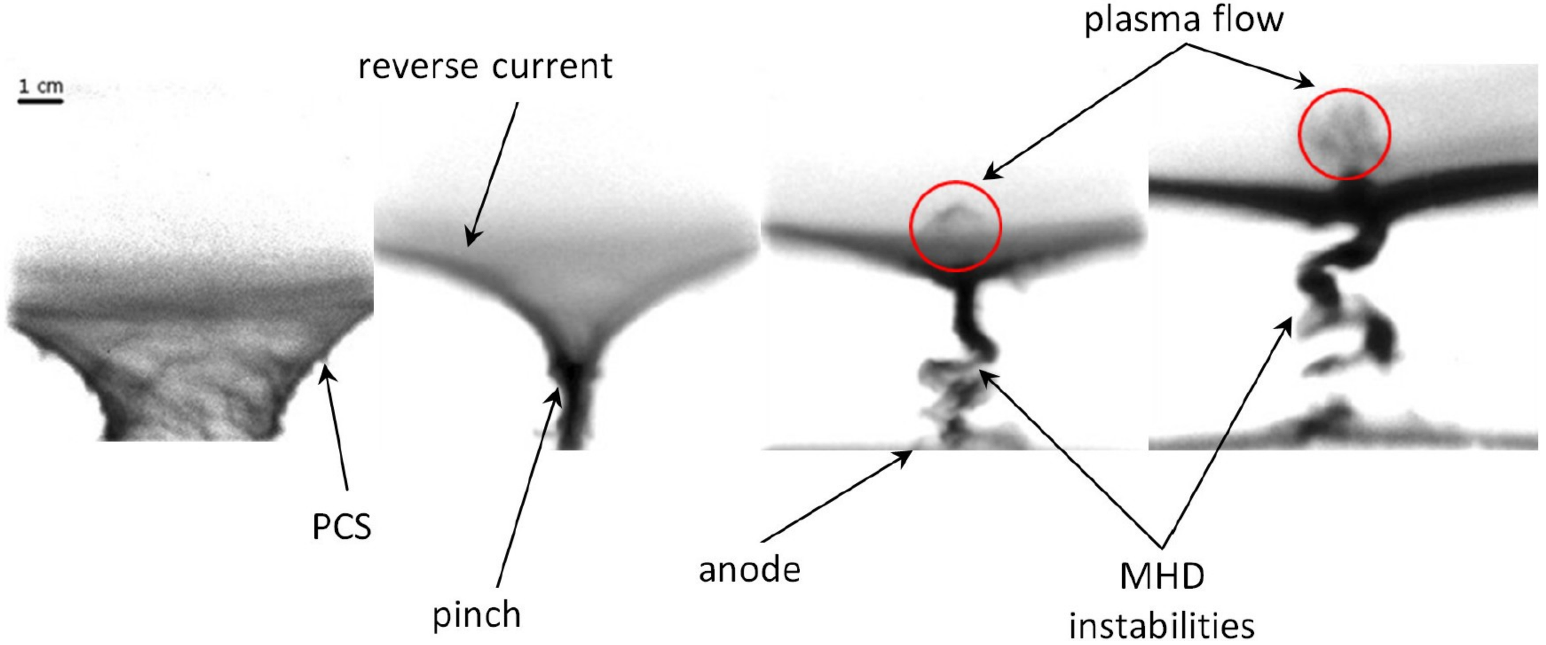
Plasma focus facility in KI

Table 1. Key dimensionless parameters

	YSO		PF-3 (35 cm above the anode)
Peclet	10^{11}	> 1, convective heat transfer	$> 10^7$
Reynolds	10^{13}	$\gg 1$, the viscosity is important	$10^4 - 10^5$
Magnetic Reynolds	10^{15}	> 1, magnetic field is frozen	~ 100
Mach ($V_{\rm jet}/V_{\rm cs}$)	$10 - 50$	> 1, the jet is supersonic	> 10 (for Ne and Ar)
β ($P_{\rm pl}/P_{\rm magn}$)	$\gg 1$ near source $\ll 1$ at 10 AU		~ 0.35 (for Ne and Ar)
density contrast ($n_{\rm jet}/n_{\rm amb}$)	> 1		$1 - 10$

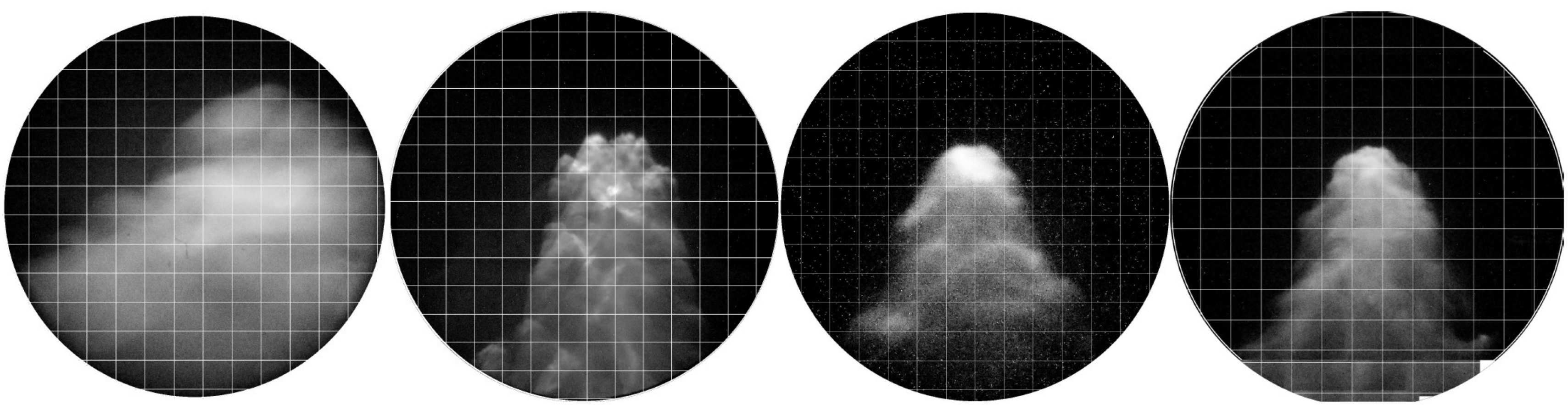
Laboratory experiment

Plasma focus facility – the very beginning



Laboratory experiment

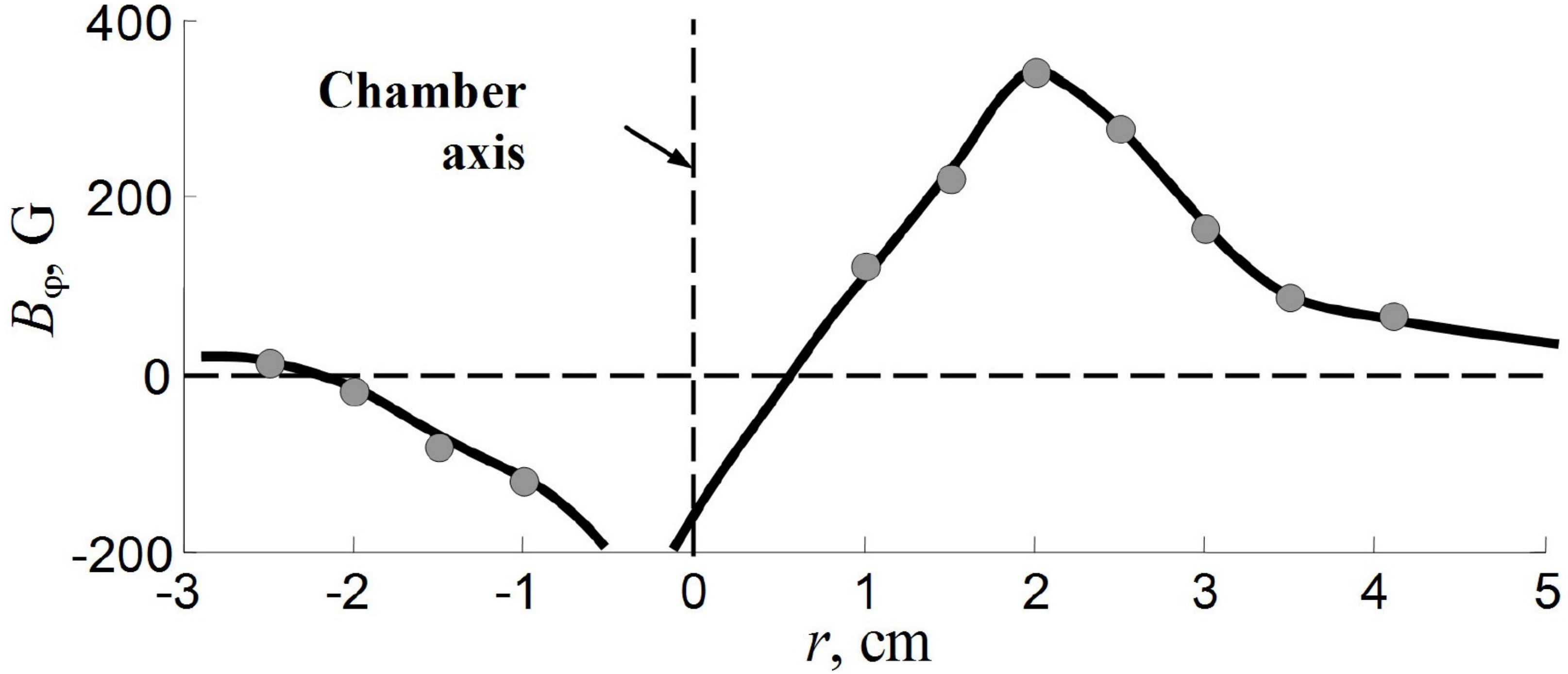
Plasma focus facility – rather stable shape, interaction with ambient gas



H_2 35 cm Ne 35 cm Ne 65 cm Ar 95 cm

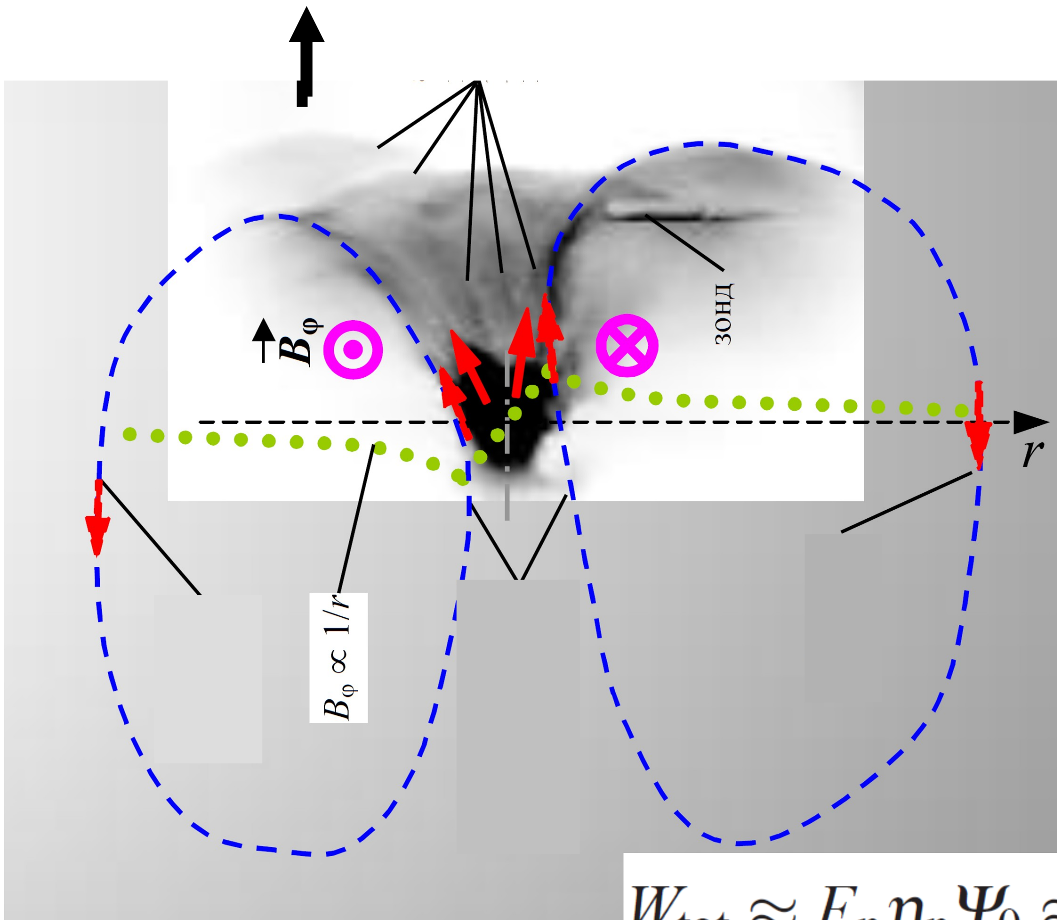
Laboratory experiment

Plasma focus facility – toroidal magnetic field



Laboratory experiment

Plasma focus facility – spheromak?



$$W_{\rm tot} \approx E_{\rm n}\eta_{\rm n}\Psi_0 \approx \Omega^{4/3}\dot{M}^{1/3}\Psi_0^{4/3}$$

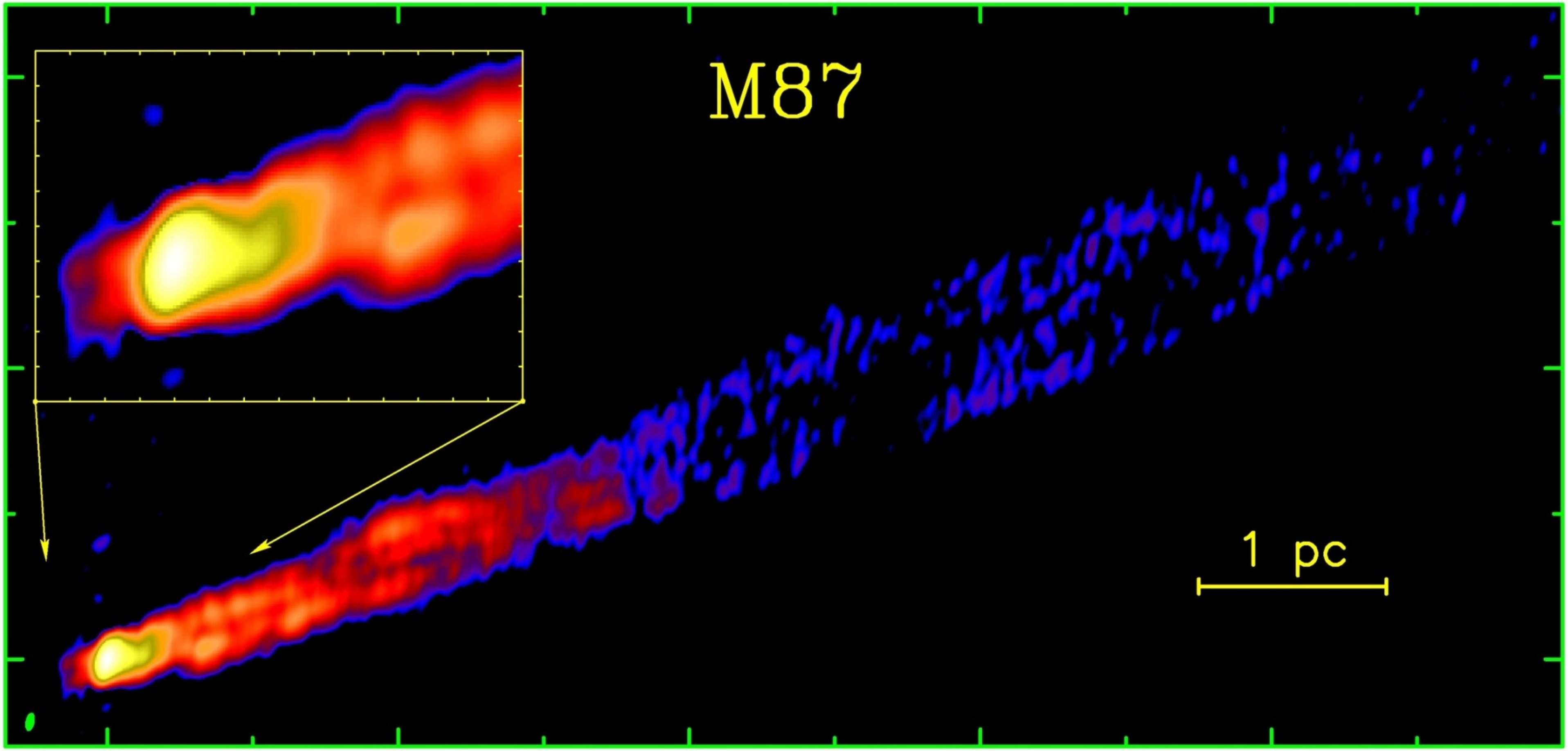
Theoretical result

Matching to ambient gas pressure –

how it affects the jet thickness?

VLBA+VLA1, 15 GHz

The inner jet structure is clearly resolved, a short counter jet is detected



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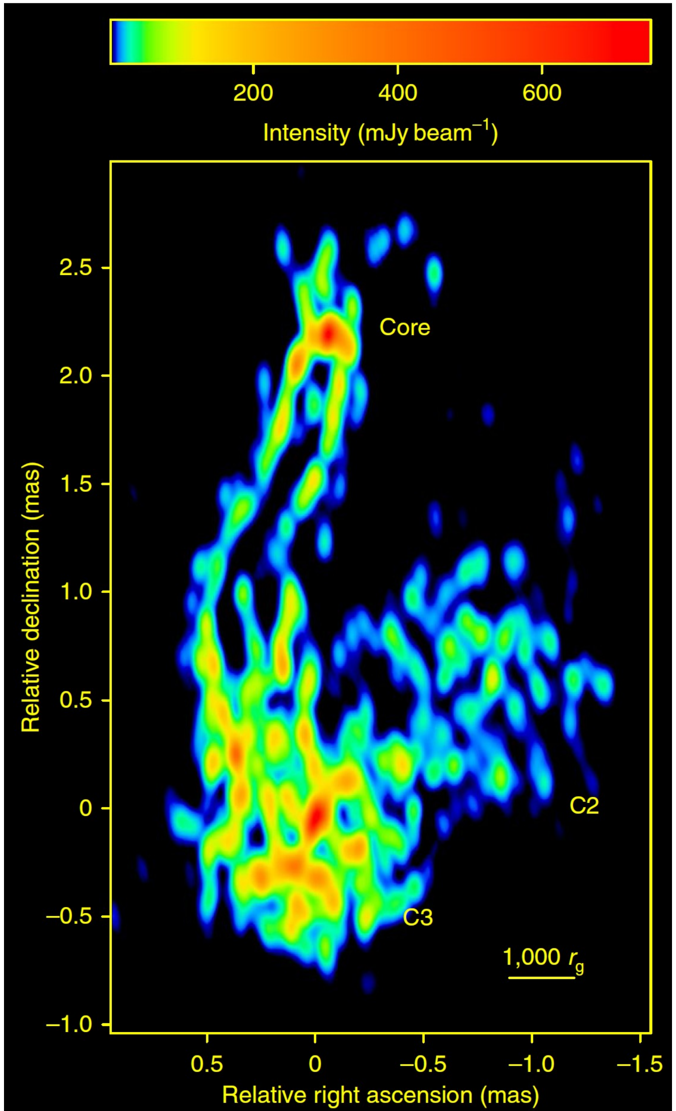
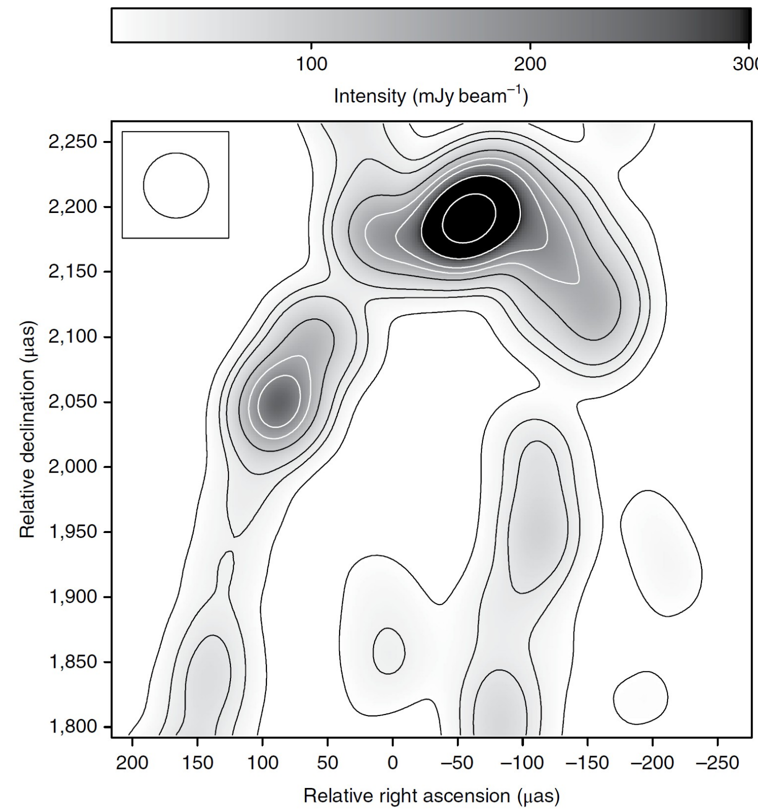


nature astronomy
LETTERS
https://doi.org/10.1038/s41550-018-0431-2

A wide and collimated radio jet in 3C84 on the scale of a few hundred gravitational radii

G. Giovannini[1,2*], T. Savolainen[3,4,5*], M. Orienti[2], M. Nakamura[6], H. Nagai[7], M. Kino[8,9], M. Giroletti[2], K. Hada[9], G. Bruni[2,5,10], Y. Y. Kovalev[5,11,12], J. M. Anderson[13], F. D'Ammando[1,2], J. Hodgson[14], M. Honma[9], T. P. Krichbaum[5], S.-S. Lee[14,15], R. Lico[1,2], M. M. Lisakov[11], A. P. Lobanov[5], L. Petrov[12,16], B. W. Sohn[14,15,17], K. V. Sokolovsky[11,18,19], P. A. Voitsik[11], J. A. Zensus[5] and S. Tingay[20]



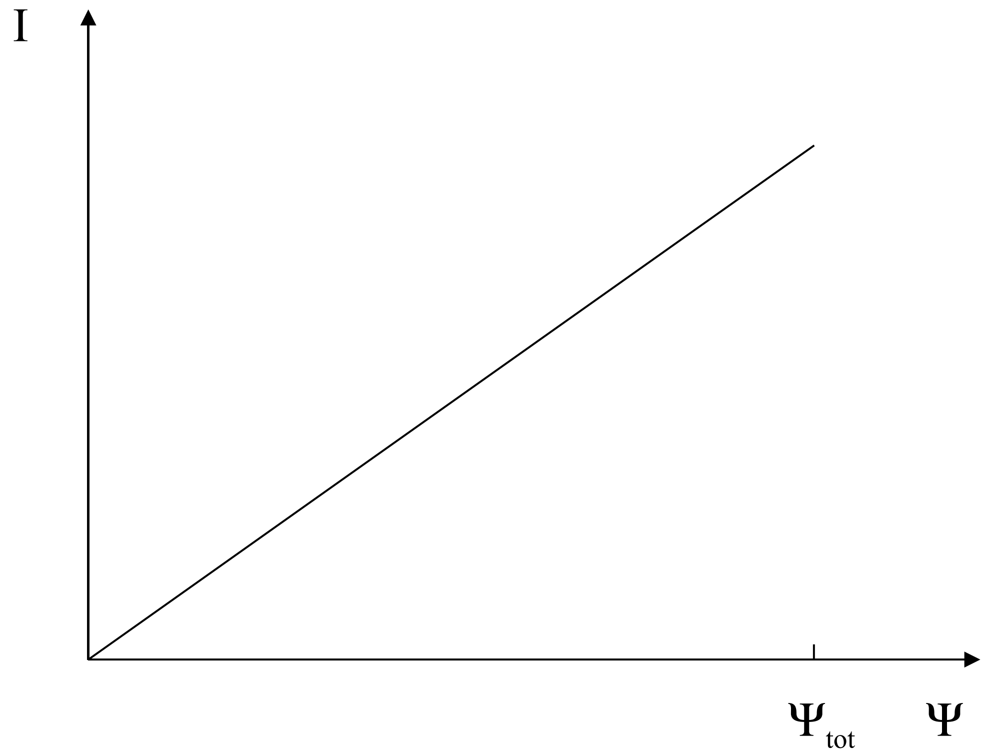
Matching to ambient gas pressure

Standard approach

$$\frac{B_\varphi^2}{8\pi} = P_{\rm ext}$$

$$B_\varphi = \frac{2I}{cr_\perp}$$

$$r_{\rm jet} = \left(\frac{I^2}{2\pi c^2 P_{\rm ext}}\right)^{1/2}$$



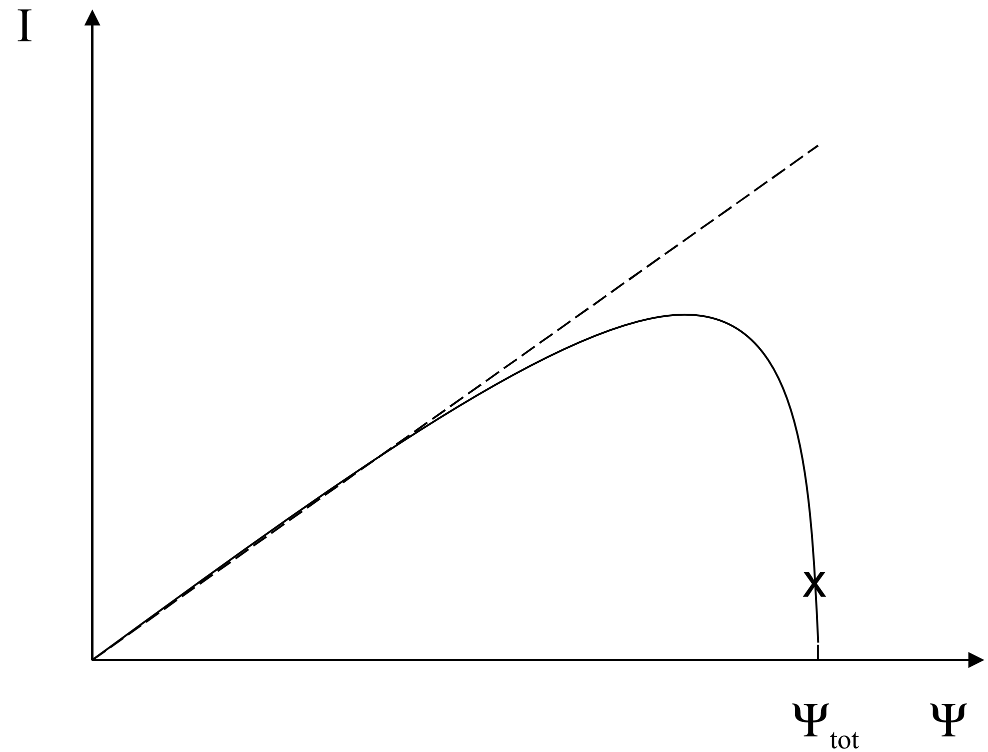
Matching to ambient gas pressure

More realistic

$$\frac{B_\varphi^2}{8\pi} = P_{\rm ext}$$

$$B_\varphi = \frac{2I}{cr_\perp}$$

$$r_{\rm jet} = \left(\frac{I^2}{2\pi c^2 P_{\rm ext}}\right)^{1/2}$$



Jets – theory

Main parameters

- Michel magnetization parameter (maximal bulk Lorentz-factor)

$$\sigma_{\rm M} = \frac{\Omega_0 e B_0 r_{\rm jet}^2}{4\lambda m_{\rm e} c^3}$$

μ now

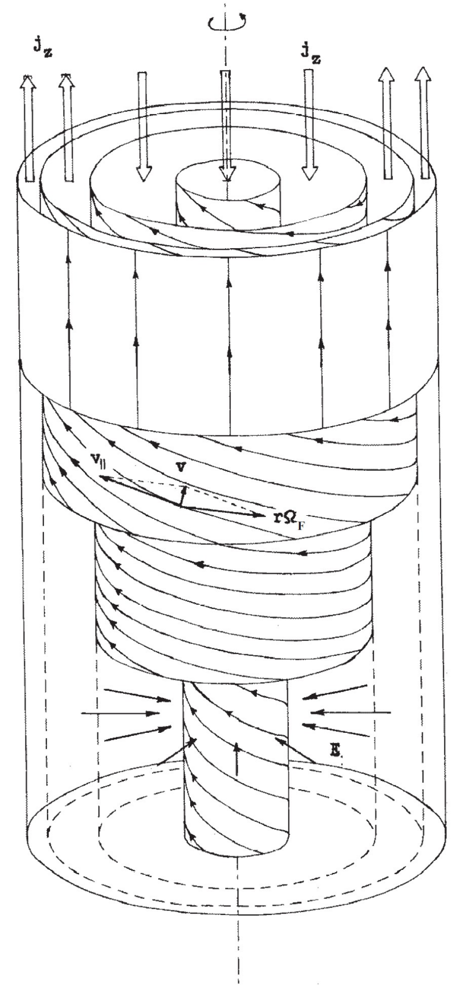
- Multiplicity parameter

$$\lambda = \frac{n^{\rm (lab)}}{n_{\rm GJ}}$$

$$\rho_{\rm GJ} = -\frac{\mathbf{\Omega}\cdot\mathbf{B}}{2\pi c}$$

- Total potential drop

$$\lambda\sigma_{\rm M} \sim \frac{e E_r r_{\rm jet}}{m_{\rm e} c^2}$$



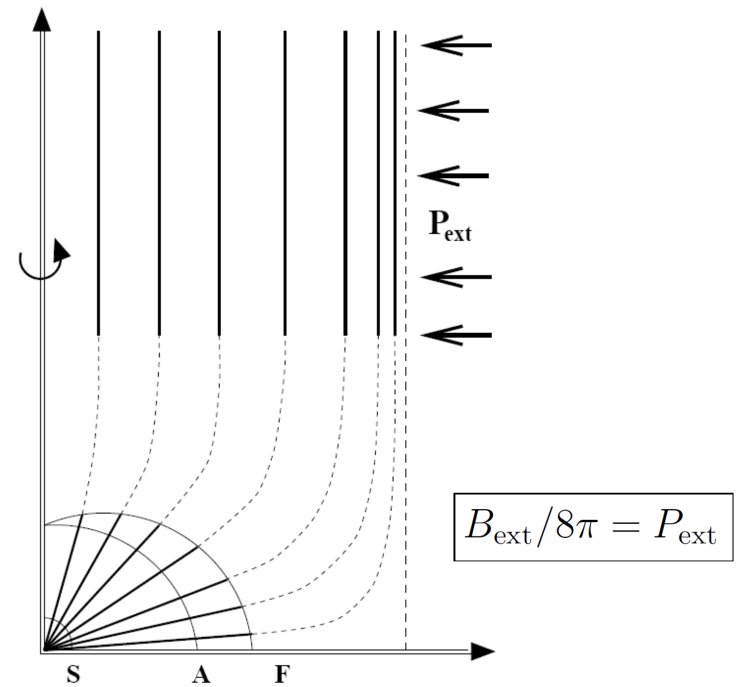
Jets – theory

It is necessary to include the external media into consideration.

It is the ambient pressure that determines the jet transverse scale and particle energy.

1D approach for cylindrical jets

$$\begin{cases} \dfrac{d\mathcal{M}^2}{dr_\perp} = F_1(\mathcal{M}^2, \Psi, r_\perp) \\ \dfrac{d\Psi}{dr_\perp} = F_2(\mathcal{M}^2, \Psi, r_\perp) \end{cases}$$



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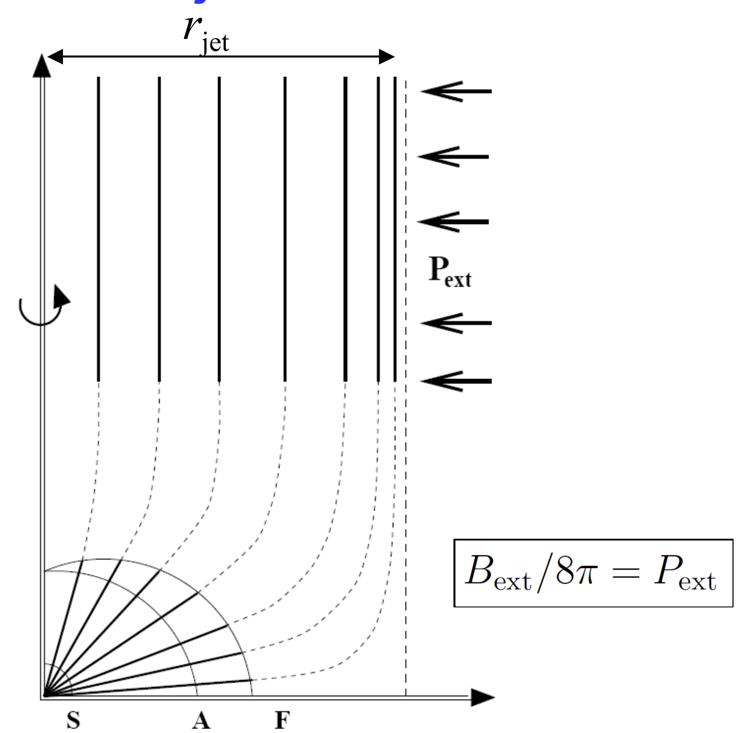
VB. Phys. Uspekhi, **40**, 659 (1997)

T.Lery, J.Heyvaerts, S.Appl, C.A.Norman. A&A, **347**, 1055 (1999)

Jets – theory

It is necessary to include the external media into consideration.
It is the ambient pressure that determines the jet transverse scale and particle energy.

$$r_{\rm jet} = \left(\frac{I^2}{2\pi c^2 P_{\rm ext}}\right)^{1/2}$$



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On the internal structure of relativistic jets collimated by ambient gas pressure

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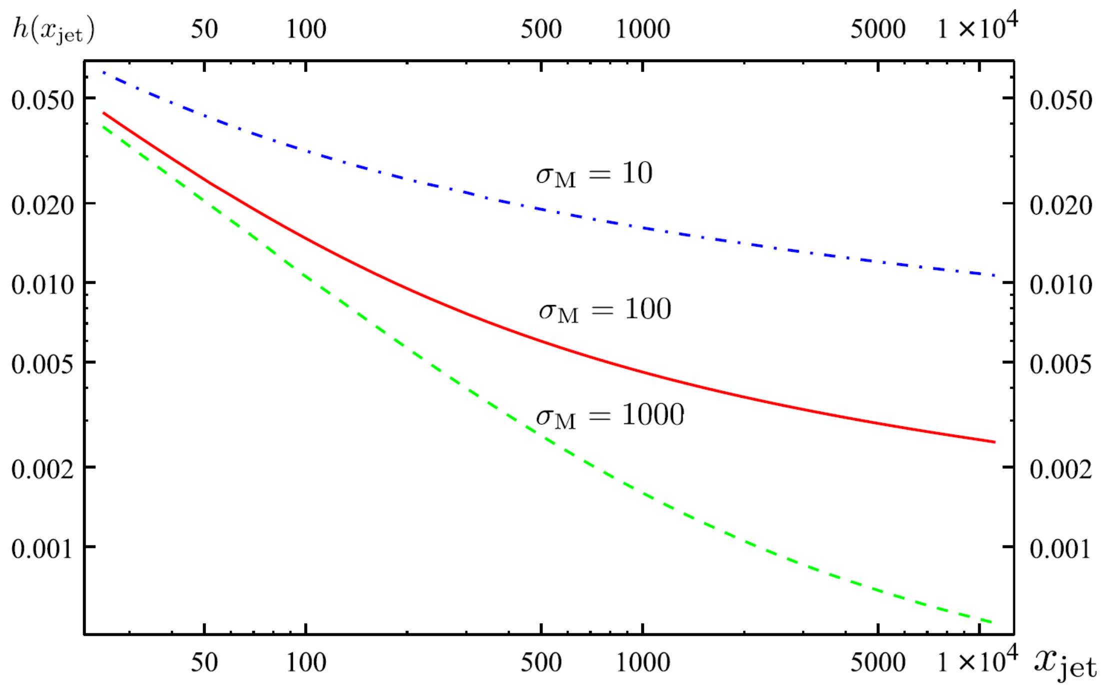
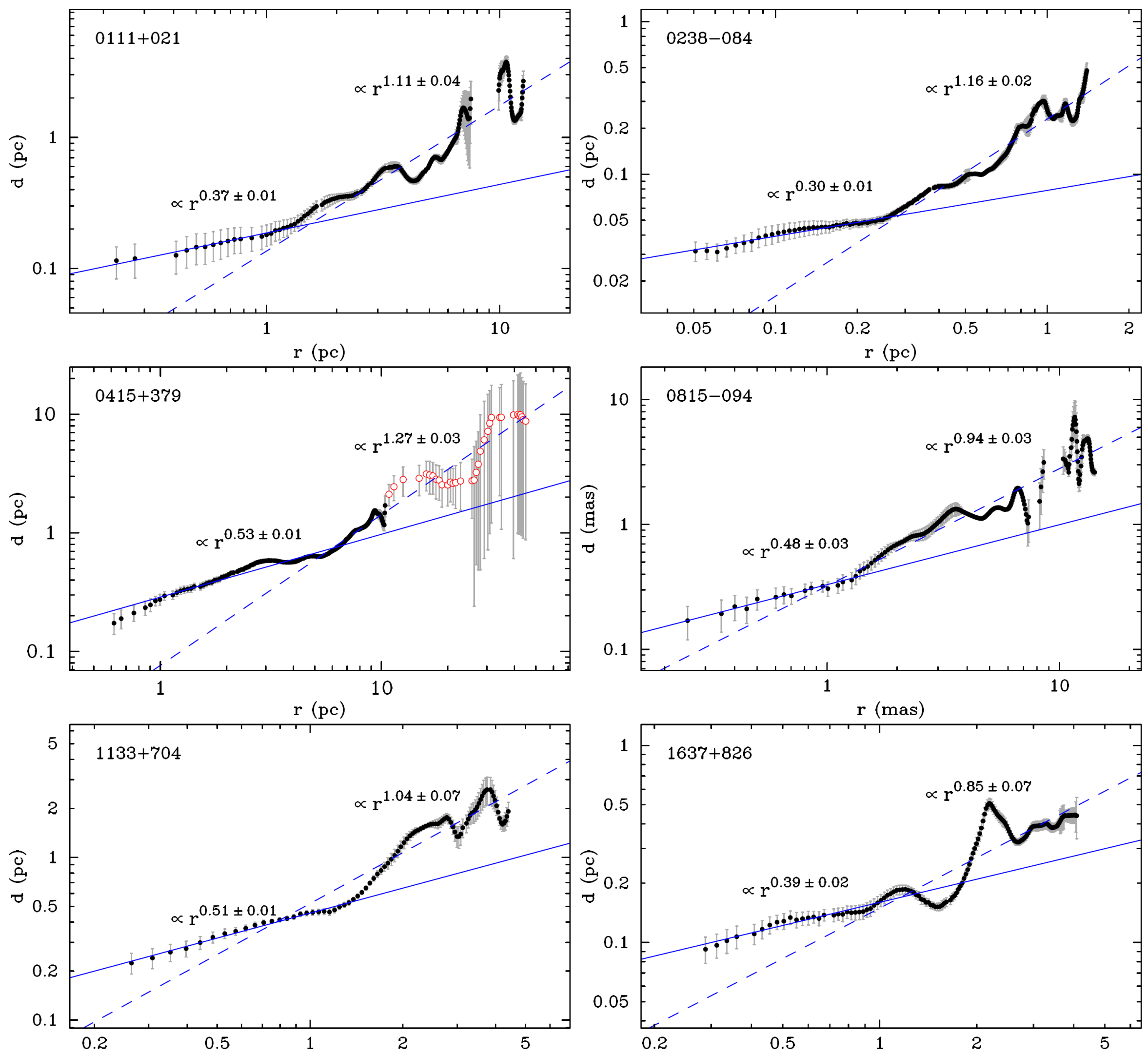
$$x_{\rm jet} = \frac{1}{2(2\pi)^{1/2}} \frac{h(x_{\rm jet})}{k_{\rm I}} \frac{B_{\rm L}}{P_{\rm ext}^{1/2}}$$

$$x = \frac{\Omega_0 r}{c}$$

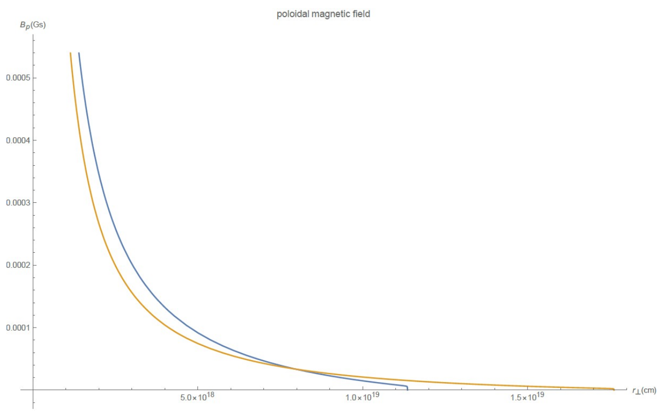


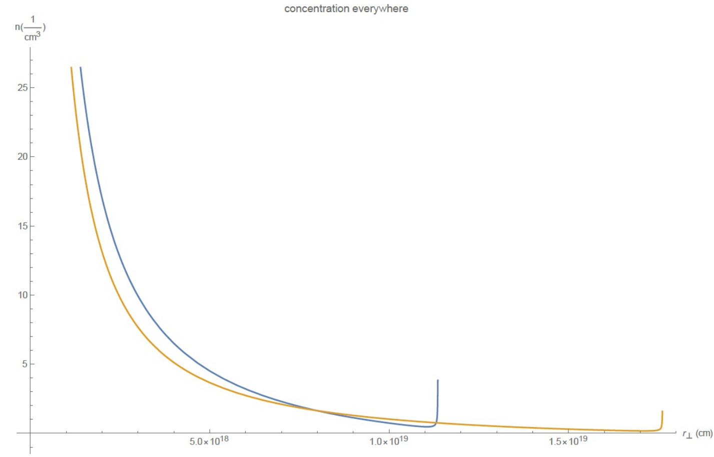
We can explain the break

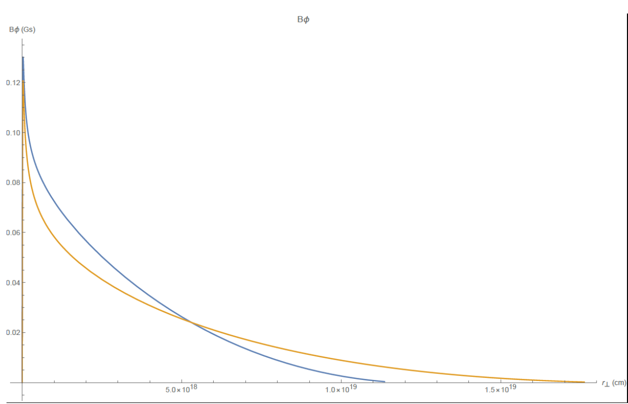


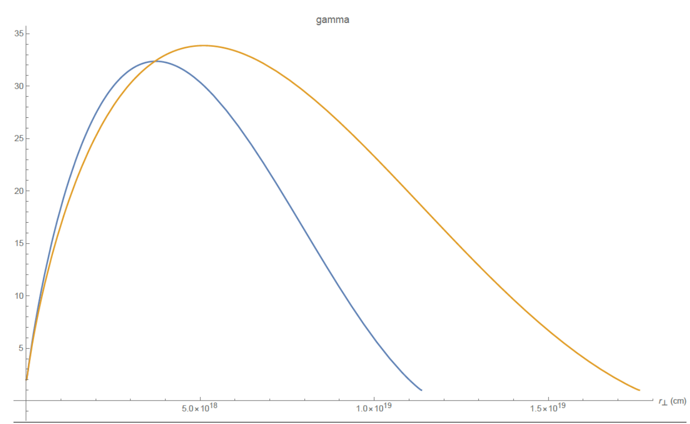
We can determine

- Internal structure



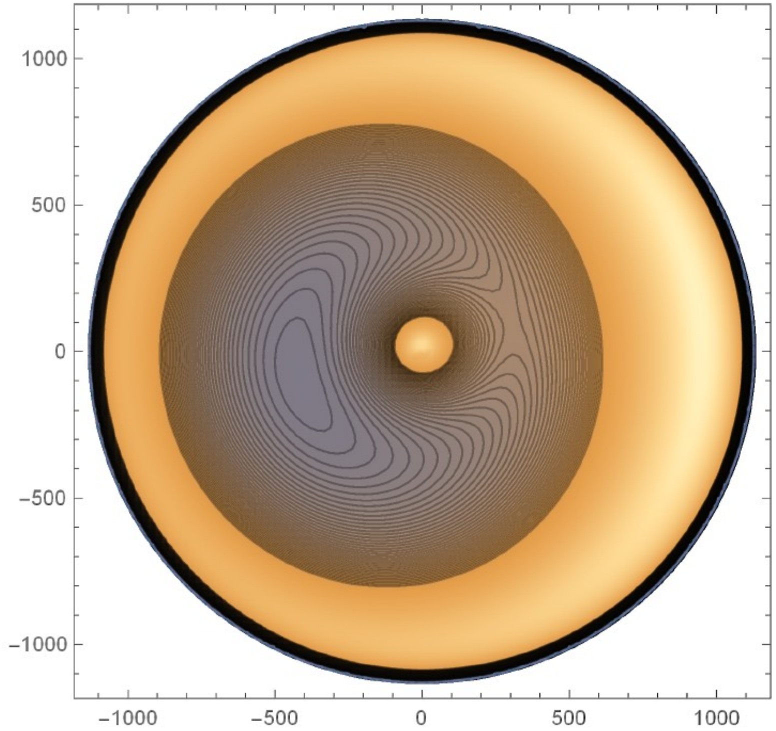






We can determine

- Doppler factor map



PSRs

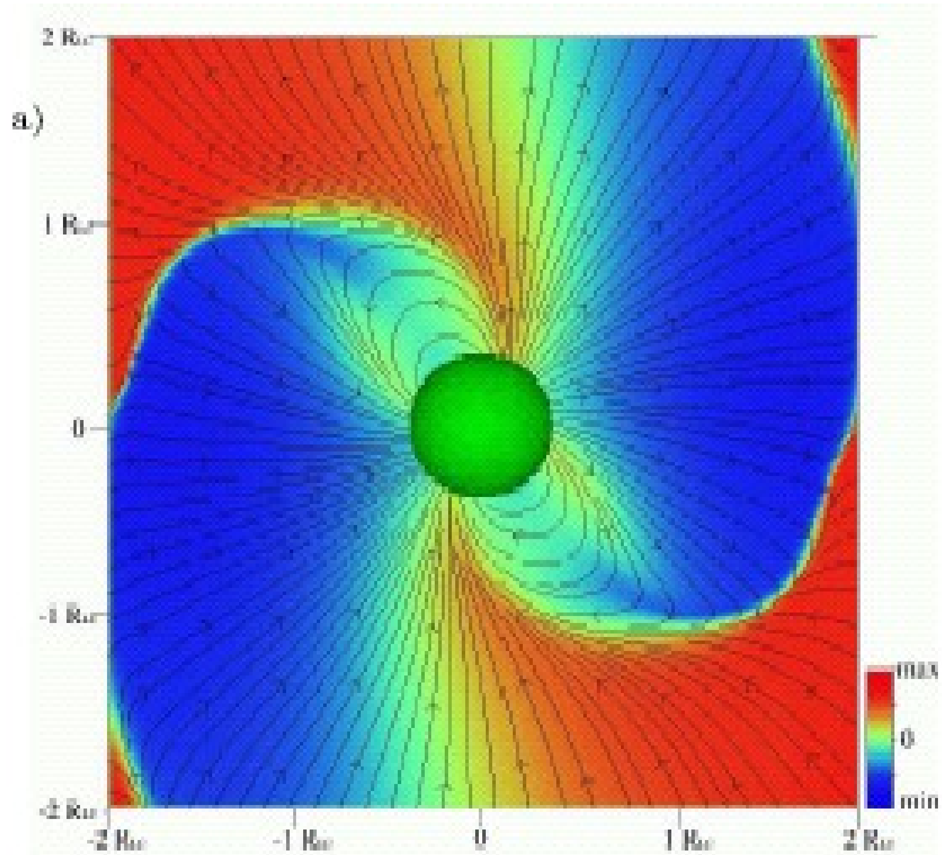
The nature of the torque

- Our theory (BGI) – GJ current
- Standard approach – as large as necessary

How does it work?

$$W_{\rm tot}^{\rm (MHD)} \approx \frac{1}{4}\frac{B_0^2\Omega^4 R^6}{c^2}(1+\sin^2\chi)$$

What is the current system?



What is the current system?

Current losses

1. Direct current losses (BGI)

$$K_{\|} = -c_{\|}\frac{B_0^2\Omega^3R^6}{c^3}i_{\mathrm{s}},$$
$$K_{\perp} = -c_{\perp}\frac{B_0^2\Omega^3R^6}{c^3}\left(\frac{\Omega R}{c}\right)i_{\mathrm{a}}.$$

2. Mismatch
(‘second term’)

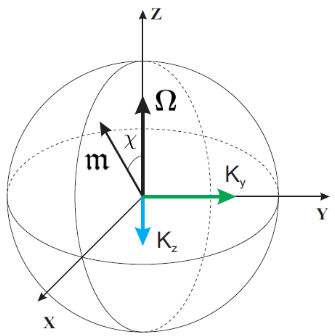
$$\mathbf{K} = \frac{R^3}{c}\int \mathbf{J}_{\mathrm{s}}\left(\mathbf{B}\mathbf{n}\right)\mathrm{d}o = \frac{R^3}{4\pi}\int\{[\mathbf{n}\times\mathbf{B}^{(3)}](\mathbf{B}^{(0)}\mathbf{n}) + [\mathbf{n}\times\mathbf{B}^{(0)}](\mathbf{B}^{(3)}\mathbf{n})\}\,\mathrm{d}o$$

3. Additional separatrix current

STEP #I

Vacuum magneto-dipole

Vacuum: magneto-dipole



$$\mathbf{K} = \frac{1}{c}\int \left[\mathbf{r} \times \left[\mathbf{J}_\mathrm{s} \times \mathbf{B}\right]\right] \mathrm{d}S$$

$$K_{z'} = \frac{2}{3}\frac{\mathfrak{m}^2}{R^3}\left(\frac{\Omega R}{c}\right)^3 \sin^2\chi$$

$$K_{x'} = \frac{2}{3}\frac{\mathfrak{m}^2}{R^3}\left(\frac{\Omega R}{c}\right)^3 \sin\chi\cos\chi$$

Vacuum: magneto-dipole

Energy losses

$$\beta_{\rm R} = \frac{\mathbf{\Omega} \times \mathbf{r}}{c}$$

$$W_{\rm tot} = \frac{c}{4\pi}\int(\beta_{\rm R}\mathbf{B})(\mathbf{B}\mathrm{d}\mathbf{S})$$

Vacuum (Deusch) 1 Ω Ω^3 1 1 $= \Omega^4$

$$\mathbf{K} = \frac{R^3}{c}\int \mathbf{J}_{\rm s}\,(\mathbf{Bn})\,\mathrm{d}o = \frac{R^3}{4\pi}\int\{[\mathbf{n}\times\mathbf{B}^{(3)}](\mathbf{B}^{(0)}\mathbf{n}) + [\mathbf{n}\times\mathbf{B}^{(0)}](\mathbf{B}^{(3)}\mathbf{n})\}\,\mathrm{d}o$$

$$\mathrm{H}_r = \mathrm{R}_1(a)\left\{\frac{a^3}{r^3}\cos\chi\cos\theta + \frac{h_1/\rho}{(h_1/\rho)_\alpha}\sin\chi\sin\theta\, e^{i\lambda}\right\}$$

Vacuum: magneto-dipole

Energy losses

$$\beta_{\rm R} = \frac{\mathbf{\Omega} \times \mathbf{r}}{c}$$

Vacuum (Deusch)

1 Ω Ω^3 1 1 $= \Omega^4$

$$H_r = R_1(a) \left\{ \frac{a^3}{r^3} \cos\chi \cos\theta + \frac{h_1/\rho}{(h_1/\rho)_\alpha} \sin\chi \sin\theta \, e^{i\lambda} \right\}$$

$$H_\theta = \frac{1}{2} R_1(a) \left\{ \frac{a^3}{r^3} \cos\chi \sin\theta + \left[\left(\frac{\rho^2}{\rho h_2' + h_2} \right)_\alpha h_2 + \left(\frac{\rho}{h_1} \right)_\alpha \left(h_1' + \frac{h_1}{\rho} \right) \right] \sin\chi \cos\theta \, e^{i\lambda} \right\}$$

$$H_\varphi = \frac{1}{2} R_1(a) \left\{ \left(\frac{\rho^2}{h_2' + h_2} \right)_\alpha h_2 \cos 2\theta + \left(\frac{\rho}{h_1} \right)_\alpha \left(h_1' + \frac{h_1}{\rho} \right) \right\} i \sin\chi \, e^{i\lambda}$$

$$E_r = \frac{1}{2} \omega\mu_0 a \, R_1(a) \left\{ -\frac{1}{2} \frac{a^4}{r^4} \cos\chi (3\cos 2\theta + 1) + 3 \left(\frac{\rho}{\rho h_2' + h_2} \right)_\alpha \frac{h_2}{\rho} \sin\chi \sin 2\theta \, e^{i\lambda} \right\}$$

$$E_\theta = \frac{1}{2} \omega\mu_0 a \, R_1(a) \left\{ -\frac{a^4}{r^4} \cos\chi \sin 2\theta + \left[\left(\frac{\rho h_2' + h_2}{\rho} \right)_\alpha \frac{\rho}{\rho h_2' + h_2} \cos 2\theta - \frac{h_1}{h_1(\alpha)} \right] \sin\chi \, e^{i\lambda} \right\}$$

$$E_\varphi = \frac{1}{2} \omega\mu_0 a \, R_1(a) \left\{ \left(\frac{\rho}{\rho h_2' + h_2} \right)_\alpha \frac{\rho h_2' + h_2}{\rho} - \frac{h_1}{h_1(\alpha)} \right\} i \sin\chi \cos\theta \, e^{i\lambda}.$$

Landau-Lifshits, Field Theory

Orthogonal rotator

$$
\begin{aligned}
B_r^{\perp} &= \frac{|\mathfrak{m}|}{r^3}\sin\theta\,\mathrm{Re}\left(2-2i\frac{\Omega r}{c}\right)\exp\left(i\frac{\Omega r}{c}+i\varphi-i\Omega t\right),\\
B_\theta^{\perp} &= \frac{|\mathfrak{m}|}{r^3}\cos\theta\,\mathrm{Re}\left(-1+i\frac{\Omega r}{c}+\frac{\Omega^2 r^2}{c^2}\right)\exp\left(i\frac{\Omega r}{c}+i\varphi-i\Omega t\right),\\
B_\varphi^{\perp} &= \frac{|\mathfrak{m}|}{r^3}\mathrm{Re}\left(-i-\frac{\Omega r}{c}+i\frac{\Omega^2 r^2}{c^2}\right)\exp\left(i\frac{\Omega r}{c}+i\varphi-i\Omega t\right),\\
E_r^{\perp} &= 0,\\
E_\theta^{\perp} &= \frac{|\mathfrak{m}|\Omega}{r^2 c}\mathrm{Re}\left(-1+i\frac{\Omega r}{c}\right)\exp\left(i\frac{\Omega r}{c}+i\varphi-i\Omega t\right),\\
E_\varphi^{\perp} &= \frac{|\mathfrak{m}|\Omega}{r^2 c}\cos\theta\,\mathrm{Re}\left(-i-\frac{\Omega r}{c}\right)\exp\left(i\frac{\Omega r}{c}+i\varphi-i\Omega t\right).
\end{aligned}
$$

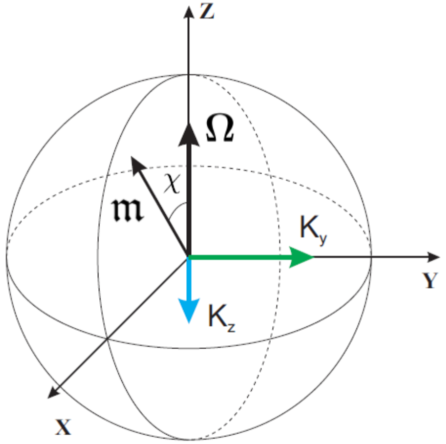
Vacuum: magneto-dipole

Energy losses

$$\beta_{\mathrm{R}} = \frac{\mathbf{\Omega} \times \mathbf{r}}{c}$$

$$W_{\mathrm{tot}} = \frac{c}{4\pi} \int (\beta_{\mathrm{R}} \mathbf{B})(\mathbf{B} d\mathbf{S})$$

Vacuum (Deusch)	1	Ω	Ω^3	1	1	$= \Omega^4$
Vacuum (L&L) (2/3)	1	Ω	Ω^3	1	1	$= \Omega^4$



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Energy losses

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$$W_{\rm tot} = \frac{c}{4\pi}\int (\beta_{\rm R}\mathbf{B})(\mathbf{B}d\mathbf{S})$$

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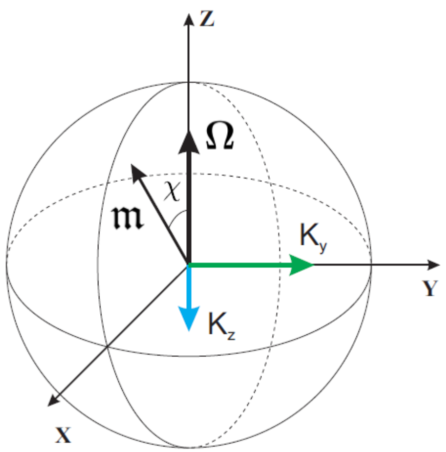
Vacuum (L&L) (2/3)	1	Ω	Ω^3	1	1	$= \Omega^4$
Vacuum (L&L) (1/3)	1	Ω	1	Ω^3	1	$= \Omega^4$



$$\mathbf{B}^{(3)} = -\frac{2}{3}\frac{\mathfrak{m}}{R^3}\left(\frac{\Omega R}{c}\right)^3 \mathbf{e}_{y'}$$

IMPORTANT CONCLUSION

Two terms can play role in energy losses

$$\mathbf{K} = \frac{R^3}{c} \int \mathbf{J}_\mathrm{s} \left(\mathbf{B}\mathbf{n}\right) \mathrm{d}o = \frac{R^3}{4\pi} \int \{[\mathbf{n} \times \mathbf{B}^{(3)}](\mathbf{B}^{(0)}\mathbf{n}) + [\mathbf{n} \times \mathbf{B}^{(0)}](\mathbf{B}^{(3)}\mathbf{n})\} \, \mathrm{d}o$$

STEP #II

Pulsar magnetosphere

Force-free approximation

One can neglect energy of particles

$$\frac{1}{c}\mathbf{j}\times\mathbf{B}+\rho_{\mathrm{e}}\mathbf{E}=0$$

Mestel equation (1973)

$$\nabla\times\tilde{\mathbf{B}}=\psi\mathbf{B}$$

$$\tilde{\mathbf{B}}=\left\{B_r\left(1-\frac{\Omega^2 r^2}{c^2}\right),\, B_\theta,\, B_z\left(1-\frac{\Omega^2 r^2}{c^2}\right)\right\}$$

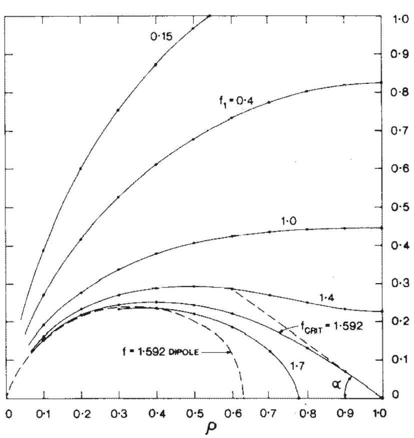
Pulsar equation

$$-\left(1-\frac{\Omega_{\mathrm{F}}^2\varpi^2}{c^2}\right)\nabla^2\Psi+\frac{2}{\varpi}\frac{\partial\Psi}{\partial\varpi}-\frac{16\pi^2}{c^2}I\frac{\mathrm{d}I}{\mathrm{d}\Psi}+\frac{\varpi^2}{c^2}\left(\nabla\Psi\right)^2\Omega_{\mathrm{F}}\frac{\mathrm{d}\Omega_{\mathrm{F}}}{\mathrm{d}\Psi}=0$$

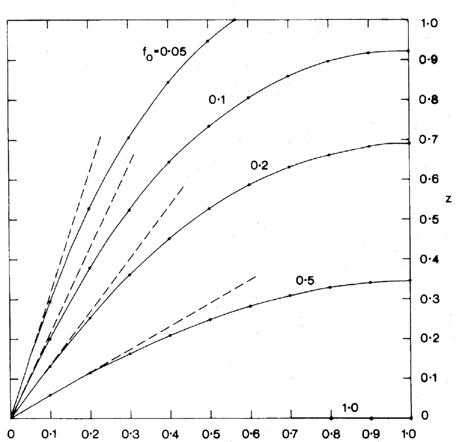
(Michel 1973, Mestel 1993, Scharlemann & Wagoner 1973, Okamoto 1974, Mestel & Wang 1979)

First solutions

$$-\left(1-\frac{\Omega_{\rm F}^2\varpi^2}{c^2}\right)\nabla^2\Psi+\frac{2}{\varpi}\frac{\partial\Psi}{\partial\varpi}-\frac{16\pi^2}{c^2}I\frac{{\rm d}I}{{\rm d}\Psi}+\frac{\varpi^2}{c^2}\left(\nabla\Psi\right)^2\Omega_{\rm F}\frac{{\rm d}\Omega_{\rm F}}{{\rm d}\Psi}=0$$



F. Michel (1973)



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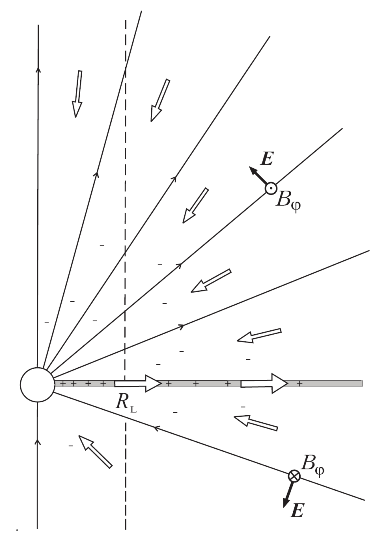

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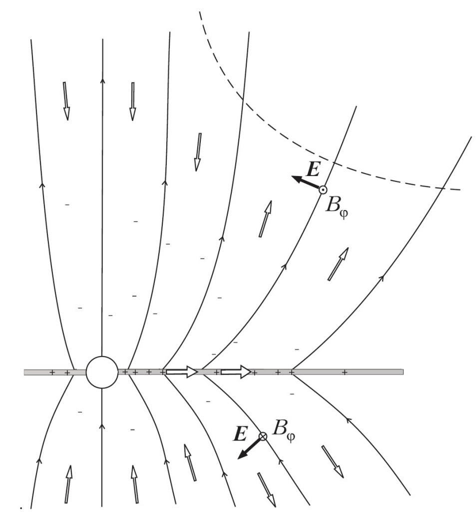
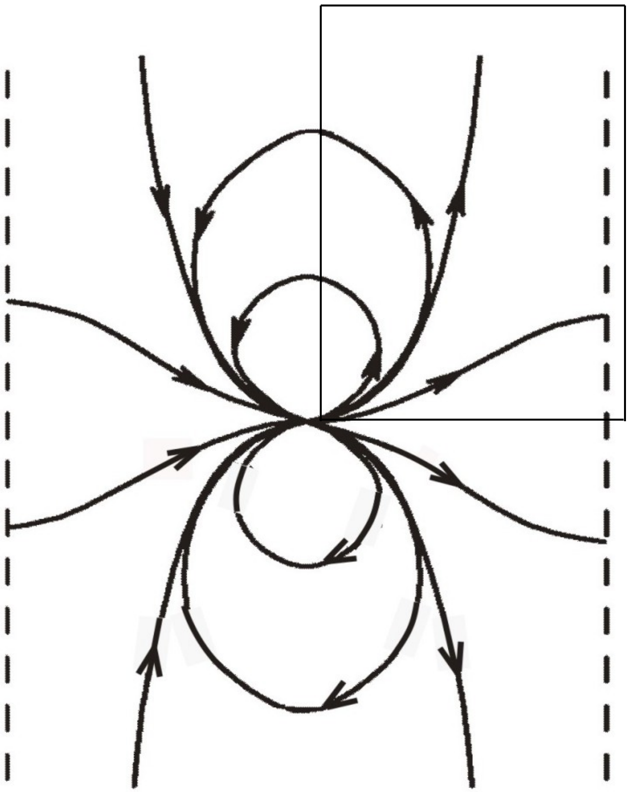


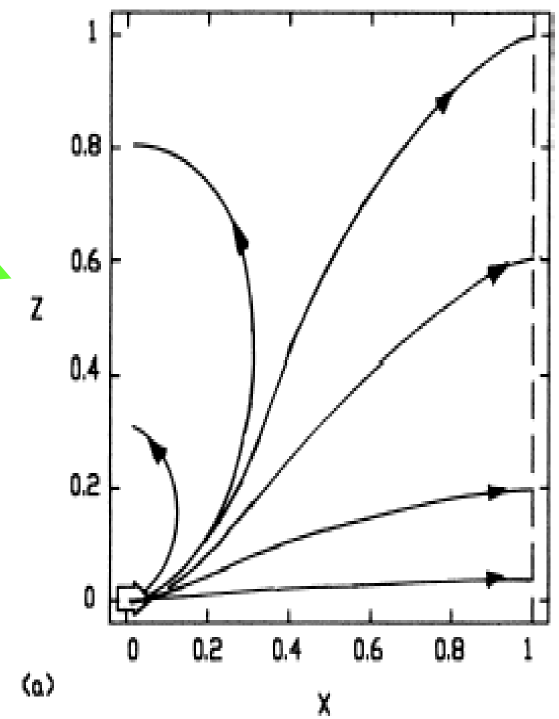
R.Blandford (1976)

Orthogonal Rotator – no currents

$$\nabla \times \tilde{\mathbf{B}} = \mathbf{0}$$



$\chi = 90^\circ$



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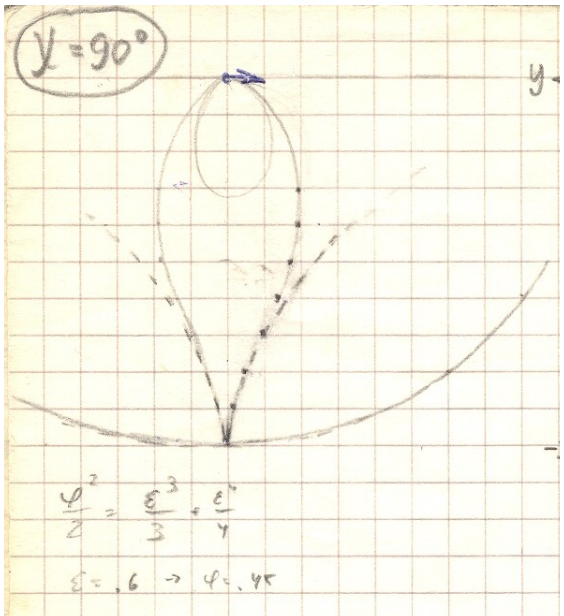
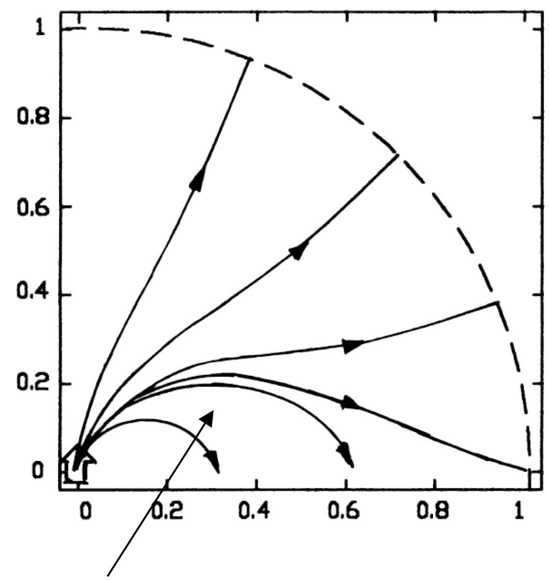
Orthogonal Rotator – no currents

VB, A.V.Gurevich, Ya.N.Istomin, JETP, **58**, 235 (1983)
L.Mestel, P.Panagi, S.Shibata, MNRAS, **309**, 388 (1999)





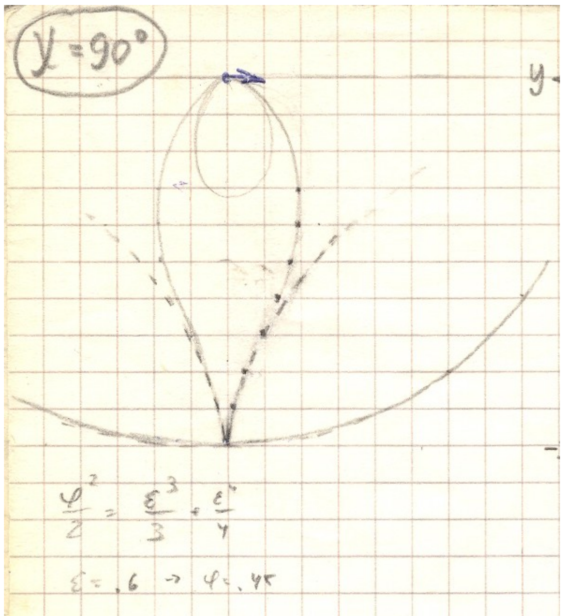
Equatorial plane

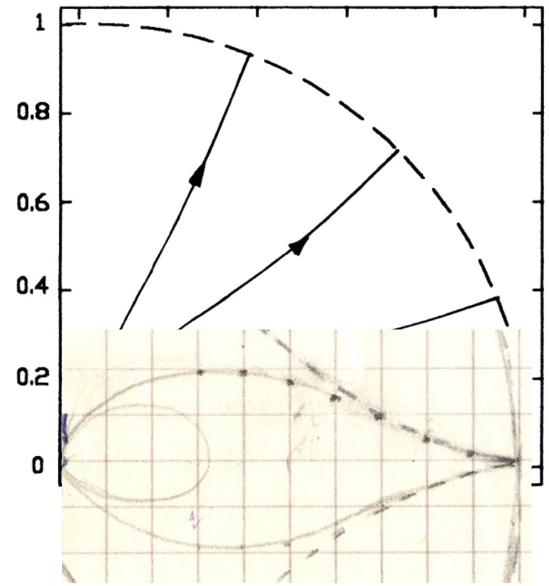
No energy flux through the light cylinder

$$B_\varphi \propto (1 - x_r^2)^2$$

Orthogonal Rotator – no currents

VB, A.V.Gurevich, Ya.N.Istomin, JETP, **58**, 235 (1983)
L.Mestel, P.Panagi, S.Shibata, MNRAS, **309**, 388 (1999)





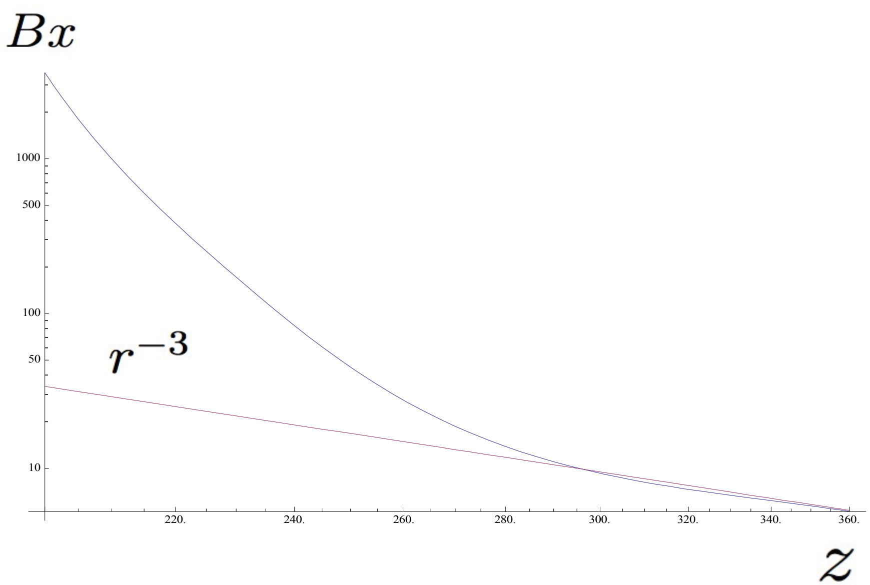
Equatorial plane

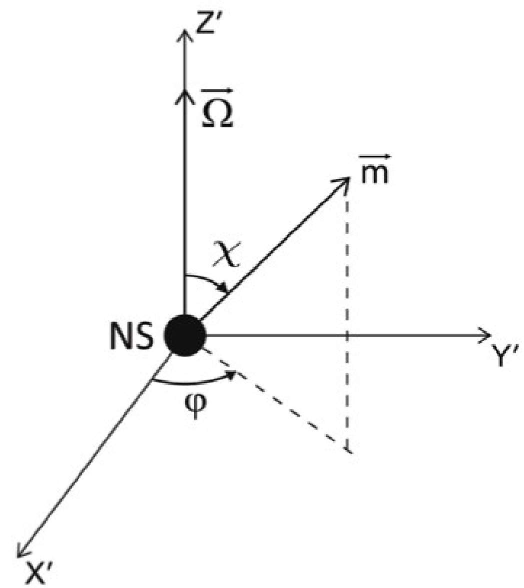
No energy flux through the light cylinder

$$B_\varphi \propto (1 - x_r^2)^2$$

Spitkovsky solution, $\chi = 60^{\rm o}$

No magnetodipole radiation





In vacuum $B_x = \frac{\ddot{d}}{cr}$

VB, YA.N.Istomin, A.A.Philippov, Phys. Uspekhi, **56**, 164 (2013)

IMPORTANT CONCLUSION

No energy losses for zero longitudinal current

STEP #III

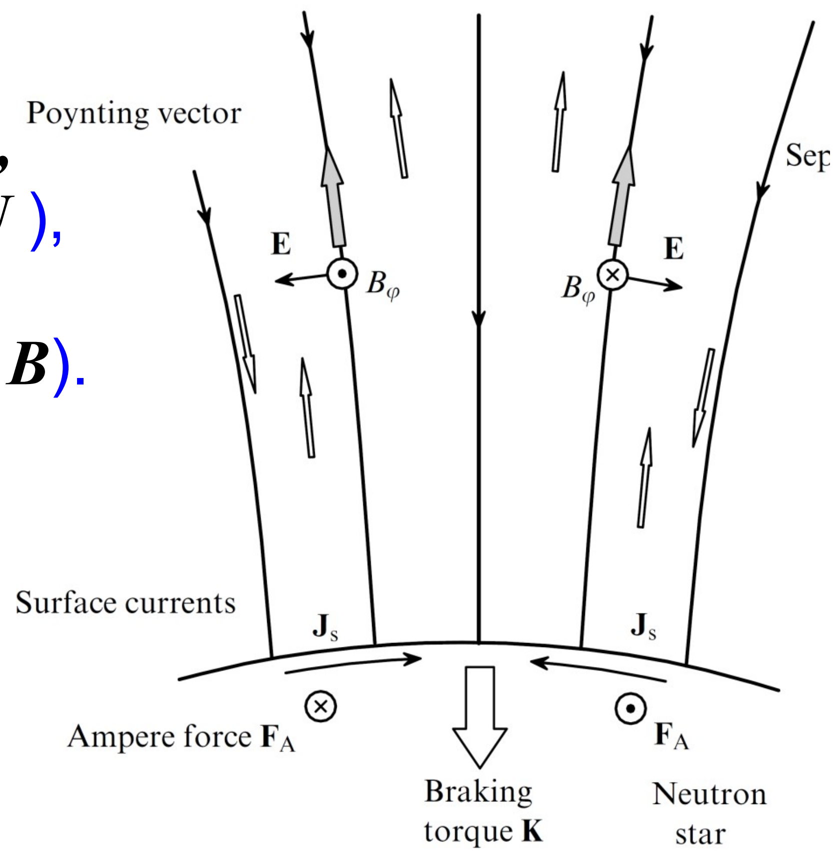
Current losses

Current losses

For current loss mechanism is necessary to have

- Plasma in the magnetosphere,
- regular poloidal magnetic field,
- rotation (inductive electric field ***E*,** EMF ***d****U*),
- longitudinal current *I* (toroidal magnetic field ***B***).

$$W_{\rm tot} = I\delta U$$



Current losses

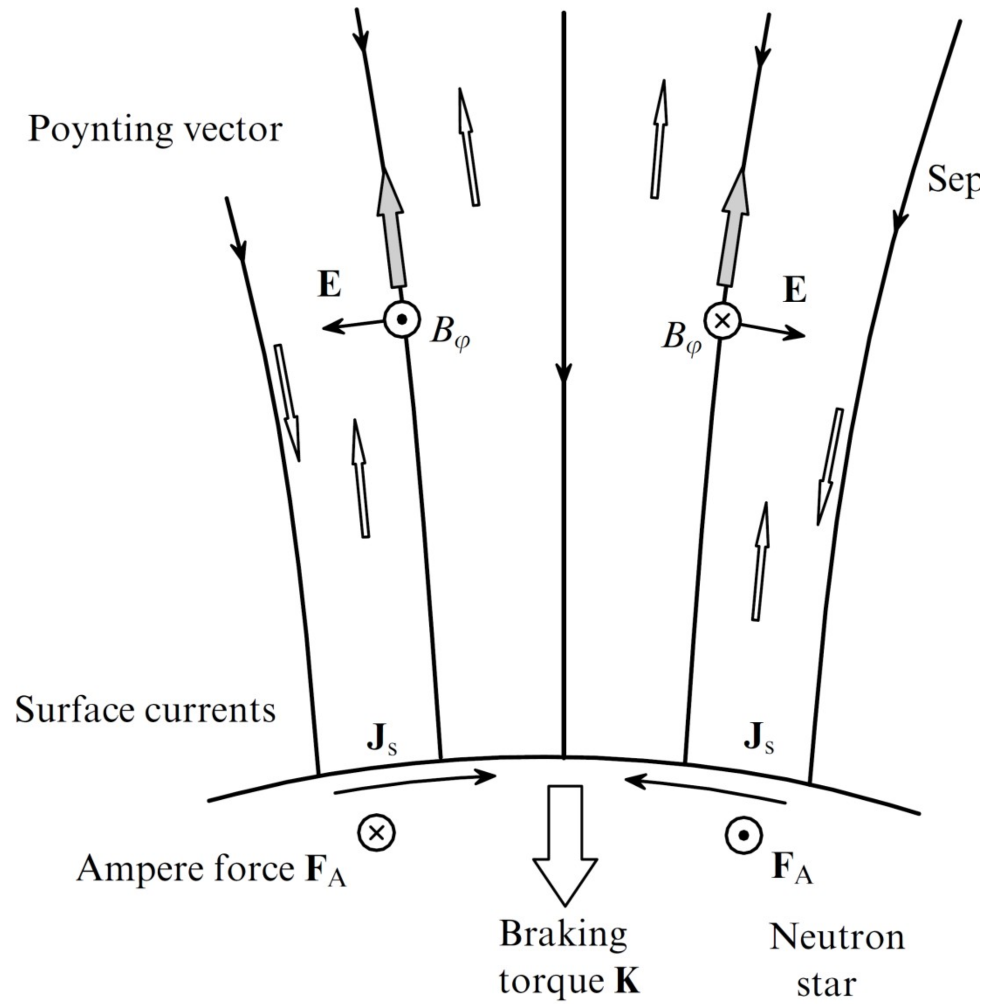
$$W_{\mathrm{tot}} = c_{\parallel} \frac{B_0^2 \Omega^4 R^6}{c^3} i_0$$

$$i_0 = j_{\parallel} / j_{\mathrm{GJ}}$$

$$W_{\mathrm{tot}}^{(\mathrm{BGI})} \approx i_{\mathrm{s}}^{\mathrm{A}} \frac{B_0^2 \Omega^4 R^6}{c^2} \cos^2 \chi$$

$$W_{\mathrm{tot}}^{(\mathrm{BGI})} \approx \frac{B_0^2 \Omega^4 R^6}{c^2} \cos^2 \chi$$

for GJ current

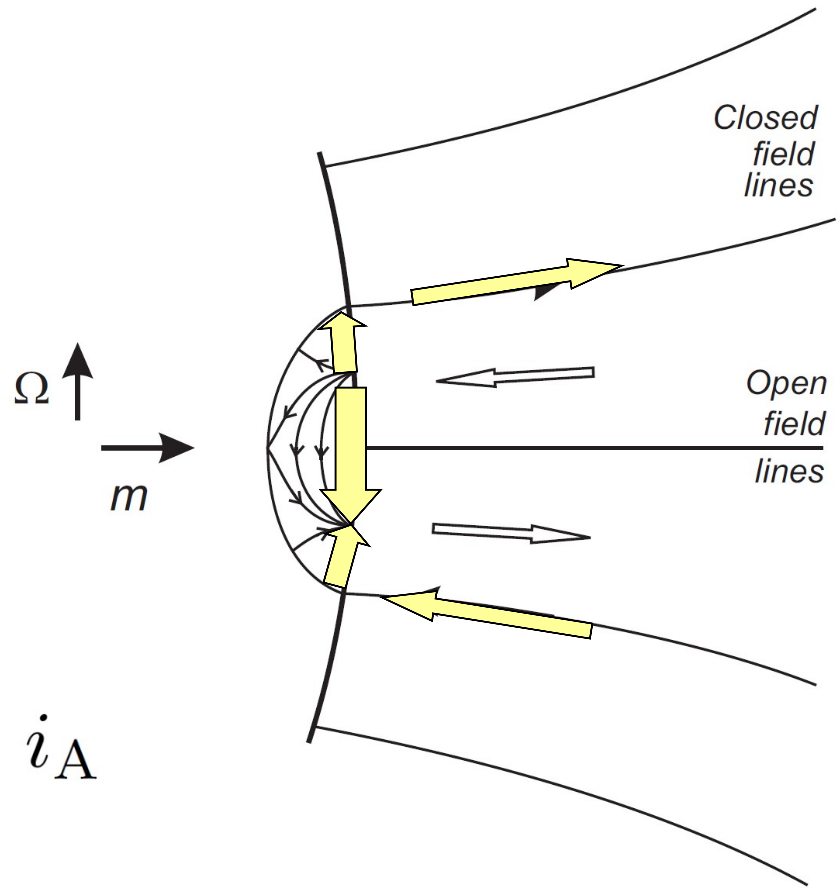


Orthogonal rotator

VB, A.V.Gurevich, Ya.N.Istomin JETP **58**, 235 (1983)

$$j_{\rm GJ} \approx \frac{\Omega B}{2\pi} \cos\theta$$

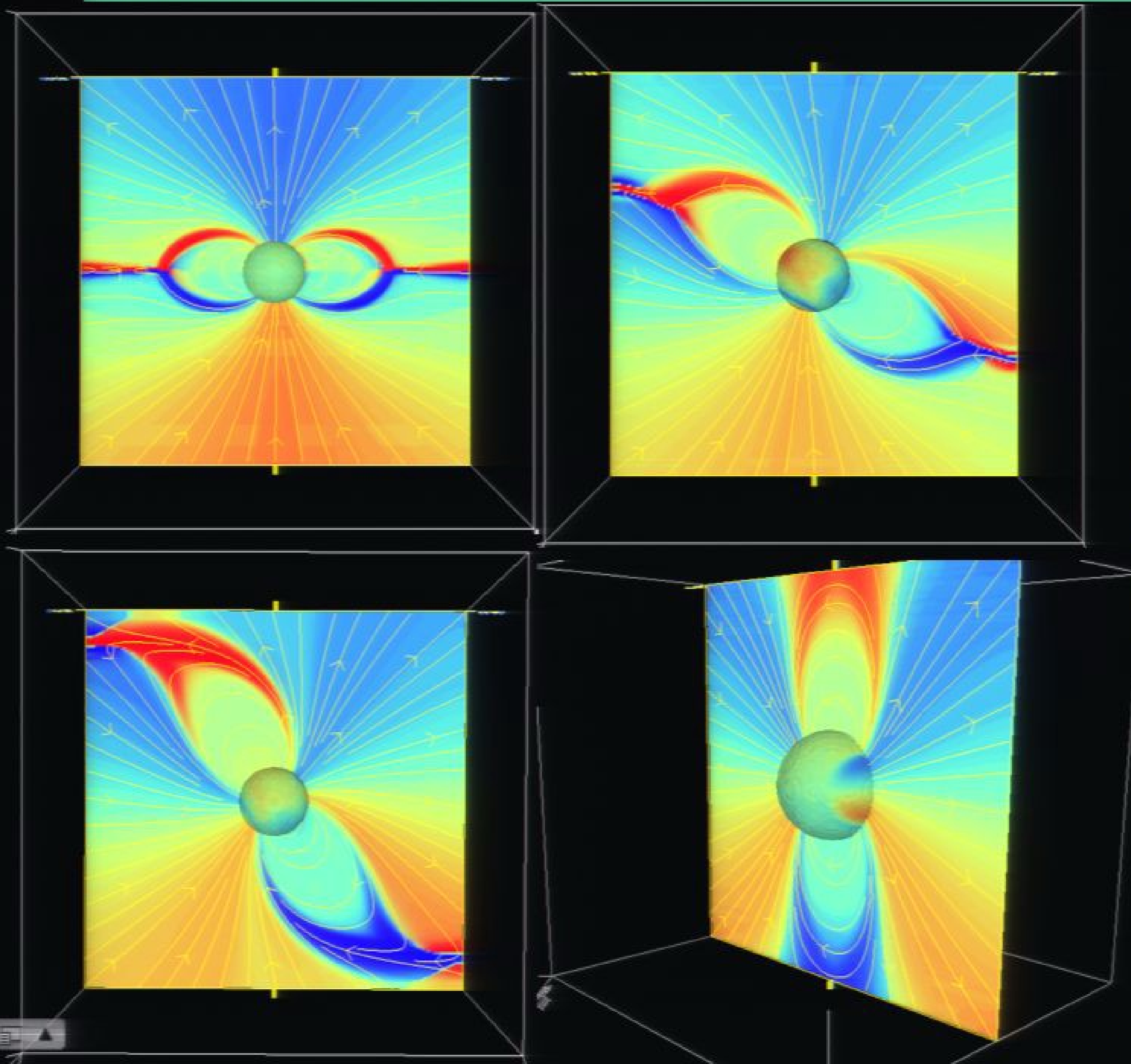
$$\mathbf{K} = \frac{1}{c} \int \left[\mathbf{r} \times [\mathbf{J}_{\rm s} \times \mathbf{B}]\right] \mathrm{d}S$$



$$W_{\rm tot} = c_{\perp} \frac{B_0^2 \Omega^4 R^6}{c^3} \left(\frac{\Omega R}{c}\right) i_{\rm A}$$

$i_{\rm A} = j_{\parallel} / \rho_{\rm GJ} c$

Magnetospheric currents

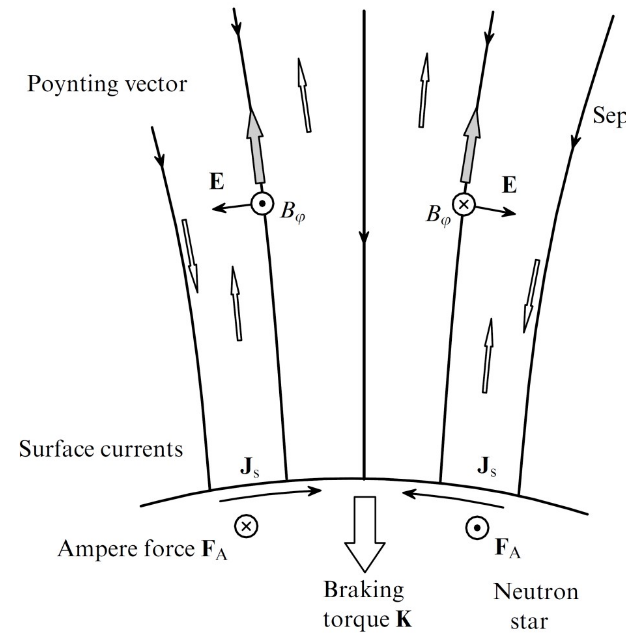


Oppositely flowing currents can occupy the same open flux tube. Does this have any obervational implications?

There is always a null-current field line in the open zone.

IMPORTANT CONCLUSION

$$W_{\rm tot} = I\delta U$$



Direct current losses correspond to first term only

$$\mathbf{K} = \frac{R^3}{c}\int \mathbf{J}_{\rm s}\,(\mathbf{Bn})\,{\rm d}o = \frac{R^3}{4\pi}\int \{[\mathbf{n}\times\mathbf{B}^{(3)}](\mathbf{B}^{(0)}\mathbf{n}) + [\mathbf{n}\times\mathbf{B}^{(0)}](\mathbf{B}^{(3)}\mathbf{n})\}\,{\rm d}o$$

STEP #IV

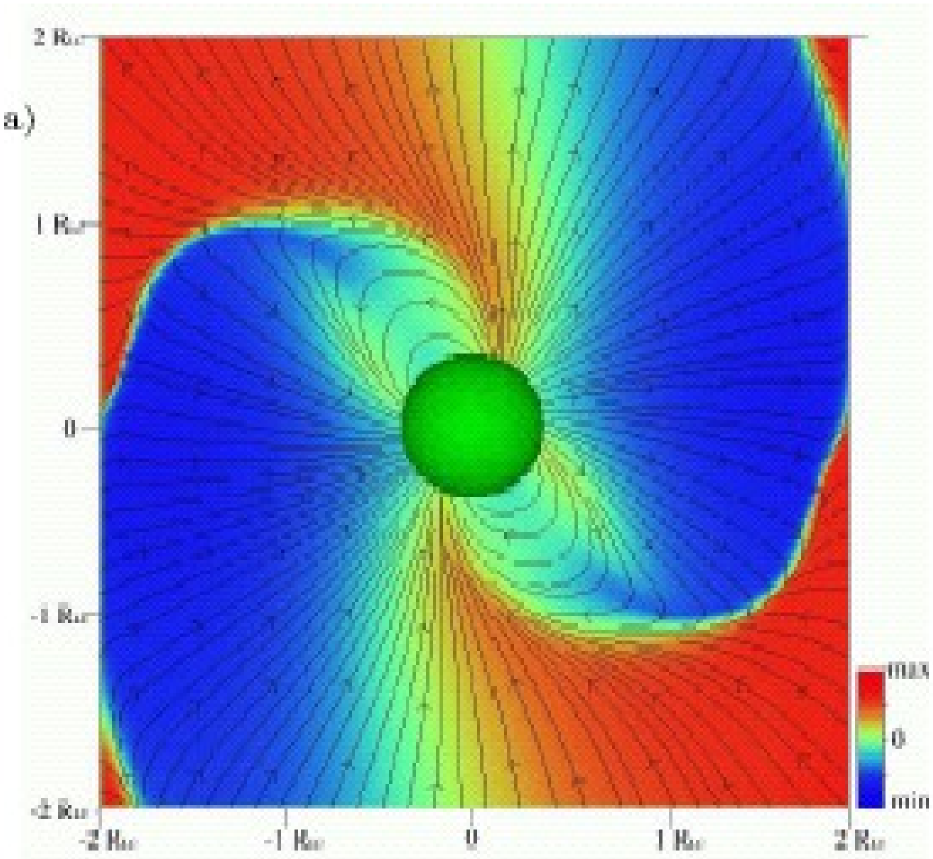
"Universal solution"

Inclined rotator

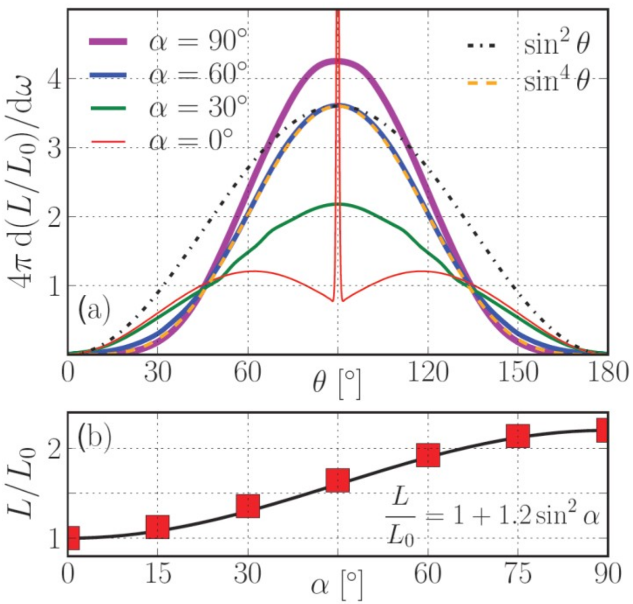
A.Spitkovsky, ApJ Lett., **648**, L51 (2006)

$$W_{\rm tot}^{\rm (MHD)} \approx \frac{1}{4}\frac{B_0^2\Omega^4R^6}{c^2}(1+\sin^2\chi)$$

a)



Inclined rotator – numerically



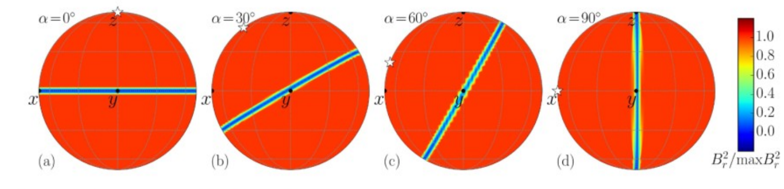
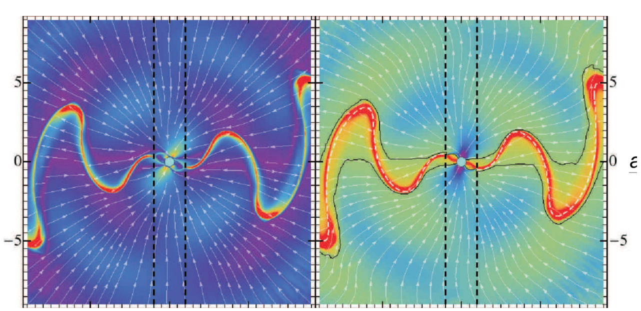


Figure 12. Colour-coded surface distribution of B_r^2 in the split-monopole solution (Bogovalov 1999). The current sheet, in which the radial magnetic field vanishes, describes the orientation of the current sheet in the numerical force-free solutions shown in Fig. 6.

A.Tchekhovskoy, A.Philippov, A.Spitkovsky, MNRAS, **457**, 3384 (2016)



I.Contopoulos et al

$$\langle B_r \rangle \sim \sin\theta$$
$$\langle E \rangle, \langle B_\varphi \rangle \sim \sin^2\theta$$

$$W_{\rm tot}(\theta) = \sin^2\theta B_r^2(\theta)$$

Wind – not a split-monopole

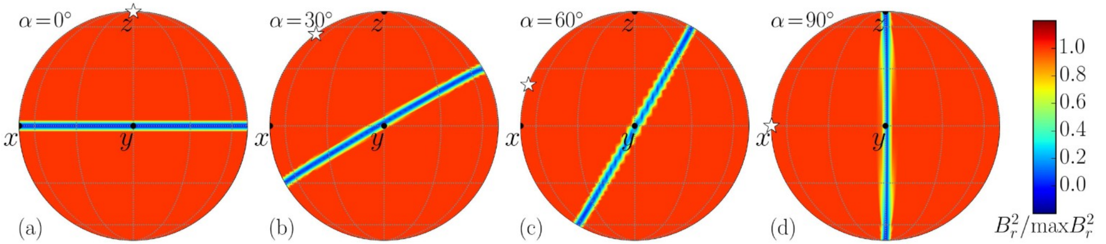
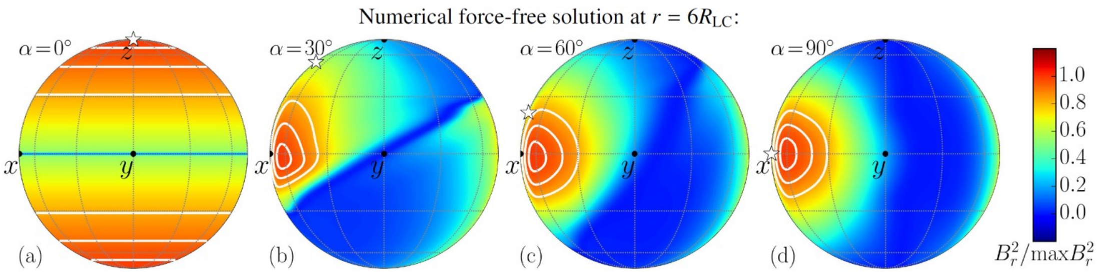


Figure 12. Colour-coded surface distribution of B_r^2 in the split-monopole solution (Bogovalov 1999). The current sheet, in which the radial magnetic field vanishes, describes the orientation of the current sheet in the numerical force-free solutions shown in Fig. 6.

$$W_{\rm tot}(\theta) = \sin^2\theta B_r^2(\theta)$$

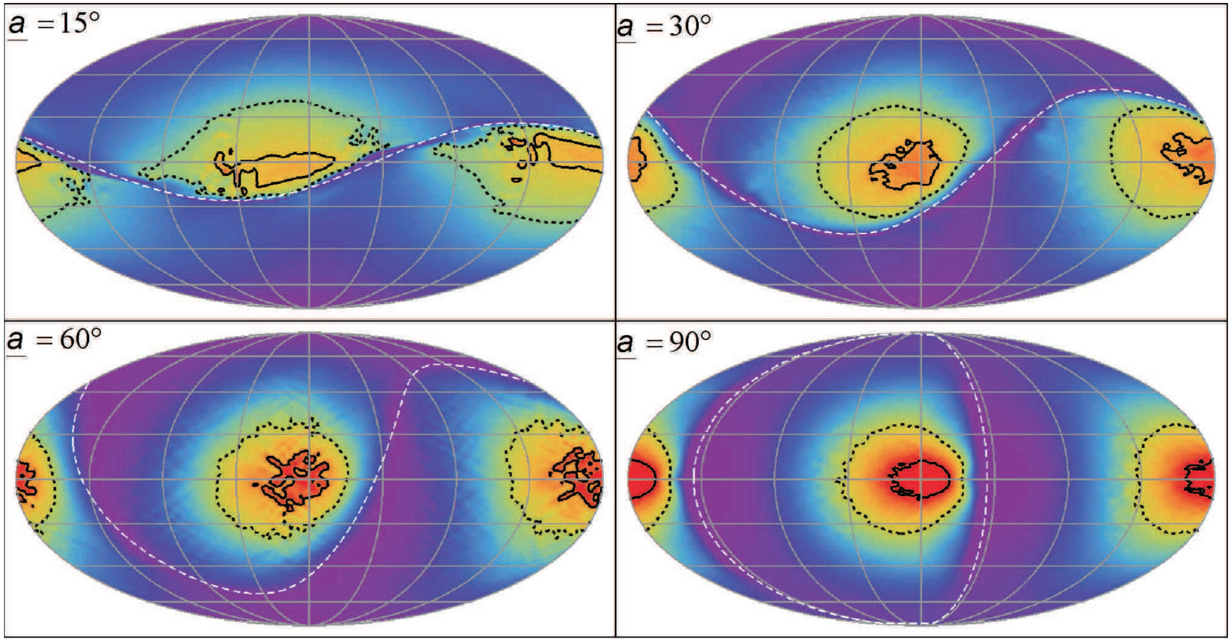


A.Tchekhovskoy, A.Philippov, A.Spitkovsky MNRAS, **457**, 3384 (2015)

$$B_r \approx B_0 \frac{R^2}{r^2} \sin\theta \cos(\varphi - \Omega t + \Omega r/c),$$

$$B_\varphi = E_\theta \approx -B_0 \frac{\Omega R^2}{cr} \sin^2\theta \cos(\varphi - \Omega t + \Omega r/c).$$

Wind – not a split-monopole



C.Kalapotharakos, I.Contopoulos, D.Kazanas, MNRAS, **420**, 2793 (2012)

$$B_r \approx B_0 \frac{R^2}{r^2} \sin\theta \cos(\varphi - \Omega t + \Omega r/c),$$

$$B_\varphi = E_\theta \approx -B_0 \frac{\Omega R^2}{cr} \sin^2\theta \cos(\varphi - \Omega t + \Omega r/c).$$

Inclined rotator – MHD

- No monopole Michel-Bogovalov poloidal field
- No magneto-dipole radiation
- Larger energy losses for orthogonal rotator

$$W_{\rm tot}^{\rm (MHD)} \approx \frac{1}{4}\frac{B_0^2\Omega^4R^6}{c^2}(1+\sin^2\chi)$$

- Alignment: inclination angle evolves to 0 deg.

Problem 5.2. Show that the relation similar to (5.24) can be obtained for the conical solutions $\Psi = \Psi(\theta)$, but only at large distances $r \gg R_{\rm L}$ from the compact object. It has the form [Ingraham, 1973, Michel, 1974]

$$4\pi I(\theta) = \Omega_{\rm F}(\theta)\sin\theta\,\frac{{\rm d}\Psi}{{\rm d}\theta}. \qquad (5.25)$$

$E_\theta = B_\varphi$

S.Gralla,T.Jacobson, G.Menon, C.Dermer ($B_{\rm p} = 0$)

Asymtotic solution for orthogonal wind

$$
\begin{aligned}
B_r &\approx B_0 \frac{R^2}{r^2} \sin\theta \cos(\varphi - \Omega t + \Omega r/c), \\
B_\varphi &= E_\theta \approx -B_0 \frac{\Omega R^2}{cr} \sin^2\theta \, \cos(\varphi - \Omega t + \Omega r/c).
\end{aligned}
$$

Radial outflow

No current sheet

Asymtotic solution for orthogonal wind

$$B_r \approx B_0 \frac{R^2}{r^2} \sin\theta \cos(\varphi - \Omega t + \Omega r/c),$$

$$B_\varphi = E_\theta \approx -B_0 \frac{\Omega R^2}{cr} \sin^2\theta \, \cos(\varphi - \Omega t + \Omega r/c).$$

Generalization

$\psi(\theta, \varphi - \Omega t + \Omega r/c)$

$$\begin{cases} B_r \approx B_\mathrm{L} \dfrac{R_\mathrm{L}^2}{r^2} \sin\theta \cos\left(\varphi - \Omega t + \dfrac{\Omega r}{c} + \varphi_0\right), \\ B_\theta \approx \dfrac{1}{r\sin\theta} \dfrac{\partial\psi}{\partial\varphi}, \\ B_\varphi \approx -B_\mathrm{L} \dfrac{\Omega R_\mathrm{L}^2}{cr} \sin^2\theta \, \cos\left(\varphi - \Omega t + \dfrac{\Omega r}{c} + \varphi_0\right) - \dfrac{1}{r} \dfrac{\partial\psi}{\partial\theta}, \\ E_r \approx 0, \\ E_\theta \approx -B_\mathrm{L} \dfrac{\Omega R_\mathrm{L}^2}{cr} \sin^2\theta \, \cos\left(\varphi - \Omega t + \dfrac{\Omega r}{c} + \varphi_0\right) - \dfrac{1}{r} \dfrac{\partial\psi}{\partial\theta}, \\ E_\varphi \approx -\dfrac{1}{r\sin\theta} \dfrac{\partial\psi}{\partial\varphi}. \end{cases}$$

What is the current system?

Current losses

Direct current losses?

$$K_{||} = -c_{||}\frac{B_0^2\Omega^3 R^6}{c^3} i_{\rm s},$$

$$K_{\perp} = -c_{\perp}\frac{B_0^2\Omega^3 R^6}{c^3}\left(\frac{\Omega R}{c}\right) i_{\rm a}.$$

Pulsar evolution: direst current losses?

$$I_{\rm r}\,\dot\Omega = K_{\parallel}\cos\chi + K_{\perp}\sin\chi,$$
$$I_{\rm r}\Omega\,\dot\chi = K_{\perp}\cos\chi - K_{\parallel}\sin\chi,$$

$$K_{\parallel} = -c_{\parallel}\frac{B_0^2\Omega^3R^6}{c^3}i_{\rm s},$$
$$K_{\perp} = -c_{\perp}\frac{B_0^2\Omega^3R^6}{c^3}\left(\frac{\Omega R}{c}\right)i_{\rm a}.$$

$$K_{\perp}^{A} \approx \left(\frac{\Omega R}{c}\right)K_{\parallel}^{A}$$

$$i_{\rm s} \approx i_{\rm a} \approx 1$$

VB, A.V.Gurevich, Ya.N.Istomin,
JETP **58**, 235 (1983)

Pulsar evolution: direst current losses?

$$I_{\rm r}\,\dot\Omega = K_{\|}\cos\chi + K_{\perp}\sin\chi,$$
$$I_{\rm r}\Omega\,\dot\chi = K_{\perp}\cos\chi - K_{\|}\sin\chi,$$

$$I_{\rm r}\dot\Omega = K_{\|}^{A} + [K_{\perp}^{A} - K_{\|}^{A}]\sin^2\chi,$$
$$I_{\rm r}\Omega\dot\chi = [K_{\perp}^{A} - K_{\|}^{A}]\sin\chi\cos\chi.$$

$$K_{\|} = -c_{\|}\frac{B_0^2\Omega^3R^6}{c^3}i_{\rm s},$$
$$K_{\perp} = -c_{\perp}\frac{B_0^2\Omega^3R^6}{c^3}\left(\frac{\Omega R}{c}\right)i_{\rm a}.$$

$$i_{\rm s} = i_{\rm s}^{A}\cos\chi,$$
$$i_{\rm a} = i_{\rm a}^{A}\sin\chi.$$

$$K_{\perp}^{A} \approx \left(\frac{\Omega R}{c}\right)K_{\|}^{A}$$

$$i_{\rm s} \approx i_{\rm a} \approx 1$$

$$i_{\rm A} \sim (\Omega R/c)^{-1}$$

BGI

Princeton (MHD)

Pulsar evolution: direst current losses?

$$
\begin{aligned}
I_{\rm r}\,\dot{\Omega} &= K_{\|}\cos\chi + K_{\perp}\sin\chi,\\
I_{\rm r}\Omega\,\dot{\chi} &= K_{\perp}\cos\chi - K_{\|}\sin\chi,
\end{aligned}
$$

$$
\begin{aligned}
K_{\|} &= -c_{\|}\frac{B_0^2\Omega^3R^6}{c^3}i_{\rm s},\\
K_{\perp} &= -c_{\perp}\frac{B_0^2\Omega^3R^6}{c^3}\left(\frac{\Omega R}{c}\right)i_{\rm a}.
\end{aligned}
$$

$$K_{\perp}^{A} \approx \left(\frac{\Omega R}{c}\right)K_{\|}^{A}$$

$$i_{\rm s} \approx i_{\rm a} \approx 1$$

BGI

$$
\begin{aligned}
I_{\rm r}\dot{\Omega} &= K_{\|}^{A} + [K_{\perp}^{A} - K_{\|}^{A}]\sin^2\chi,\\
I_{\rm r}\Omega\dot{\chi} &= [K_{\perp}^{A} - K_{\|}^{A}]\sin\chi\cos\chi.
\end{aligned}
$$

$$
\begin{aligned}
i_{\rm s} &= i_{\rm s}^{A}\cos\chi,\\
i_{\rm a} &= i_{\rm a}^{A}\sin\chi.
\end{aligned}
$$

$$i_{\rm A} \sim (\Omega R/c)^{-1}$$

?

Princeton (MHD)

How to write down the current

Drift approximation

$$\mathbf{j} = c\,\rho_{\rm e}\frac{[\mathbf{E}\times\mathbf{B}]}{B^2} + a\,\mathbf{B}$$

$$\boxed{\mathbf{j} = \rho_{\rm e}\,[\mathbf{\Omega}\times\mathbf{r}] + i_{\|}\,\mathbf{B}}$$

$$\boxed{\mathbf{j} = \frac{(\mathbf{B}\cdot\nabla\times\mathbf{B} - \mathbf{E}\cdot\nabla\times\mathbf{E})\mathbf{B} + (\nabla\cdot\mathbf{E})\mathbf{E}\times\mathbf{B}}{B^2}}$$

$$(\nabla i_{\|}\,\mathbf{B}) = 0$$

Mestel, BGI

Gruzinov

$$i_{\rm a} \sim \left(\frac{\Omega R}{c}\right)^{-1/2}$$

No point 1

$$B_r \approx B_0 \frac{R^2}{r^2} \sin\theta \cos(\varphi - \Omega t + \Omega r/c),$$

$$B_\varphi = E_\theta \approx -B_0 \frac{\Omega R^2}{cr} \sin^2\theta \, \cos(\varphi - \Omega t + \Omega r/c).$$

In the wind

$$i_{\|} = -3 \frac{\Omega}{c} \cos\theta$$

Polar cap

$$i_{\rm a}^{\rm A} \approx f_*^{-1/2} \left(\frac{\Omega R}{c}\right)^{-1/2}$$

Current is too small!

What is the current system?

Current losses

1. Direct current losses

$$K_{\parallel} = -c_{\parallel} \frac{B_0^2 \Omega^3 R^6}{c^3} i_{\rm s},$$

$$K_{\perp} = -c_{\perp} \frac{B_0^2 \Omega^3 R^6}{c^3} \left(\frac{\Omega R}{c}\right) i_{\rm a}.$$

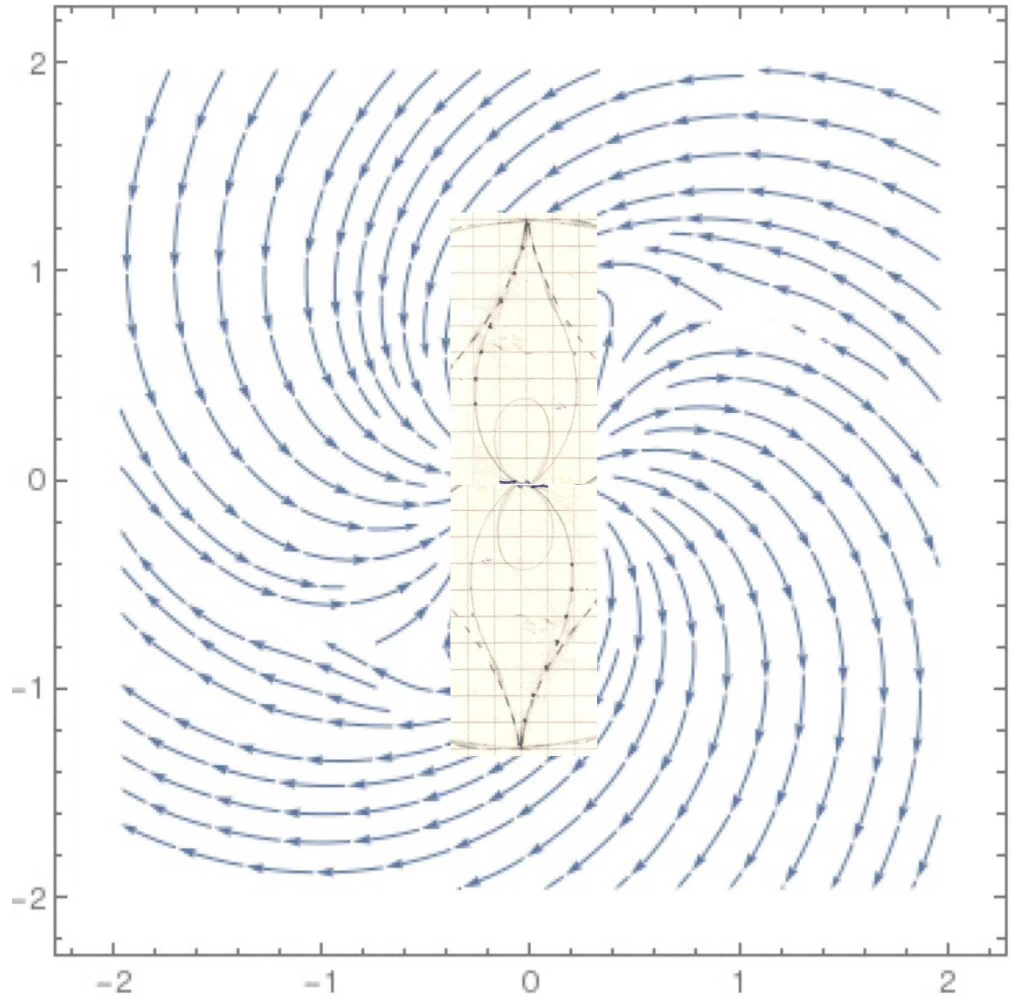
2. Mismatch
(‘second term’)

$$\mathbf{K} = \frac{R^3}{c} \int \mathbf{J}_{\rm s} \left(\mathbf{B}\mathbf{n}\right) {\rm d}o = \frac{R^3}{4\pi} \int \{[\mathbf{n} \times \mathbf{B}^{(3)}](\mathbf{B}^{(0)}\mathbf{n}) + [\mathbf{n} \times \mathbf{B}^{(0)}](\mathbf{B}^{(3)}\mathbf{n})\} \, {\rm d}o$$

3. Additional separatrix current

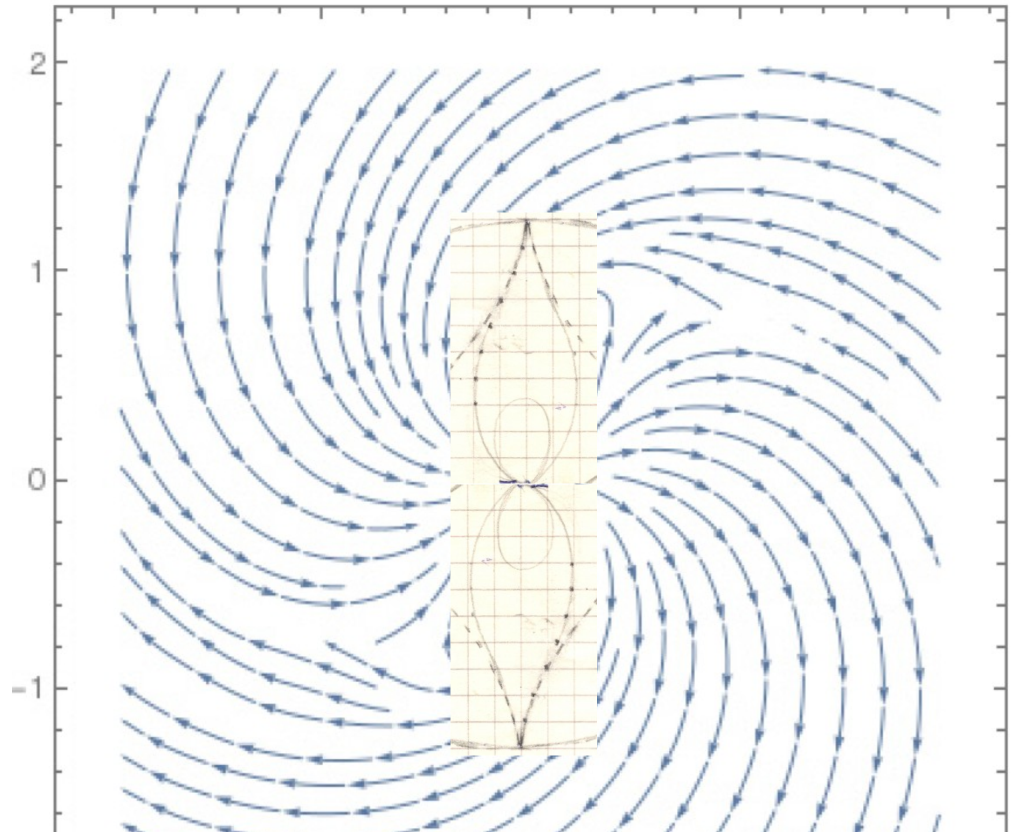
Point 2?

Mismatch



Point 2?

"Second term"
(all NS surface works)



$$\mathbf{K} = \frac{R^3}{c} \int \mathbf{J}_{\mathrm{s}} \left(\mathbf{B} \mathbf{n} \right) \mathrm{d}o = \frac{R^3}{4\pi} \int \{ [\mathbf{n} \times \mathbf{B}^{(3)}] (\mathbf{B}^{(0)} \mathbf{n}) + [\mathbf{n} \times \mathbf{B}^{(0)}] (\mathbf{B}^{(3)} \mathbf{n}) \} \, \mathrm{d}o$$

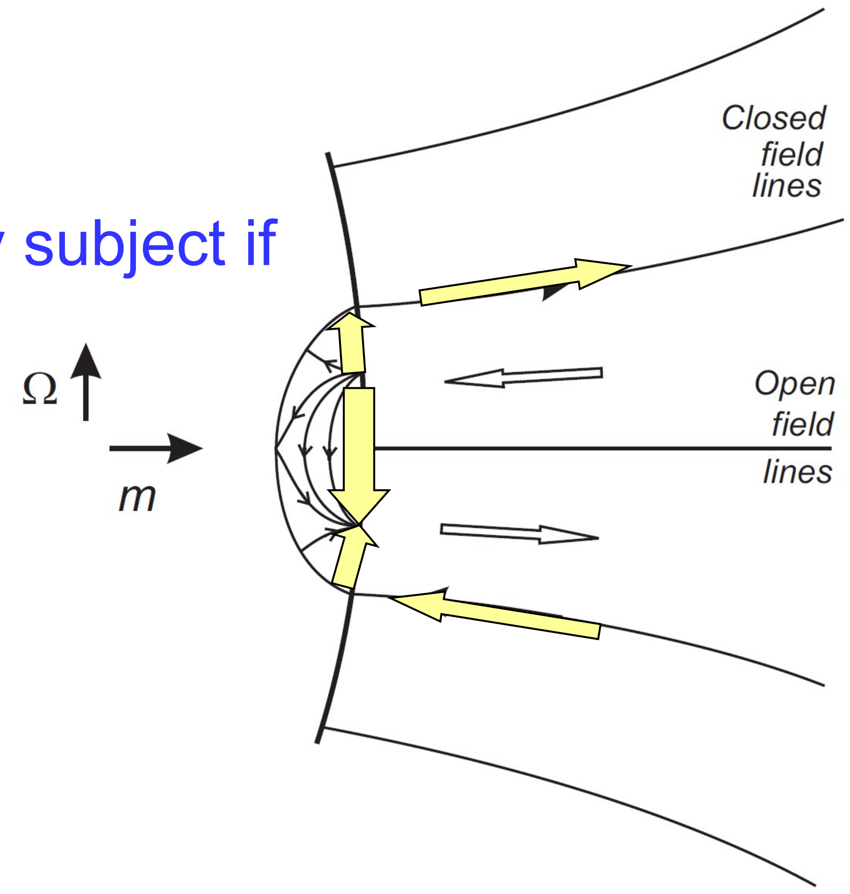
Point 3?

VB, E.E.Nokhrina. Astron. Letters, **30**, 685 (2004)

$$W_{\rm tot} = \frac{\Omega R^3}{c} \int J_\theta B_n {\rm d}o$$

Direct current losses are the only subject if

- No longitudinal currents
  in close magnetosphere.
- No additional current
  along the separatrix.



$$I_{\rm sep} = 3/4\, I_{\rm vol}$$

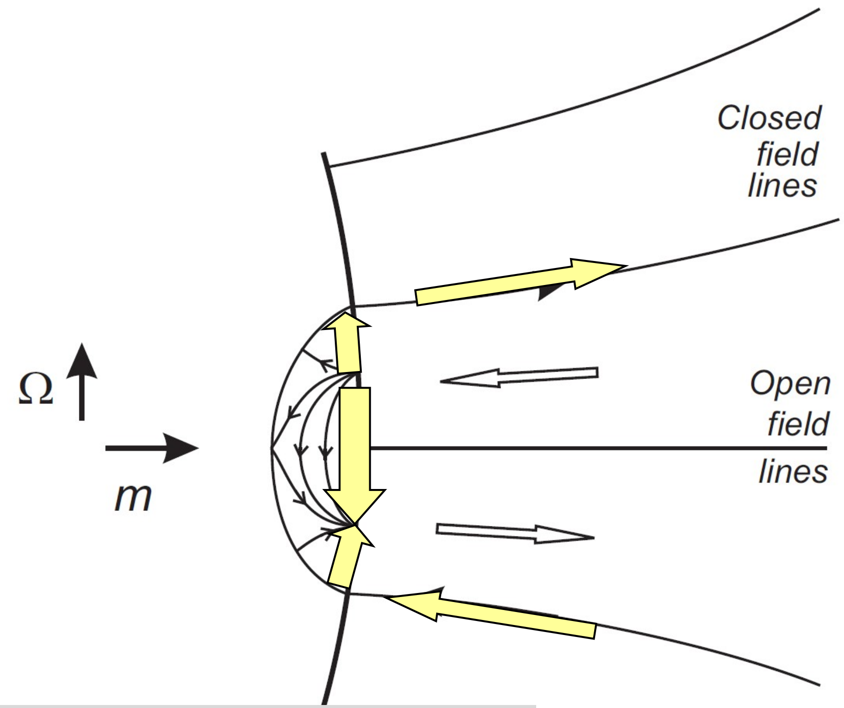
$$< J_\theta > = 0$$

$$< B_t > = 0$$

Point 3?

VB, MHD Flows in Compact Astrophysical Objects, Springer (2010)

$$W_{\rm tot} = \frac{\Omega R^3}{c} \int J_\theta B_n {\rm d}o$$



$$I_{\rm sep} = 3/4\, I_{\rm vol}$$

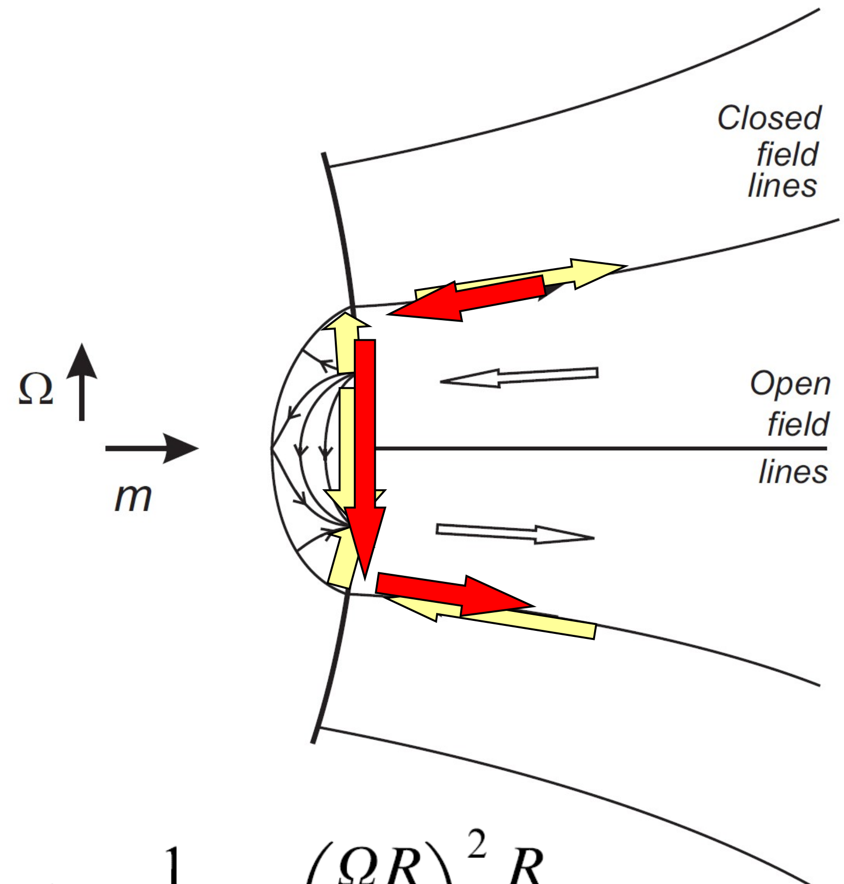
Problem 2.16. Show that in this case the total current $I_{\rm sep}$ flowing along the separatrix is 3/4 the total bulk current $I_{\rm bulk}$ flowing in the region of the open field lines:

$$\frac{I_{\rm sep}}{I_{\rm bulk}} = -\frac{3}{4}. \qquad (2.160)$$

Point 3?

$$W_{\rm tot} = \frac{\Omega R^3}{c}\int J_\theta B_n {\rm d}o$$

Additional current along the separatrix.



$$I_{\rm sep} < 3/4\, I_{\rm vol}$$

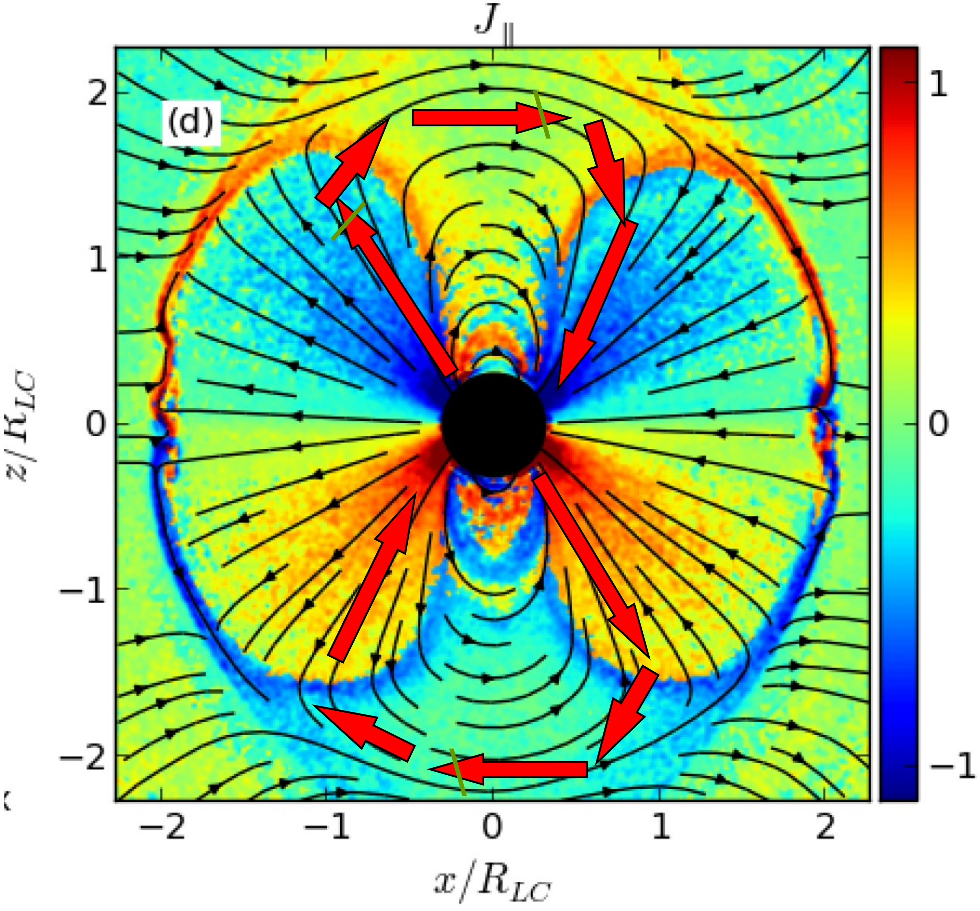
$$< J_\theta > \neq 0$$

$$\langle B_\varphi \rangle = \frac{1}{f_*} B_0 \left(\frac{\Omega R}{c}\right)^2 \frac{R}{r}$$

Point 3?

$$W_{\rm tot} = \frac{\Omega R^3}{c} \int J_\theta B_n {\rm d}o$$

Direction corresponds to energy losses.

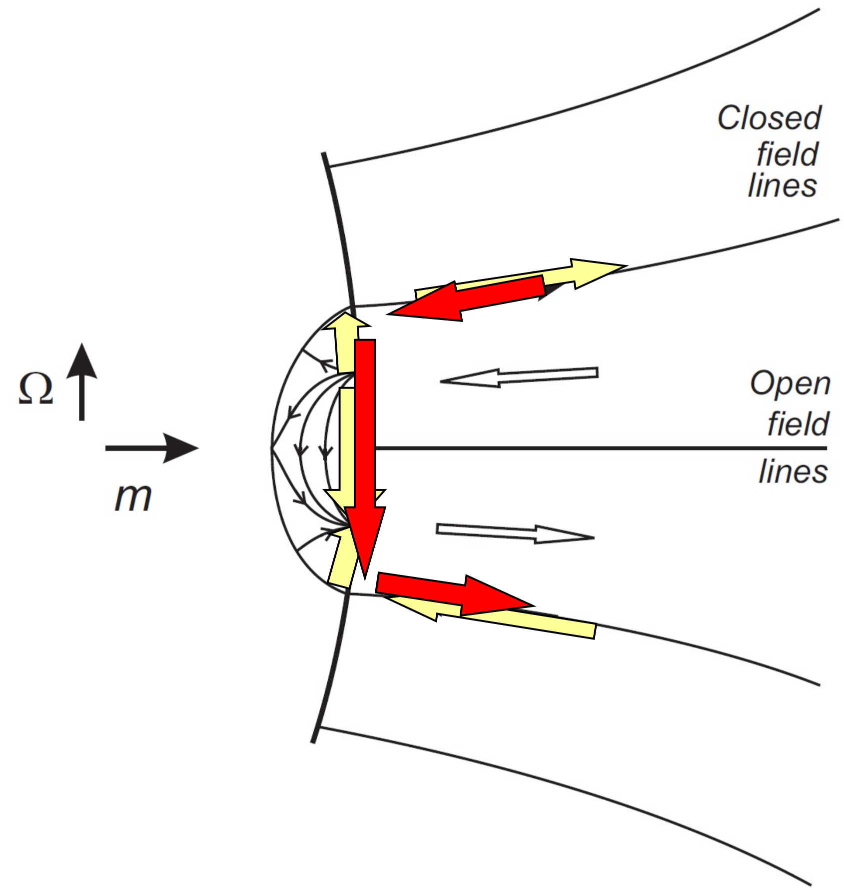


Point 3?

X.-N. Bai, A.Spitkovsky ApJ, **715**, 1282 (2010)

$$I_{\mathrm{sep}} = 20\%\ I_{\mathrm{vol}}$$

$$I_{\mathrm{sep}} = 3/4\ I_{\mathrm{vol}}$$



How to check?

Current losses

1. Direct current losses

$$K_{\|} = -c_{\|} \frac{B_0^2 \Omega^3 R^6}{c^3} i_{\mathrm{s}},$$

$$K_{\perp} = -c_{\perp} \frac{B_0^2 \Omega^3 R^6}{c^3} \left(\frac{\Omega R}{c} \right) i_{\mathrm{a}}.$$

2. Mismatch
('second term')

$$\mathbf{K} = \frac{R^3}{c} \int \mathbf{J}_{\mathrm{s}} \left(\mathbf{B} \mathbf{n} \right) \mathrm{d}o = \frac{R^3}{4\pi} \int \{ [\mathbf{n} \times \mathbf{B}^{(3)}](\mathbf{B}^{(0)} \mathbf{n}) + [\mathbf{n} \times \mathbf{B}^{(0)}](\mathbf{B}^{(3)} \mathbf{n}) \} \, \mathrm{d}o$$

3. Additional separatrix current

How to check?

Current losses

1. Direct current losses

$$K_{\parallel} = -c_{\parallel} \frac{B_0^2 \Omega^3 R^6}{c^3} i_{\mathrm{s}},$$

$$K_{\perp} = -c_{\perp} \frac{B_0^2 \Omega^3 R^6}{c^3} \left(\frac{\Omega R}{c}\right) i_{\mathrm{a}}.$$

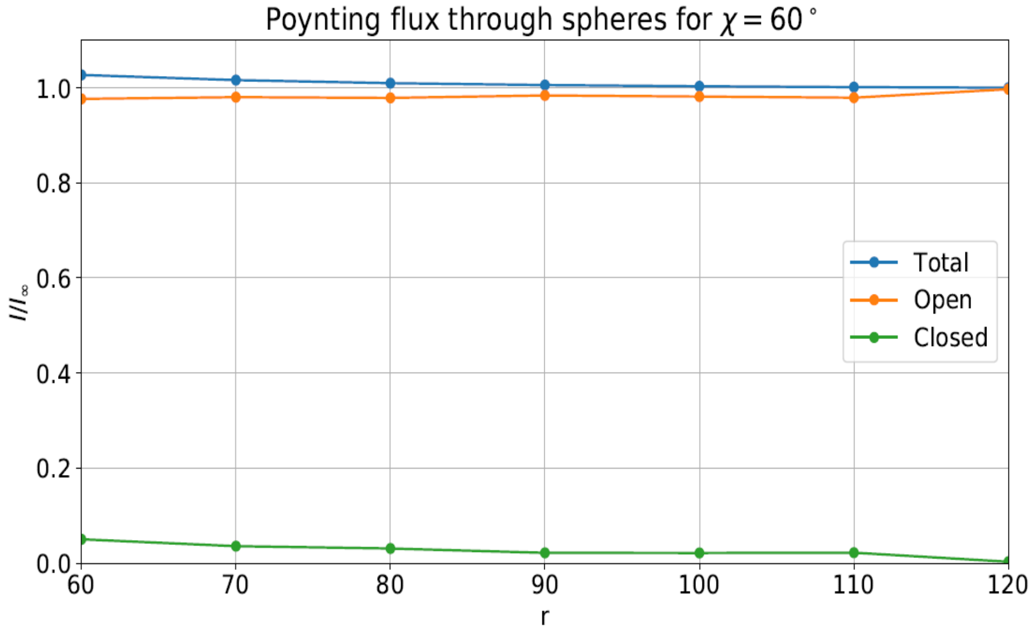
2. Mismatch
('second term') ALL SURFACE WORKS

$$\mathbf{K} = \frac{R^3}{c} \int \mathbf{J}_{\mathrm{s}} \left(\mathbf{B}\mathbf{n}\right) \mathrm{d}o = \frac{R^3}{4\pi} \int \{[\mathbf{n} \times \mathbf{B}^{(3)}](\mathbf{B}^{(0)}\mathbf{n}) + [\mathbf{n} \times \mathbf{B}^{(0)}](\mathbf{B}^{(3)}\mathbf{n})\} \, \mathrm{d}o$$

3. Additional separatrix current
POLAR CAP ONLY

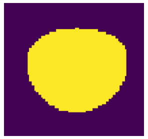
Direct check

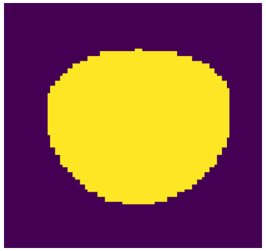
VB, A.K.Galishnikova, E.M.Novoselov, A.A.Philippov, M.M.Rashkovetskyi JPhys: Conf. Series, **932**, 012012 (2017)



R = 50

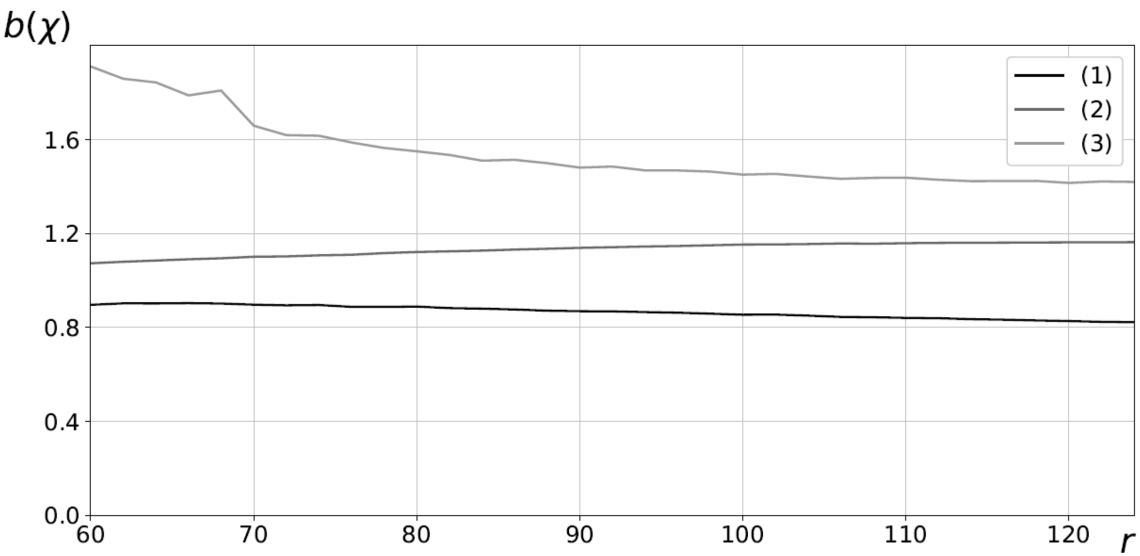
R_L = 500





Direct check

M.M.Rashkovetskyi, VB, A.K.Galishnikova, E.M.Novoselov, A.A.Philippov (2018)



$$< B_\varphi >= b(\chi)\frac{1}{f_*}\,B_0\left(\frac{\Omega R}{c}\right)^2\frac{R}{r}$$

$$b(\chi) = \left[\frac{k_1+k_2}{2} - \frac{f_*^{5/2}}{32}\left(\frac{\Omega R}{c}\right)^{1/2}\right]\sin\chi$$

$$W_{\rm tot}^{\rm MHD} \approx \frac{1}{4}\,\frac{B_0^2\Omega^4R^6}{c^3}(k_1+k_2\,\sin^2\chi)$$

χ	30°	60°	90°
$b(\chi)$ (num)	0.8	1.2	1.4
$b(\chi)$ (anal)	0.6 ± 0.1	1.0 ± 0.1	1.2 ± 0.1

Direct check

M.M.Rashkovetskyi, VB, A.K.Galishnikova, E.M.Novoselov, A.A.Philippov (2018)

$$\frac{I_{\rm sep}}{I_{\rm vol}} = \frac{3}{4} - \frac{2}{f_*^{3/2}} \left(\frac{\Omega R}{c}\right)^{1/2}$$

$$I_{\rm sep} \sim 0.3\, I_{\rm vol}$$

X.-N. Bai, A.Spitkovsky ApJ **715**, 1282 (2010)

$$\boxed{I_{\rm sep} = 20\%\, I_{\rm vol}}$$

Fortunately for us

$$\begin{aligned} I_{\rm r}\dot{\Omega} &= K_{||}^{A} + [K_{\perp}^{A} - K_{||}^{A}]\sin^2\chi, \\ I_{\rm r}\Omega\dot{\chi} &= [K_{\perp}^{A} - K_{||}^{A}]\sin\chi\cos\chi. \end{aligned}$$

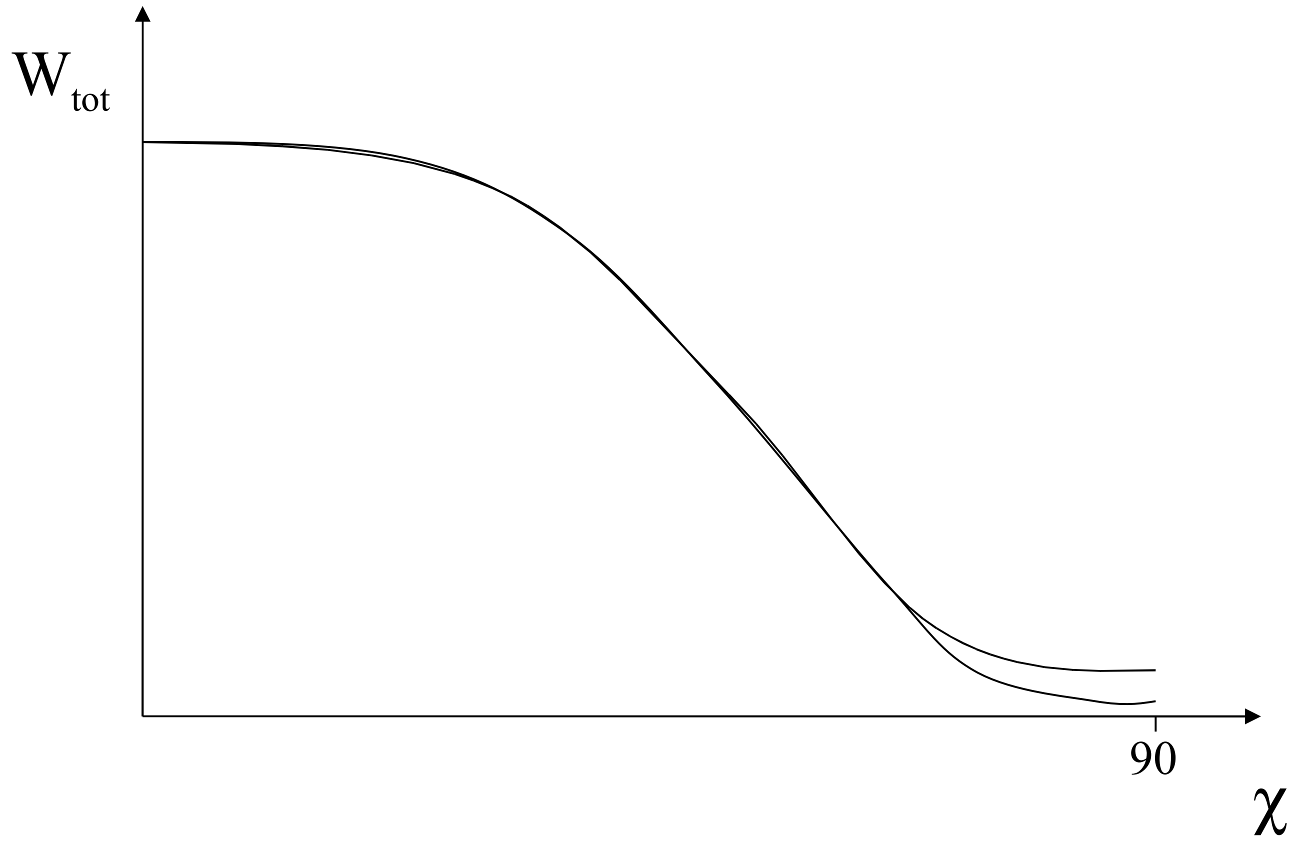
$$K_{\perp}^{\rm mag} = -A\,\frac{B_0^2\Omega^3 R^6}{c^3}\,i_{\rm a}$$

$$\boxed{A \approx 2\left(\frac{\Omega R}{c}\right)^{1/2}}$$

One can neglect additional losses for GJ current

BGI correction

Some difference for orthogonal pulsars only



Conclusion

THANKS AGAIN!