So How Do Radio Pulsars Slow-Down?

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with

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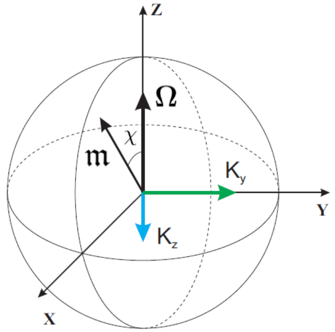
under the supervision of A.Philippov

STEP #I

Vacuum: magneto-dipole radiation

Vacuum: magneto-dipole losses



$$W_{\text{tot}} = -\mathbf{\Omega K}$$

$$\mathbf{K} = \frac{1}{c}\int \left[\mathbf{r} \times \left[\mathbf{J}_{\text{s}} \times \mathbf{B}\right]\right] \mathrm{d}S$$

$$K_{z'} = \frac{2}{3}\frac{\mathfrak{m}^2}{R^3}\left(\frac{\Omega R}{c}\right)^3 \sin^2\chi$$

$$K_{x'} = \frac{2}{3}\frac{\mathfrak{m}^2}{R^3}\left(\frac{\Omega R}{c}\right)^3 \sin\chi\cos\chi$$

Vacuum: magneto-dipole

Energy losses $$W_{\rm tot} = -\mathbf{\Omega}\mathbf{K}$$

$$K_{z'} = \frac{2}{3}\frac{\mathfrak{m}^2}{R^3}\left(\frac{\Omega R}{c}\right)^3 \sin^2\chi$$

$$\varepsilon = \frac{\Omega R}{c} \qquad \beta_{\rm R} = \frac{\mathbf{\Omega}\times\mathbf{r}}{c} \qquad K_{x'} = \frac{2}{3}\frac{\mathfrak{m}^2}{R^3}\left(\frac{\Omega R}{c}\right)^3 \sin\chi\cos\chi$$

$$\mathbf{K} = \frac{R^3}{c}\int \mathbf{J}_{\rm s}\,(\mathbf{B}\mathbf{n})\,{\rm d}o = \frac{R^3}{4\pi}\int \{[\mathbf{n}\times\mathbf{B}^{(3)}](\mathbf{B}^{(0)}\mathbf{n}) + [\mathbf{n}\times\mathbf{B}^{(0)}](\mathbf{B}^{(3)}\mathbf{n})\}\,{\rm d}o$$

$$W_{\rm tot} = \frac{c}{4\pi}\int(\beta_{\rm R}\mathbf{B})(\mathbf{B}{\rm d}\mathbf{S})$$

Vacuum: magneto-dipole

Energy losses

$$\beta_{\rm R} = \frac{\mathbf{\Omega} \times \mathbf{r}}{c}$$

$$W_{\rm tot} = \frac{c}{4\pi} \int (\beta_{\rm R} \mathbf{B})(\mathbf{B} {\rm d}\mathbf{S})$$

Vacuum (Deusch) 1 Ω Ω^3 1 1 $= \Omega^4$

$$\mathbf{K} = \frac{R^3}{c} \int \mathbf{J}_{\rm s} \, (\mathbf{B}\mathbf{n}) \, {\rm d}o = \frac{R^3}{4\pi} \int \{[\mathbf{n} \times \mathbf{B}^{(3)}](\mathbf{B}^{(0)}\mathbf{n}) + [\mathbf{n} \times \mathbf{B}^{(0)}](\mathbf{B}^{(3)}\mathbf{n})\} \, {\rm d}o$$

Landau-Lifshits, Field Theory

$$
\begin{aligned}
B_r^\perp &= \frac{|\mathfrak{m}|}{r^3}\sin\theta\,\mathrm{Re}\left(2-2i\frac{\Omega r}{c}\right)\exp\left(i\frac{\Omega r}{c}+i\varphi-i\Omega t\right),\\
B_\theta^\perp &= \frac{|\mathfrak{m}|}{r^3}\cos\theta\,\mathrm{Re}\left(-1+i\frac{\Omega r}{c}+\frac{\Omega^2 r^2}{c^2}\right)\exp\left(i\frac{\Omega r}{c}+i\varphi-i\Omega t\right),\\
B_\varphi^\perp &= \frac{|\mathfrak{m}|}{r^3}\,\mathrm{Re}\left(-i-\frac{\Omega r}{c}+i\frac{\Omega^2 r^2}{c^2}\right)\exp\left(i\frac{\Omega r}{c}+i\varphi-i\Omega t\right),\\
E_r^\perp &= 0,\\
E_\theta^\perp &= \frac{|\mathfrak{m}|\Omega}{r^2 c}\,\mathrm{Re}\left(-1+i\frac{\Omega r}{c}\right)\exp\left(i\frac{\Omega r}{c}+i\varphi-i\Omega t\right),\\
E_\varphi^\perp &= \frac{|\mathfrak{m}|\Omega}{r^2 c}\cos\theta\,\mathrm{Re}\left(-i-\frac{\Omega r}{c}\right)\exp\left(i\frac{\Omega r}{c}+i\varphi-i\Omega t\right).
\end{aligned}
$$

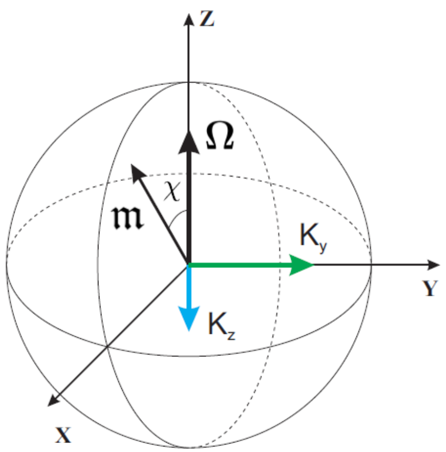
Vacuum: magneto-dipole

Energy losses

$$\beta_{\rm R} = \frac{\mathbf{\Omega} \times \mathbf{r}}{c}$$

$$W_{\rm tot} = \frac{c}{4\pi} \int (\beta_{\rm R}\mathbf{B})(\mathbf{B}d\mathbf{S})$$

Vacuum (Deusch)	1	Ω	Ω^3	1	1	$= \Omega^4$
Vacuum (L&L) (2/3)	1	Ω	Ω^3	1	1	$= \Omega^4$



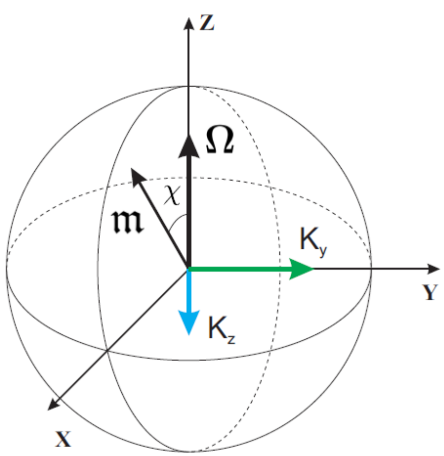
Vacuum: magneto-dipole

Energy losses

$$\beta_{\rm R} = \frac{\mathbf{\Omega} \times \mathbf{r}}{c}$$

$$W_{\rm tot} = \frac{c}{4\pi}\int(\beta_{\rm R}\mathbf{B})(\mathbf{B}d\mathbf{S})$$

Vacuum (Deusch)	1	Ω	Ω^3	1	1	$= \Omega^4$
Vacuum (L&L) (2/3)	1	Ω	Ω^3	1	1	$= \Omega^4$
Vacuum (L&L) (1/3)	1	Ω	1	Ω^3	1	$= \Omega^4$



$$\mathbf{B}^{(3)} = -\frac{2}{3}\frac{\mathfrak{m}}{R^3}\left(\frac{\Omega R}{c}\right)^3 \mathbf{e}_{y'}$$

IMPORTANT CONCLUSION

Two terms can play role in energy losses

$$\mathbf{K} = \frac{R^3}{c}\int \mathbf{J}_{\mathrm{s}}\,(\mathbf{B}\mathbf{n})\,\mathrm{d}o = \frac{R^3}{4\pi}\int \{[\mathbf{n}\times\mathbf{B}^{(3)}](\mathbf{B}^{(0)}\mathbf{n}) + [\mathbf{n}\times\mathbf{B}^{(0)}](\mathbf{B}^{(3)}\mathbf{n})\}\,\mathrm{d}o$$

STEP #II

Pulsar magnetosphere

Force-free approximation

One can neglect energy of particles

$$\frac{1}{c}\mathbf{j} \times \mathbf{B} + \rho_{\mathrm{e}}\mathbf{E} = 0$$

Mestel equation (1973)

$$\nabla \times \tilde{\mathbf{B}} = \psi \mathbf{B}$$

$$\tilde{\mathbf{B}} = \left\{ B_r\left(1 - \frac{\Omega^2 r^2}{c^2}\right), B_\theta, B_z\left(1 - \frac{\Omega^2 r^2}{c^2}\right) \right\}$$

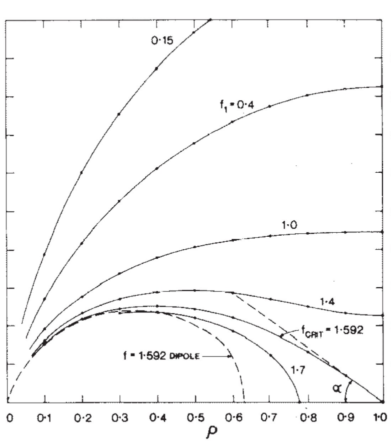
Pulsar equation

$$-\left(1 - \frac{\Omega_{\mathrm{F}}^2 \varpi^2}{c^2}\right)\nabla^2\Psi + \frac{2}{\varpi}\frac{\partial \Psi}{\partial \varpi} - \frac{16\pi^2}{c^2} I \frac{\mathrm{d}I}{\mathrm{d}\Psi} + \frac{\varpi^2}{c^2}(\nabla\Psi)^2\,\Omega_{\mathrm{F}}\frac{\mathrm{d}\Omega_{\mathrm{F}}}{\mathrm{d}\Psi} = 0$$

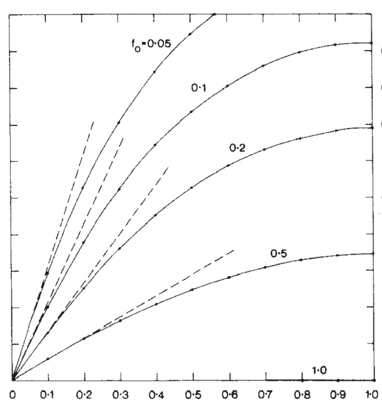
(Michel 1973, Mestel 1993, Scharlemann & Wagoner 1973,
Okamoto 1974, Mestel & Wang 1979)

First solutions

$$-\left(1-\frac{\Omega_{\rm F}^2\varpi^2}{c^2}\right)\nabla^2\Psi+\frac{2}{\varpi}\frac{\partial\Psi}{\partial\varpi}-\frac{16\pi^2}{c^2}I\frac{{\rm d}I}{{\rm d}\Psi}+\frac{\varpi^2}{c^2}\left(\nabla\Psi\right)^2\Omega_{\rm F}\frac{{\rm d}\Omega_{\rm F}}{{\rm d}\Psi}=0$$



F. Michel (1973)



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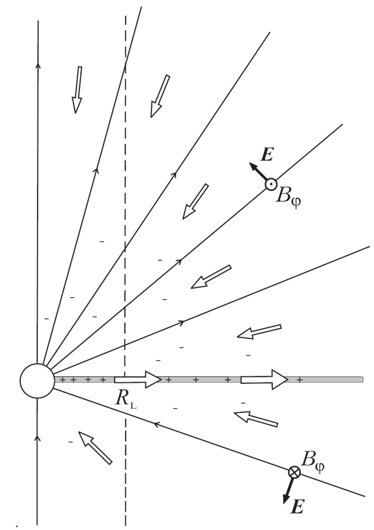
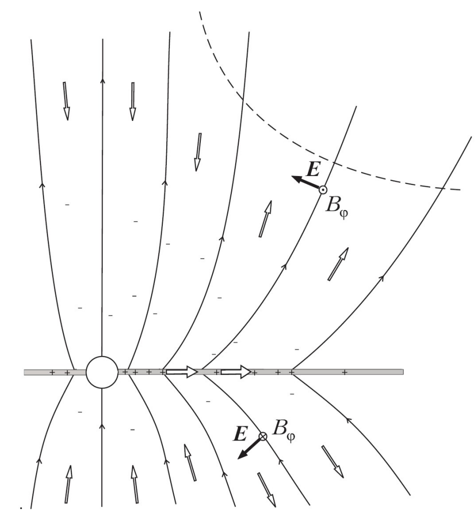


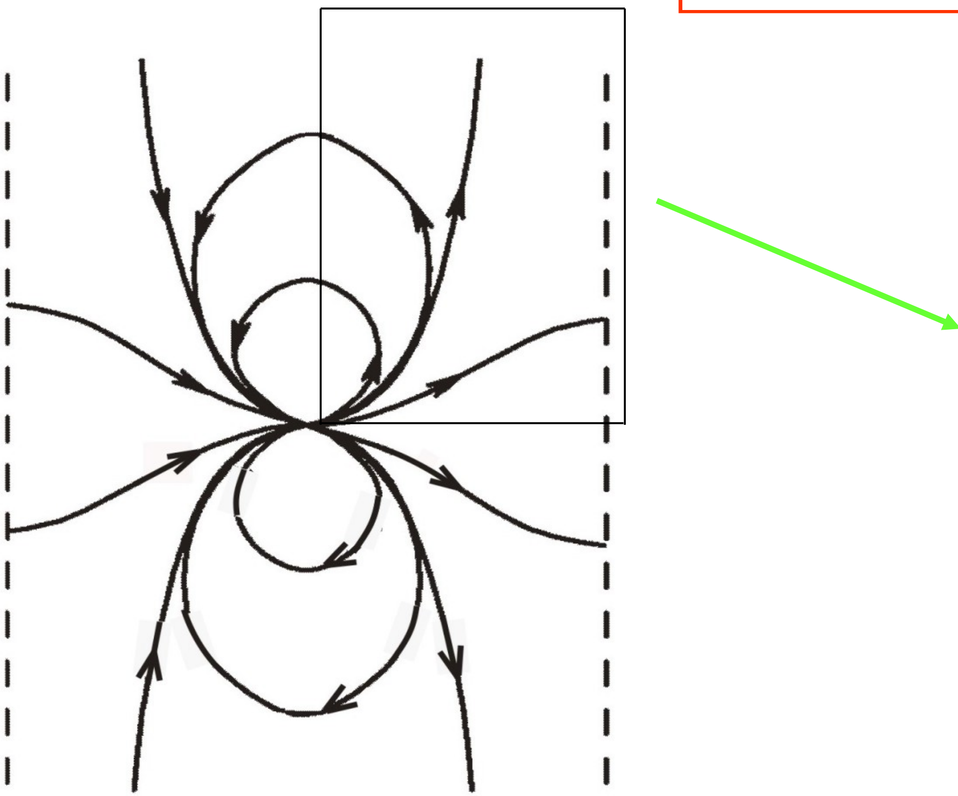
F. Michel (1973)



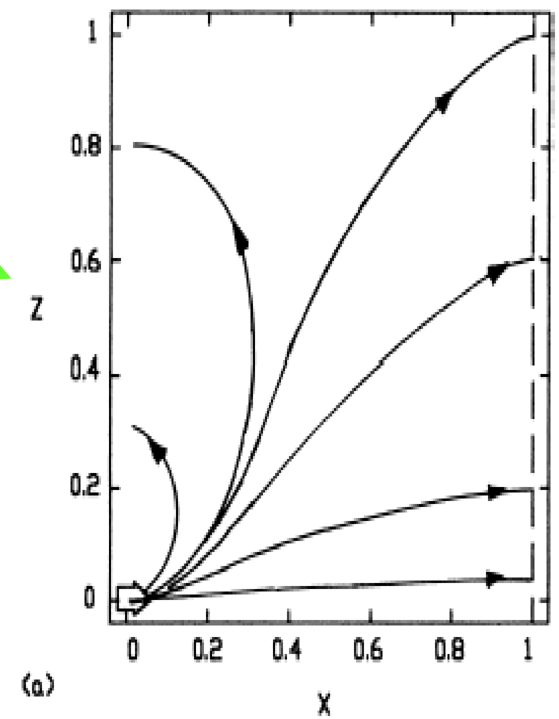
R.Blandford (1976)

Orthogonal Rotator

$$\nabla \times \tilde{\mathbf{B}} = 0$$



$\chi = 90^\circ$

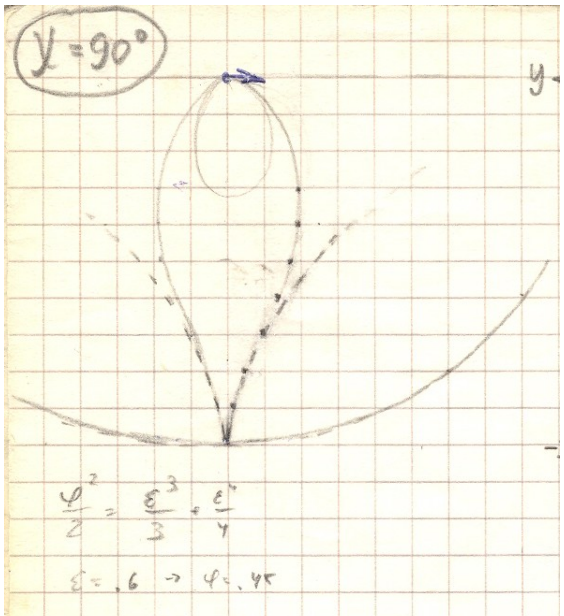


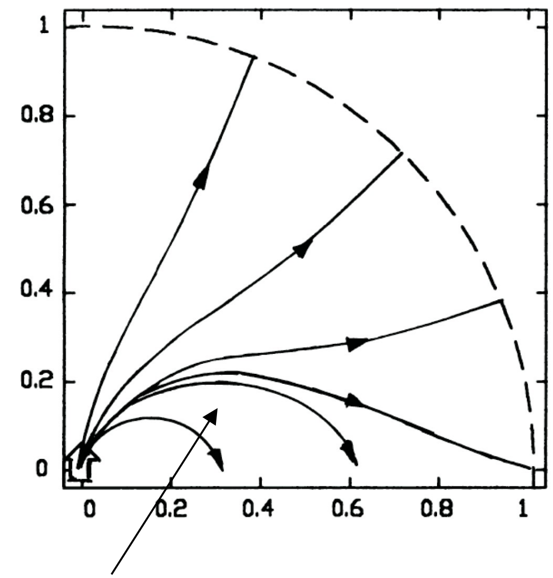
VB, A.V.Gurevich, Ya.N.Istomin,
Sov. Phys. JETP, **58**, 235 (1983)

L.Mestel, P.Panagi, S.Shibata,
MNRAS, **309**, 388 (1999)

Orthogonal Rotator

VB, A.V.Gurevich, Ya.N.Istomin, JETP, **58**, 235 (1983)
L.Mestel, P.Panagi, S.Shibata, MNRAS, **309**, 388 (1999)





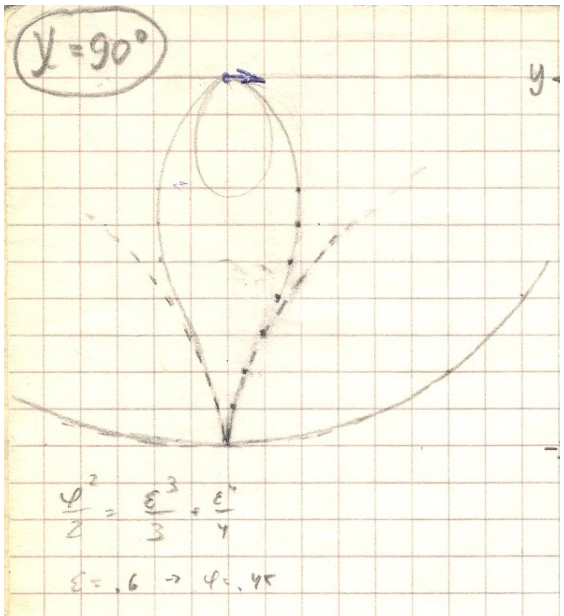
Equatorial plane

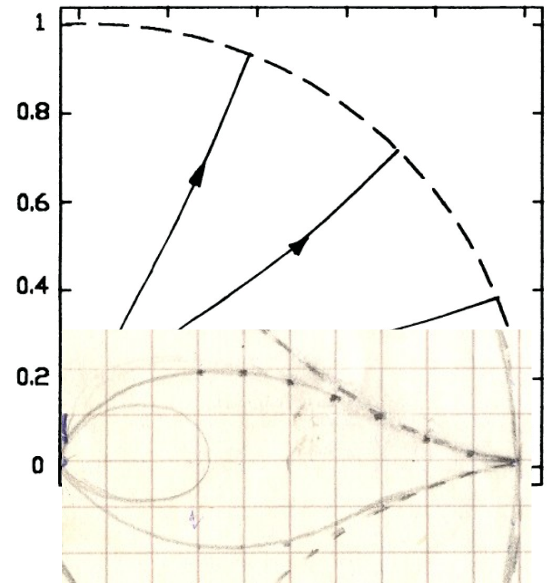
No energy flux through the light cylinder

$$B_\varphi \propto (1 - x_r^2)^2$$

Orthogonal Rotator

VB, A.V.Gurevich, Ya.N.Istomin, JETP, **58**, 235 (1983)
L.Mestel, P.Panagi, S.Shibata, MNRAS, **309**, 388 (1999)





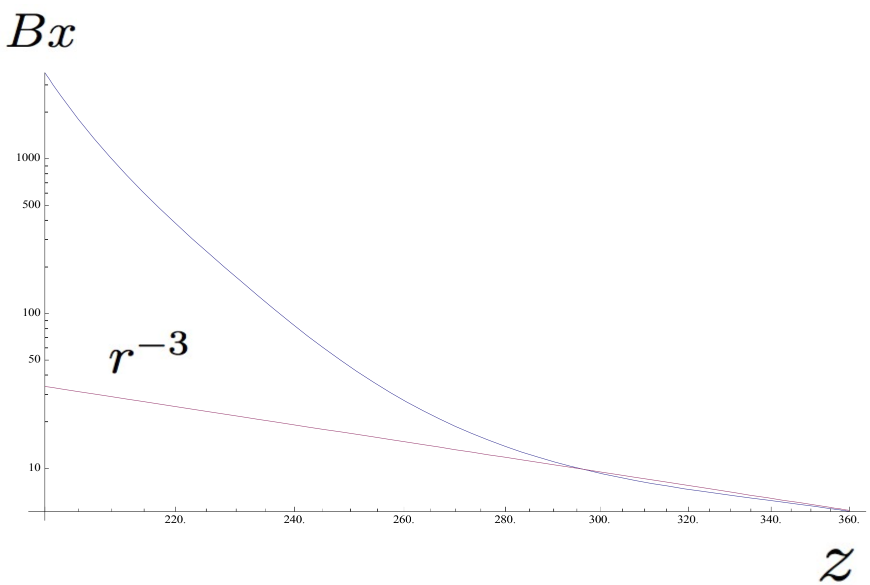
Equatorial plane

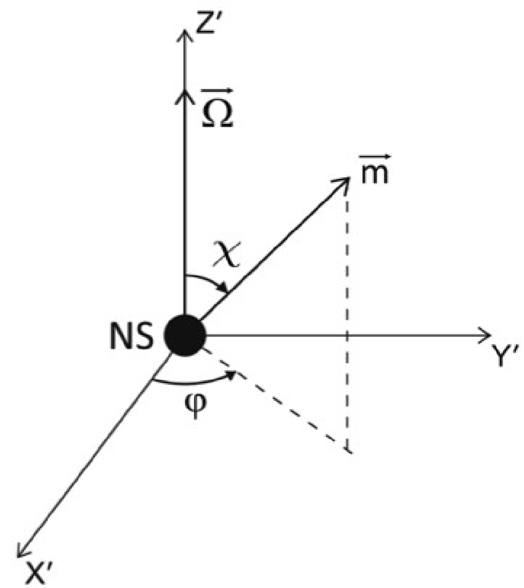
No energy flux through the light cylinder

$$B_\varphi \propto (1 - x_r^2)^2$$

Spitkovsky solution, $\chi = 60^{o}$

No magnetodipole radiation





In vacuum $B_x = \frac{\ddot{d}}{cr}$

VB, YA.N.Istomin, A.A.Philippov, Phys. Uspekhi, **56**, 164 (2013)

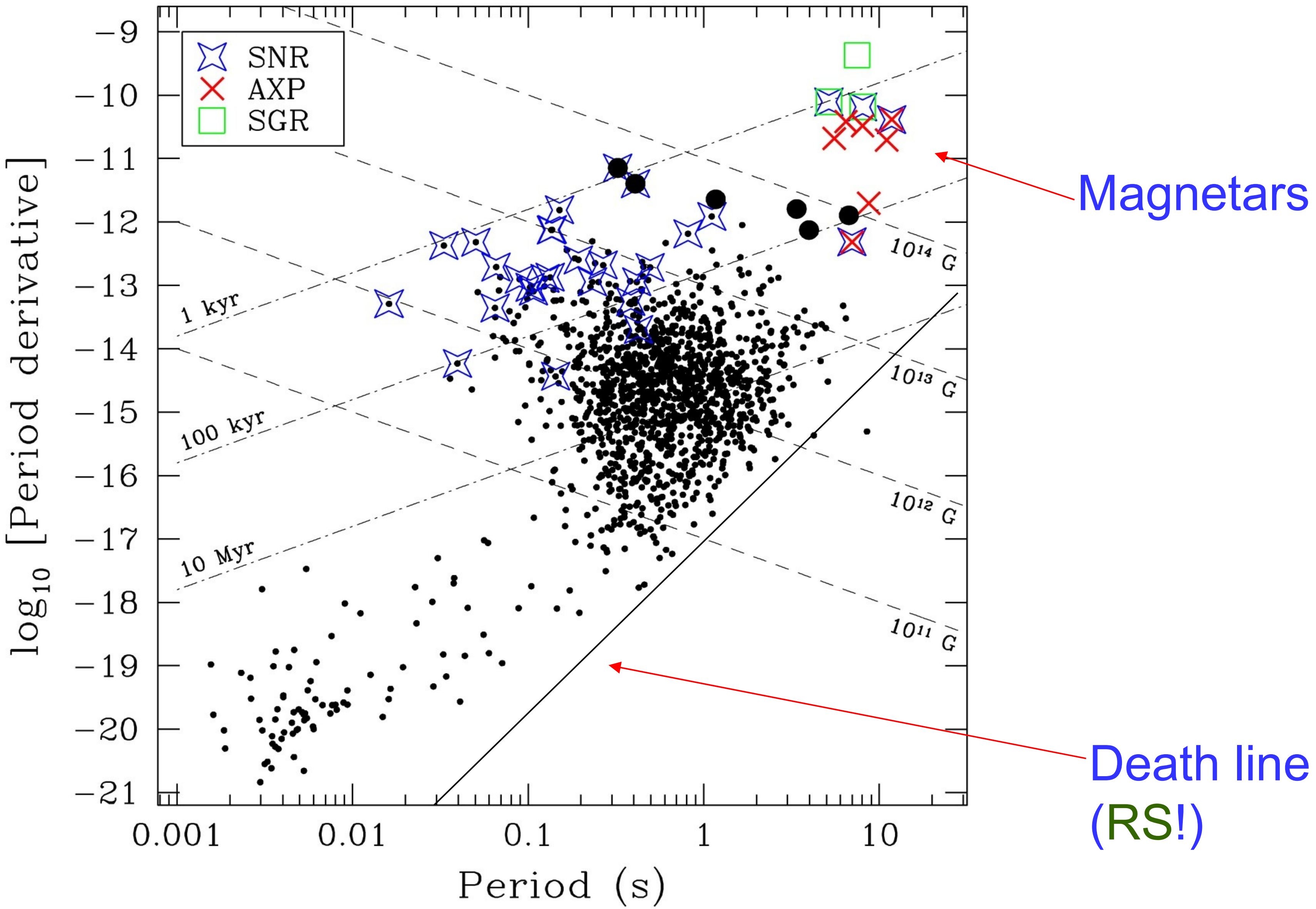
IMPORTANT CONCLUSION

No energy losses for zero longitudinal current

STEP #III

Current losses

PPdot – death line

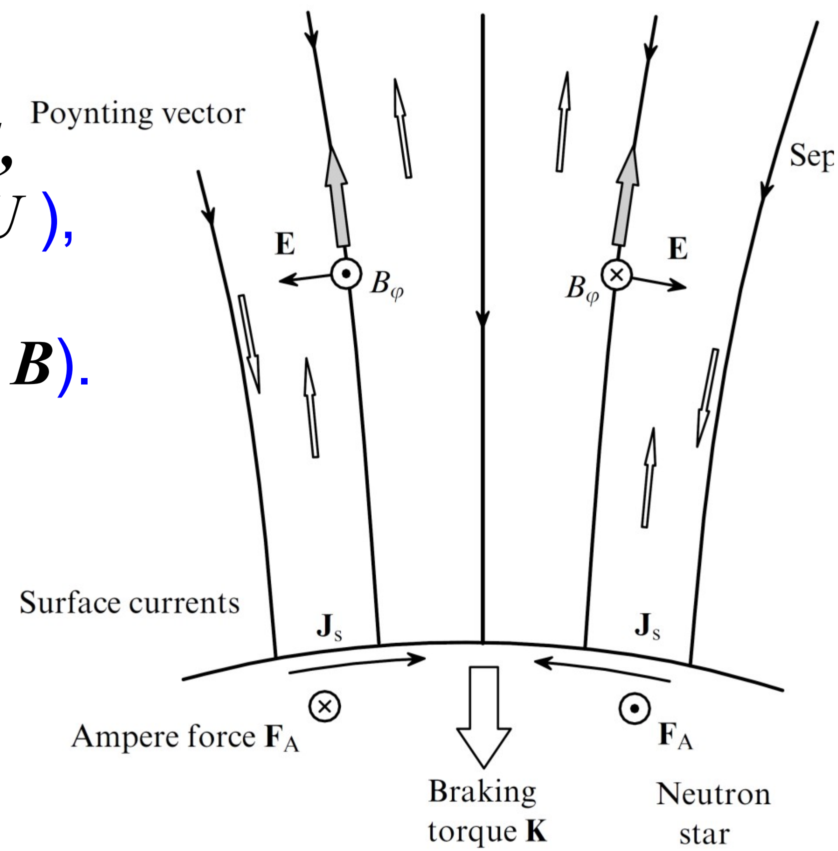


Current losses

For current losses mechanism is necessary to have

- Plasma in the magnetosphere,
- regular poloidal magnetic field,
- rotation (inductive electric field ***E***, EMF ΔU),
- longitudinal current I (toroidal magnetic field ***B***).

$$W_{\rm tot} = I\delta U$$



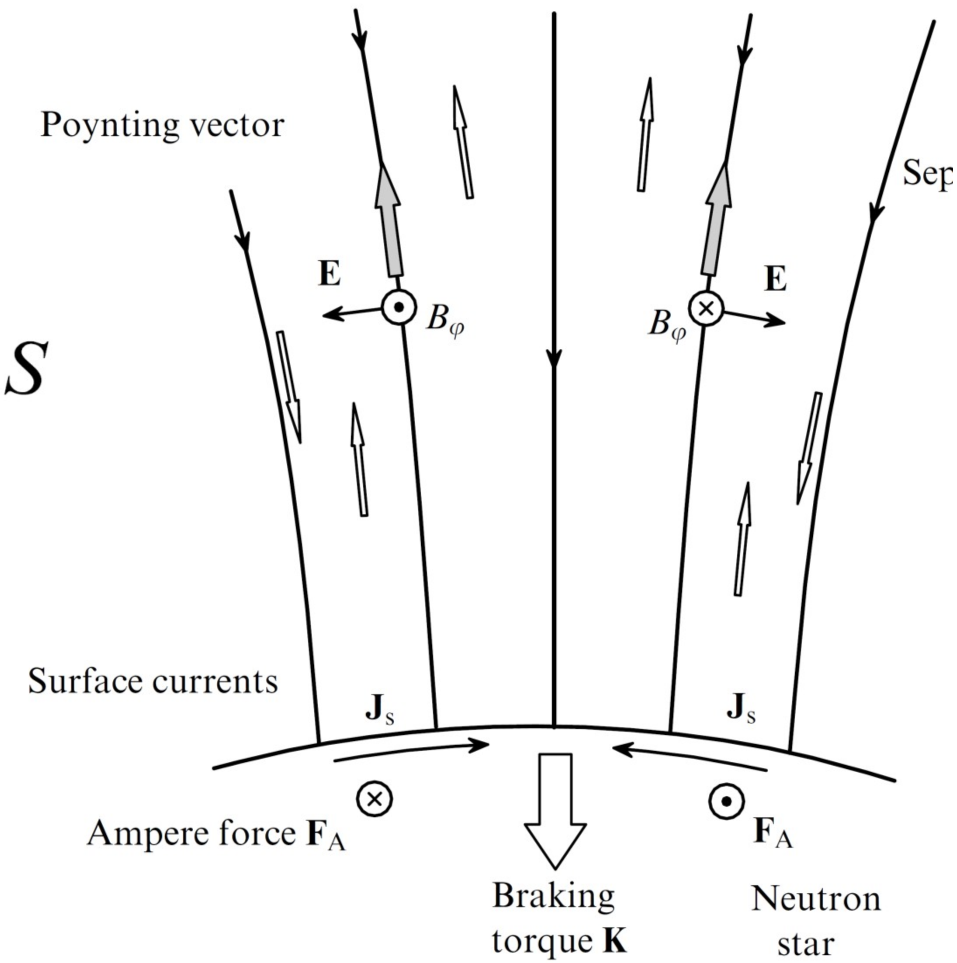
Current losses

$$W_{\rm tot} = -\mathbf{\Omega K}$$

$$\mathbf{K} = \frac{1}{c}\int \left[\mathbf{r} \times [\mathbf{J}_{\rm s} \times \mathbf{B}]\right] \mathrm{d}S$$

$$\nabla_2 \mathbf{J}_{\rm s} = j_{\rm n}$$

$$\mathbf{J}_{\rm s} = \frac{I}{2\pi R \sin\theta}\mathbf{e}_\theta$$



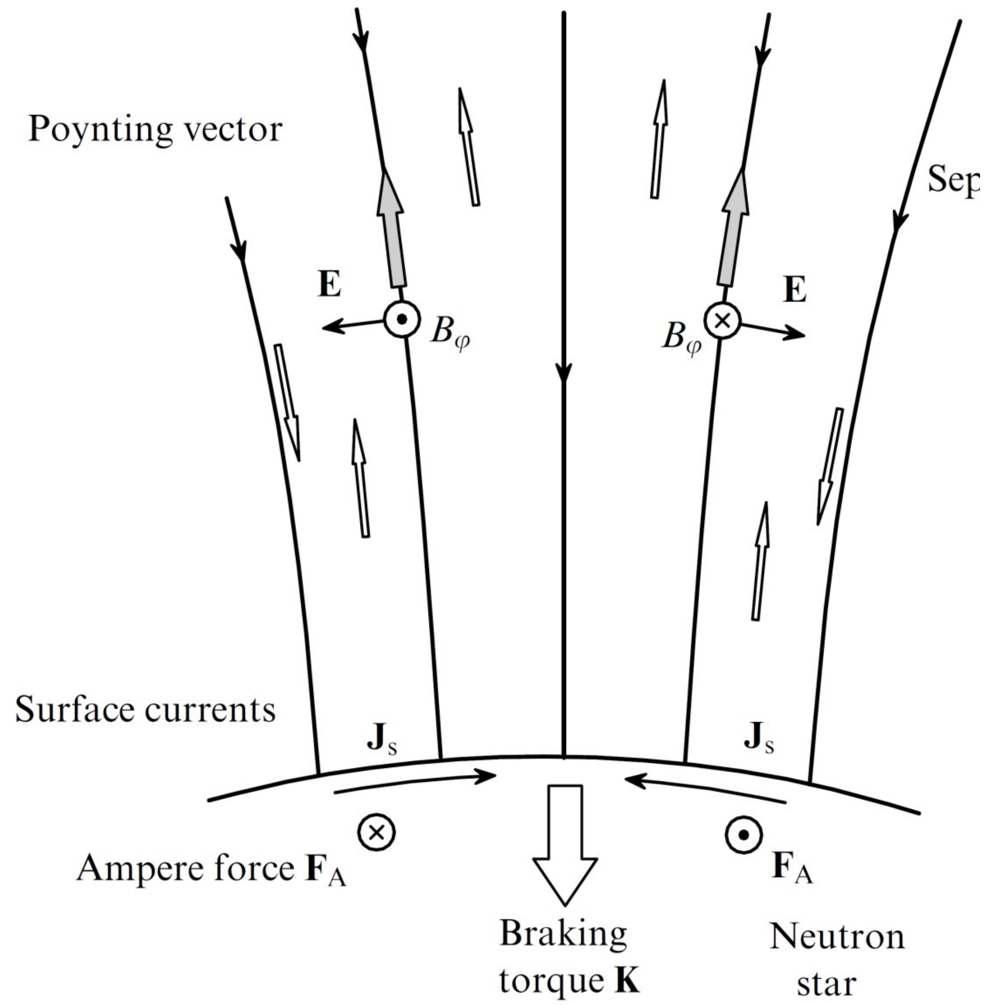
Current losses

$$W_{\rm tot} = c_{\parallel}\frac{B_0^2\Omega^4R^6}{c^3}i_0$$

$$i_0 = j_{\parallel}/j_{\rm GJ}$$

$$W_{\rm tot}^{\rm (BGI)} \approx i_{\rm s}^{\rm A}\frac{B_0^2\Omega^4R^6}{c^2}\cos^2\chi$$

for GJ current

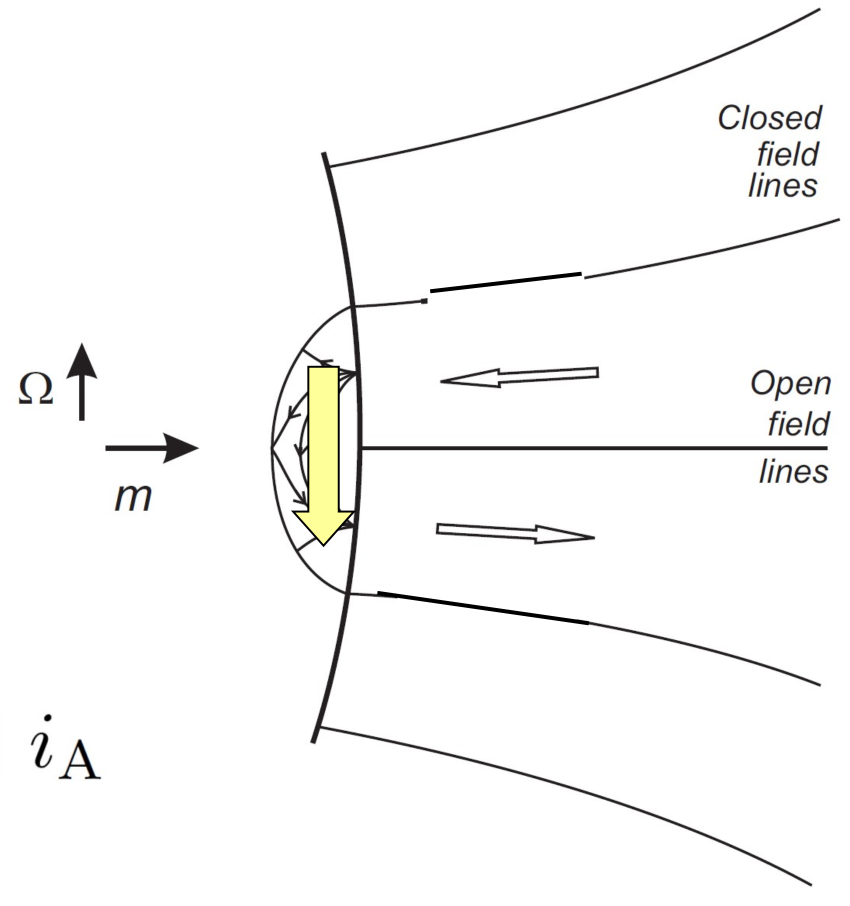


Orthogonal rotator

$$j_{\rm GJ} \approx \frac{\Omega B}{2\pi} \cos\theta$$

$$\mathbf{K} = \frac{1}{c} \int \left[\mathbf{r} \times [\mathbf{J}_{\rm s} \times \mathbf{B}]\right] \mathrm{d}S$$

$$W_{\rm tot} = c_{\perp} \frac{B_0^2 \Omega^4 R^6}{c^3} \left(\frac{\Omega R}{c}\right) i_{\rm A}$$



$i_{\rm A} = j_{\parallel} / \rho_{\rm GJ} c$

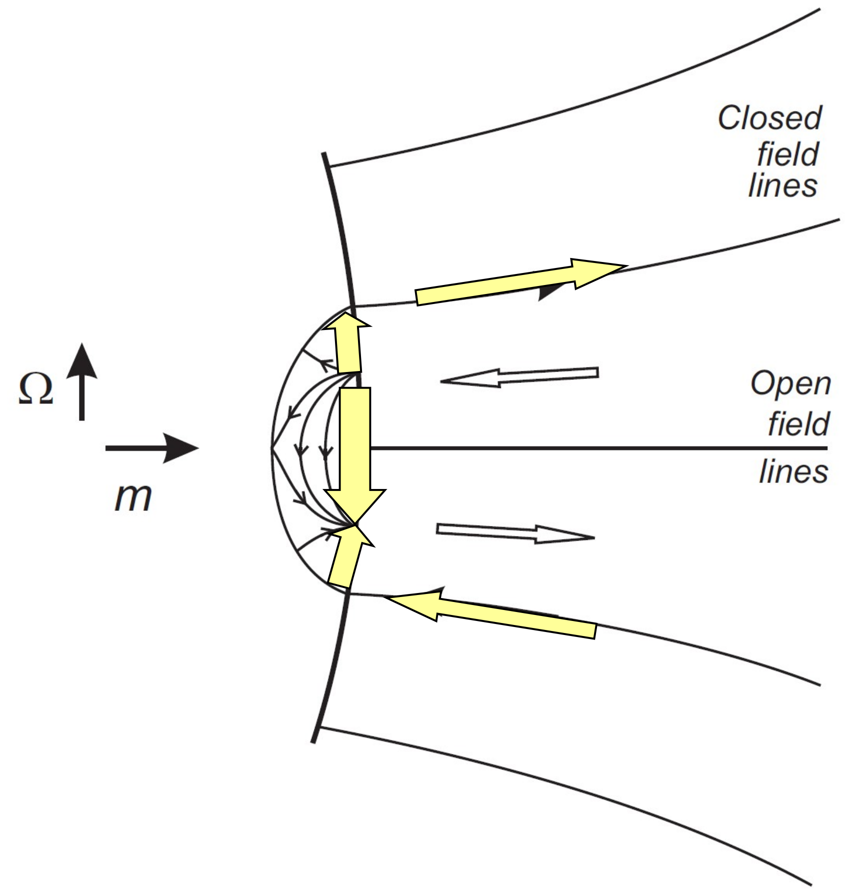
Orthogonal rotator

VB, A.V.Gurevich, Ya.N.Istomin JETP **58**, 235 (1983)

$$\mathbf{K} = \frac{1}{c}\int \left[\mathbf{r} \times [\mathbf{J}_{\mathrm{s}} \times \mathbf{B}]\right] \mathrm{d}S$$

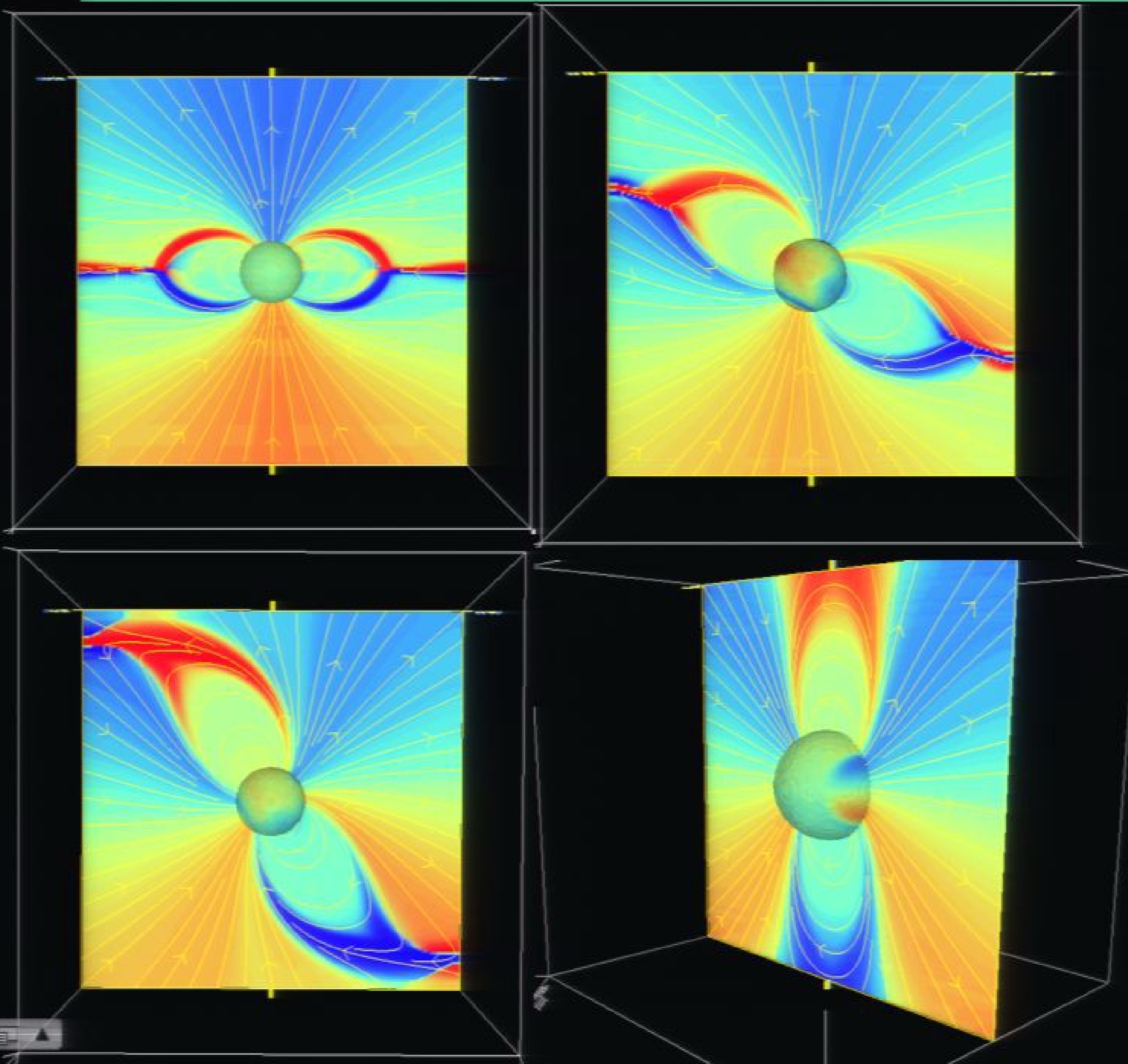
$$W_{\mathrm{tot}} = c_{\perp} \frac{B_0^2 \Omega^4 R^6}{c^3} \left(\frac{\Omega R}{c}\right) i_{\mathrm{A}}$$

$$\mathrm{I}_{\mathrm{sep}} = 3/4\ \mathrm{I}_{\mathrm{vol}}$$



$i_A = j_{\|} / \rho_{GJ} c$

Magnetospheric currents



Oppositely flowing currents can occupy the same open flux tube. Does this hav any obervational implications?

There is always a null-current field lin in the open zone.

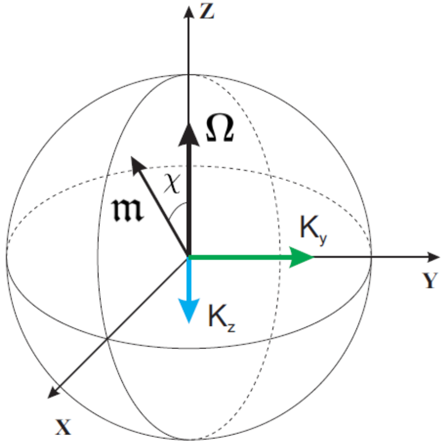
Current losses

$$\beta_{\mathrm{R}} = \frac{\mathbf{\Omega} \times \mathbf{r}}{c}$$

Energy losses

$$W_{\mathrm{tot}} = \frac{c}{4\pi} \int (\beta_{\mathrm{R}}\mathbf{B})(\mathbf{B}d\mathbf{S})$$

Current axisymmetric	$i_s \sim 1$	1	$\Omega^{3/2}$	$\Omega^{3/2}$	1	Ω	$= \Omega^4$
Current orthogonal	$i_a \sim 1$	Ω	Ω	Ω^2	1	Ω	$= \Omega^5$
Vacuum (L&L) (2/3)		1	Ω	Ω^3	1	1	$= \Omega^4$



Current losses

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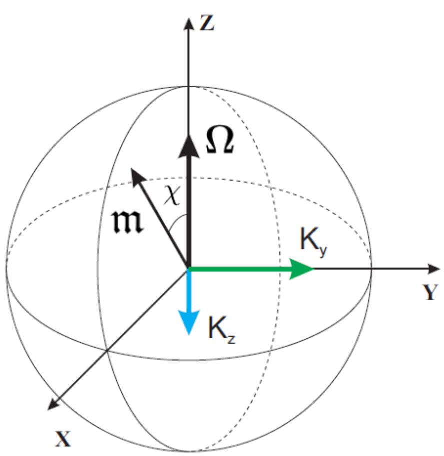
Vacuum (L&L) (1/3)		1	Ω	1	Ω^3	1	$= \Omega^4$



$$\mathbf{B}^{(3)} = -\frac{2}{3}\frac{\mathfrak{m}}{R^3}\left(\frac{\Omega R}{c}\right)^3 \mathbf{e}_{y'}$$

Current losses

$$\beta_{\mathrm{R}} = \frac{\mathbf{\Omega} \times \mathbf{r}}{c}$$

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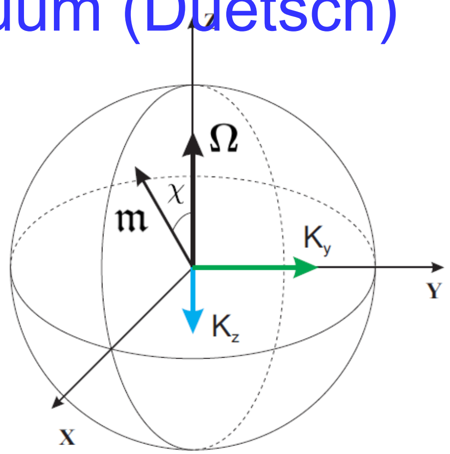
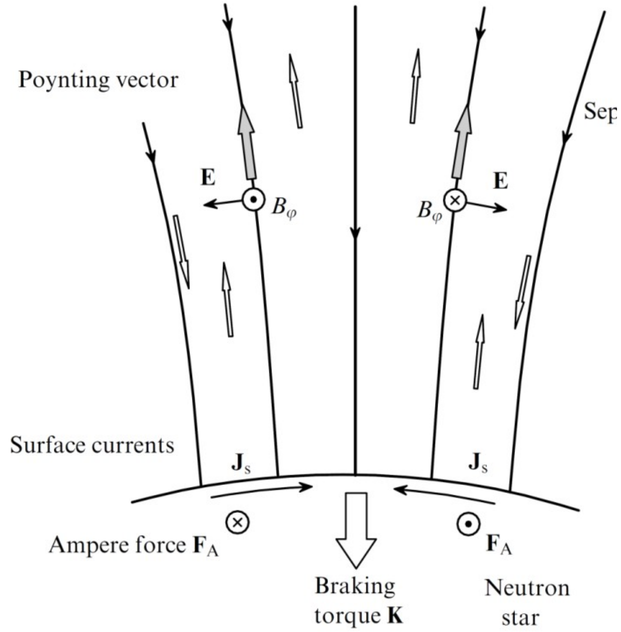
Vacuum (L&L) (2/3)		1	Ω	Ω^3	1	1	$= \Omega^4$
Vacuum (Duetsch)		1	Ω	Ω^3	1	1	$= \Omega^4$



IMPORTANT CONCLUSION

$$W_{\text{tot}} = I\delta U$$



Current losses correspond to first term only

$$\mathbf{K} = \frac{R^3}{c}\int \mathbf{J}_{\rm s}\,(\mathbf{B}\mathbf{n})\,{\rm d}o = \frac{R^3}{4\pi}\int \{[\mathbf{n}\times\mathbf{B}^{(3)}](\mathbf{B}^{(0)}\mathbf{n}) + [\mathbf{n}\times\mathbf{B}^{(0)}](\mathbf{B}^{(3)}\mathbf{n})\}\,{\rm d}o$$

IMPORTANT REMARK

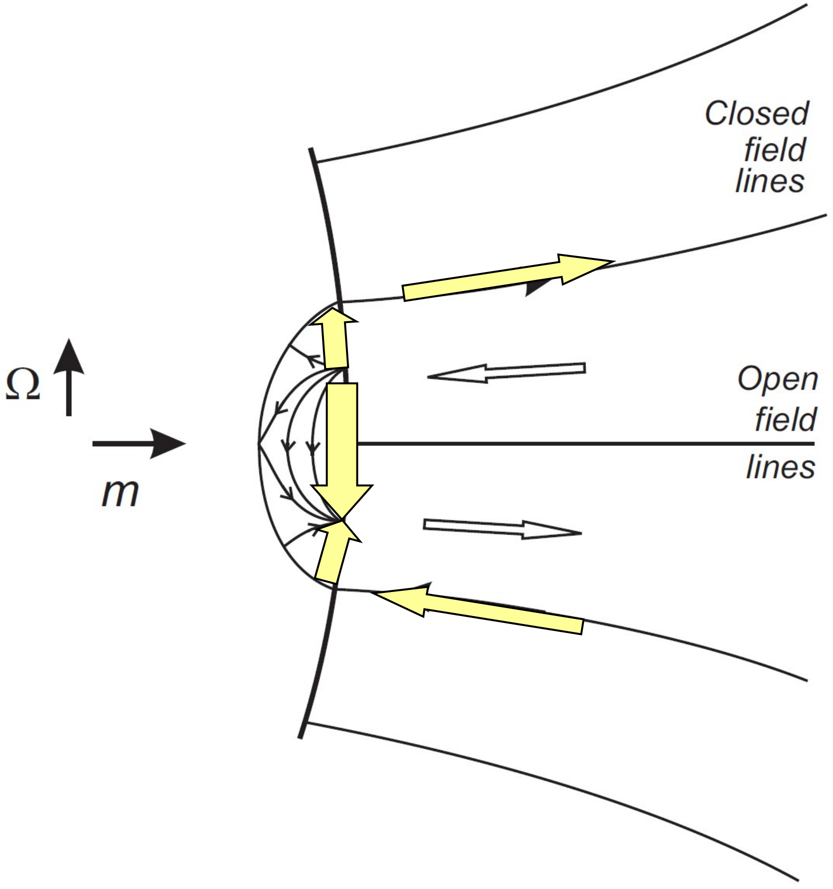
VB, E.E.Nokhrina. Astron. Letters, **30**, 685 (2004)

$$W_{\rm tot} = \frac{\Omega R^3}{c}\int J_\theta B_n {\rm d}o$$

No longitudinal currents in close magnetosphere.
No additional currents along the separatrix.

$$I_{sep} = 3/4 \; I_{vol}$$

$$< J_\theta > = 0$$



IMPORTANT REMARK

VB, E.E.Nokhrina. Astron. Letters, **30**, 685 (2004)

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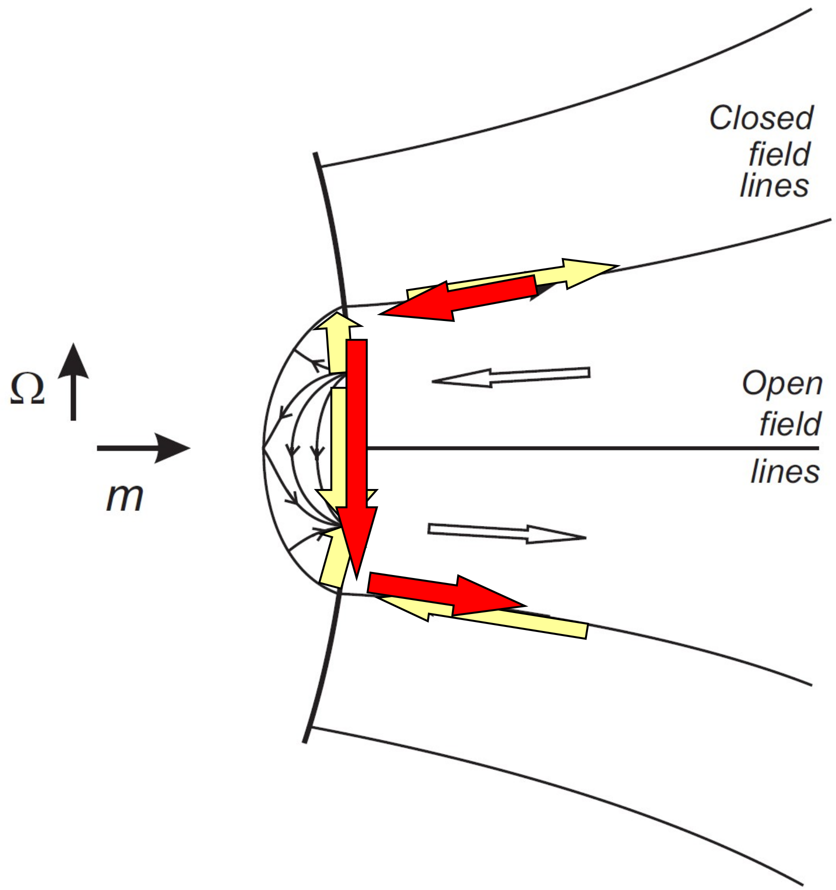


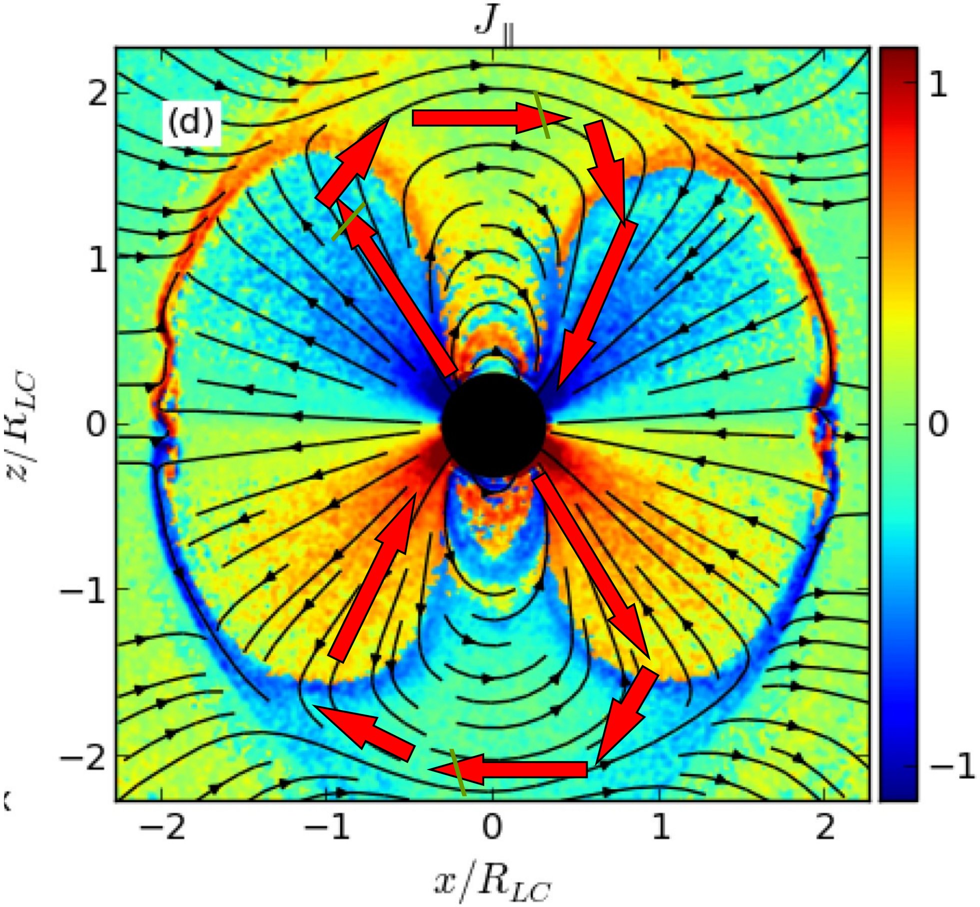
$$I_{sep} < 3/4\ I_{vol}$$

$$< J_\theta > \neq 0$$

IMPORTANT REMARK

VB, E.E.Nokhrina. Astron. Letters, **30**, 685 (2004)

$$W_{\rm tot} = \frac{\Omega R^3}{c}\int J_\theta B_n {\rm d}o$$



Current direction corresponds to energy losses.

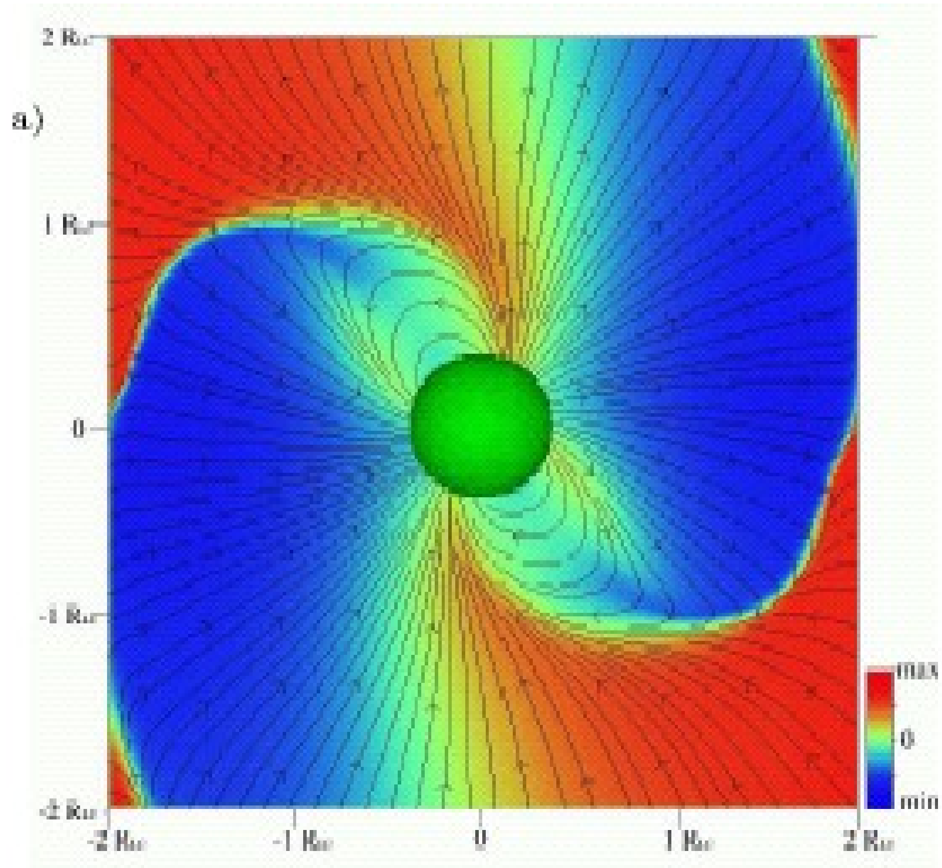
STEP #IV

"Universal solution"

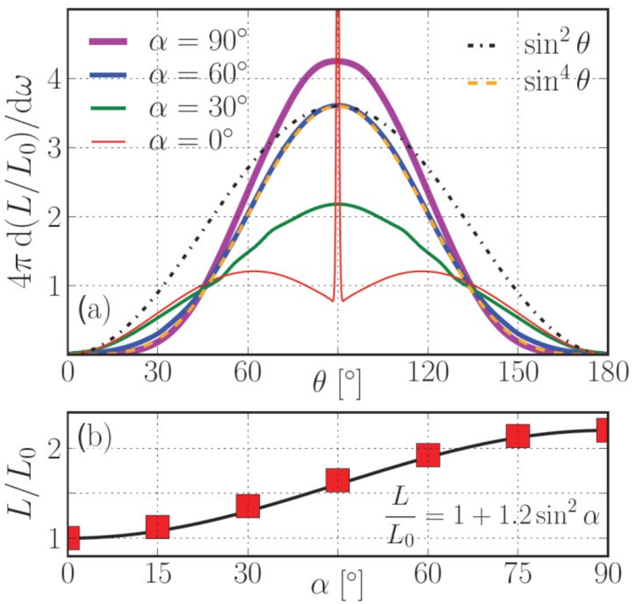
Inclined rotator

A.Spitkovsky, ApJ Lett., **648**, L51 (2006)

$$W_{\rm tot}^{\rm (MHD)} \approx \frac{1}{4}\frac{B_0^2\Omega^4 R^6}{c^2}(1+\sin^2\chi)$$



Inclined rotator – numerically



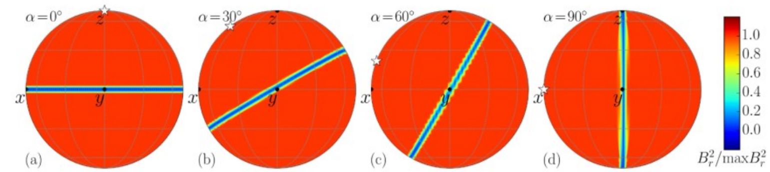
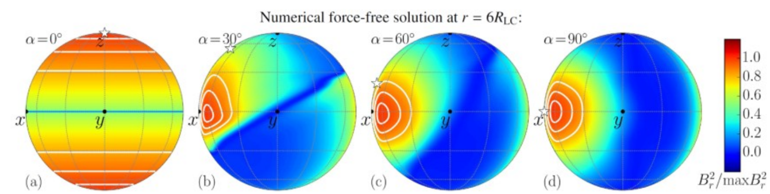
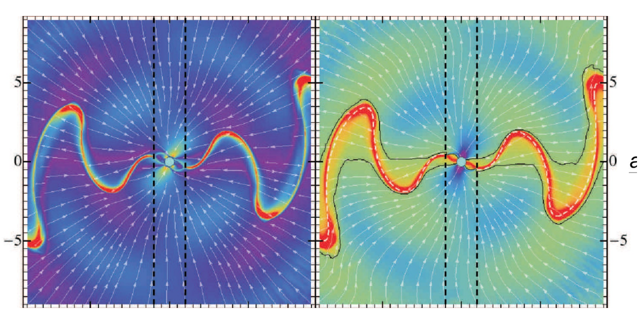


Figure 12. Colour-coded surface distribution of B_r^2 in the split-monopole solution (Bogovalov 1999). The current sheet, in which the radial magnetic field vanishes, describes the orientation of the current sheet in the numerical force-free solutions shown in Fig. 6.



A.Tchekhovskoy, A.Philippov, A.Spitkovsky, MNRAS, **457**, 3384 (2016)



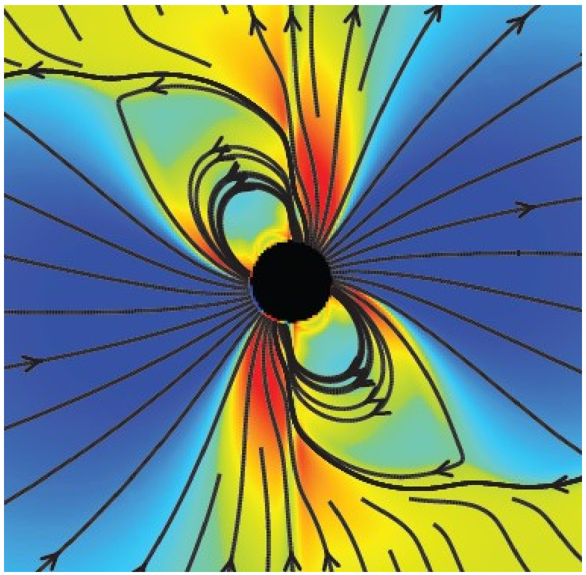
I.Contopoulos et al

$$\langle B_r \rangle \sim \sin\theta$$

$$\langle E \rangle, \langle B_\varphi \rangle \sim \sin^2\theta$$

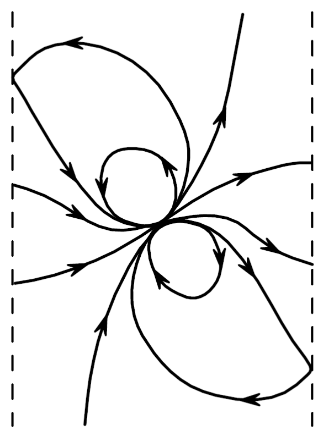
$$W_{\rm tot}(\theta) = \sin^2\theta B_r^2(\theta)$$

Inclined rotator



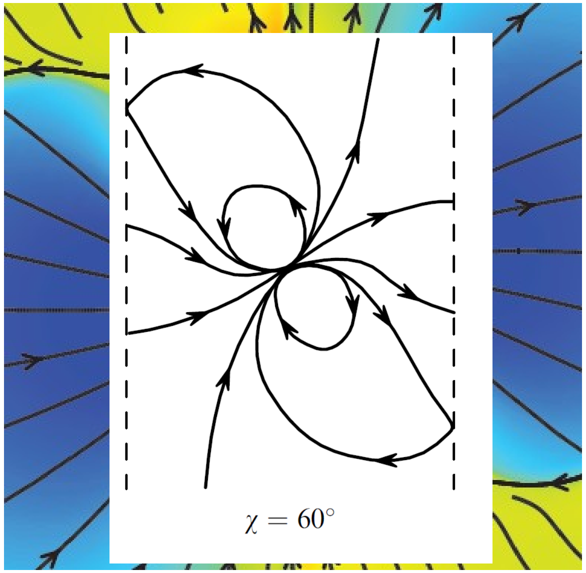
A.Tchekhovskoy,

A.Spitkovsky, J.Li,
MNRAS, **431**, 1 (2013)



$\chi = 60^\circ$

Inclined rotator



A.Tchekhovskoy,

A.Spitkovsky, J.Li,
MNRAS, **431**, 1 (2013)

Polar cap

BGI

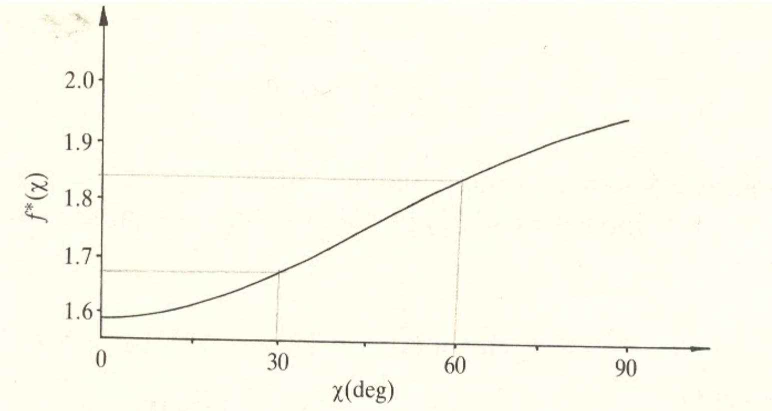
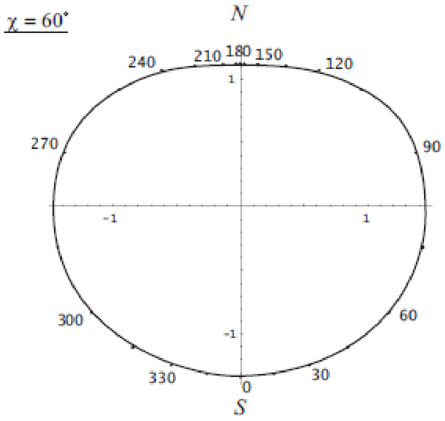
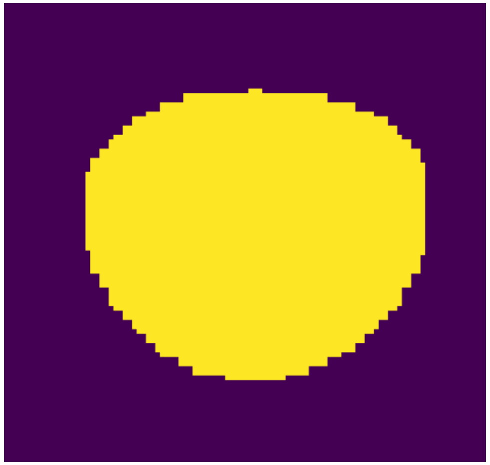


Fig. 4.12. Dependence of the parameter $f^*(\chi)$ on the angle χ.





A.Tchekhovskoy,

A.Spitkovsky, J.Li,
MNRAS, **431**, 1 (2013)

10% precision!

Orthogonal rotator – numerically

$$<B_r> \sim \sin\theta$$
$$<E><B_\varphi> \sim \sin^2\theta$$

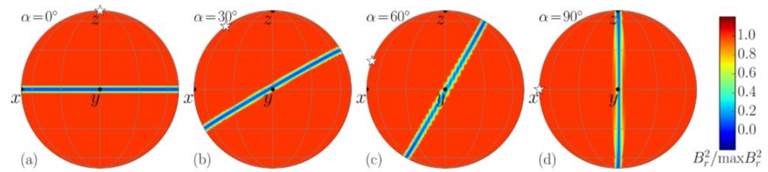
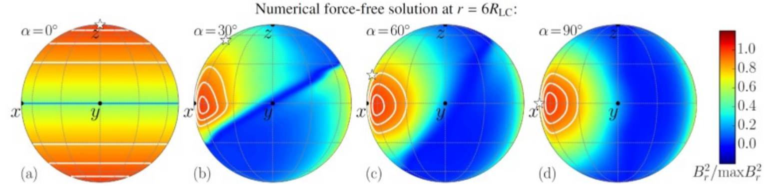


Figure 12. Colour-coded surface distribution of B_r^2 in the split-monopole solution (Bogovalov 1999). The current sheet, in which the radial magnetic field vanishes, describes the orientation of the current sheet in the numerical force-free solutions shown in Fig. 6.



$$W_{\rm tot}(\theta) = \sin^2\theta B_r^2(\theta)$$

No current sheet!

$$\begin{aligned} B_r &\approx B_0 \frac{R^2}{r^2} \sin\theta \cos(\varphi - \Omega t + \Omega r/c), \\ B_\varphi &= E_\theta \approx -B_0 \frac{\Omega R^2}{cr} \sin^2\theta \cos(\varphi - \Omega t + \Omega r/c). \end{aligned}$$

STEP #V

Pulsar braking

Pulsar braking

$$
\begin{aligned}
I_{\rm r}\,\dot\Omega &= K_{\|}\cos\chi + K_{\perp}\sin\chi,\\
I_{\rm r}\Omega\,\dot\chi &= K_{\perp}\cos\chi - K_{\|}\sin\chi,
\end{aligned}
$$

$$
\begin{aligned}
I_{\rm r}\dot\Omega &= K_{\|}^A + [K_{\perp}^A - K_{\|}^A]\sin^2\chi,\\
I_{\rm r}\Omega\dot\chi &= [K_{\perp}^A - K_{\|}^A]\sin\chi\cos\chi.
\end{aligned}
$$

$$
\begin{aligned}
K_{\|} &= -c_{\|}\frac{B_0^2\Omega^3R^6}{c^3}i_{\rm s},\\
K_{\perp} &= -c_{\perp}\frac{B_0^2\Omega^3R^6}{c^3}\left(\frac{\Omega R}{c}\right)i_{\rm a}.
\end{aligned}
$$

$$
\begin{aligned}
i_{\rm s} &= i_{\rm s}^A\cos\chi,\\
i_{\rm a} &= i_{\rm a}^A\sin\chi.
\end{aligned}
$$

$$K_{\perp}^A \approx \left(\frac{\Omega R}{c}\right)K_{\|}^A$$

$$i_{\rm s} \approx i_{\rm a} \approx 1$$

$$i_{\rm A} \sim (\Omega R/c)^{-1}$$

BGI

Princeton (MHD)?

Pulsar braking

Current losses

- Direct current losses

$$K_{\|} = -c_{\|}\frac{B_0^2\Omega^3R^6}{c^3}i_{\rm s},$$

$$K_{\perp} = -c_{\perp}\frac{B_0^2\Omega^3R^6}{c^3}\left(\frac{\Omega R}{c}\right)i_{\rm a}.$$

- Mismatch ('second term')

$$\mathbf{K} = \frac{R^3}{c}\int \mathbf{J}_{\rm s}\,(\mathbf{Bn})\,{\rm d}o = \frac{R^3}{4\pi}\int\{[\mathbf{n}\times\mathbf{B}^{(3)}](\mathbf{B}^{(0)}\mathbf{n}) + [\mathbf{n}\times\mathbf{B}^{(0)}](\mathbf{B}^{(3)}\mathbf{n})\}\,{\rm d}o$$

- Separatrix currents

Pulsar braking

$$I_{\rm r}\,\dot\Omega = K_{\parallel}\cos\chi + K_{\perp}\sin\chi,$$
$$I_{\rm r}\Omega\,\dot\chi = K_{\perp}\cos\chi - K_{\parallel}\sin\chi,$$

$$K_{\parallel} = -c_{\parallel}\frac{B_0^2\Omega^3R^6}{c^3}i_{\rm s},$$
$$K_{\perp} = -c_{\perp}\frac{B_0^2\Omega^3R^6}{c^3}\left(\frac{\Omega R}{c}\right)i_{\rm a}.$$

$$K_{\perp}^{A} \approx \left(\frac{\Omega R}{c}\right)K_{\parallel}^{A}$$

$$i_{\rm s} \approx i_{\rm a} \approx 1$$

VB, A.V.Gurevich, Ya.N.Istomin,
JETP **58**, 235 (1983)

How to write down the current

Drift approximation

$$\mathbf{j} = c\,\rho_{\rm e}\frac{[\mathbf{E}\times\mathbf{B}]}{B^2} + a\,\mathbf{B}$$

$$\boxed{\mathbf{j} = \rho_{\rm e}\,[\mathbf{\Omega}\times\mathbf{r}] + i_{||}\,\mathbf{B}}$$

$$\boxed{\mathbf{j} = \frac{(\mathbf{B}\cdot\nabla\times\mathbf{B} - \mathbf{E}\cdot\nabla\times\mathbf{E})\mathbf{B} + (\nabla\cdot\mathbf{E})\mathbf{E}\times\mathbf{B}}{B^2}}$$

$$(\nabla i_{||}\,\mathbf{B}) = 0$$

Mestel, BGI

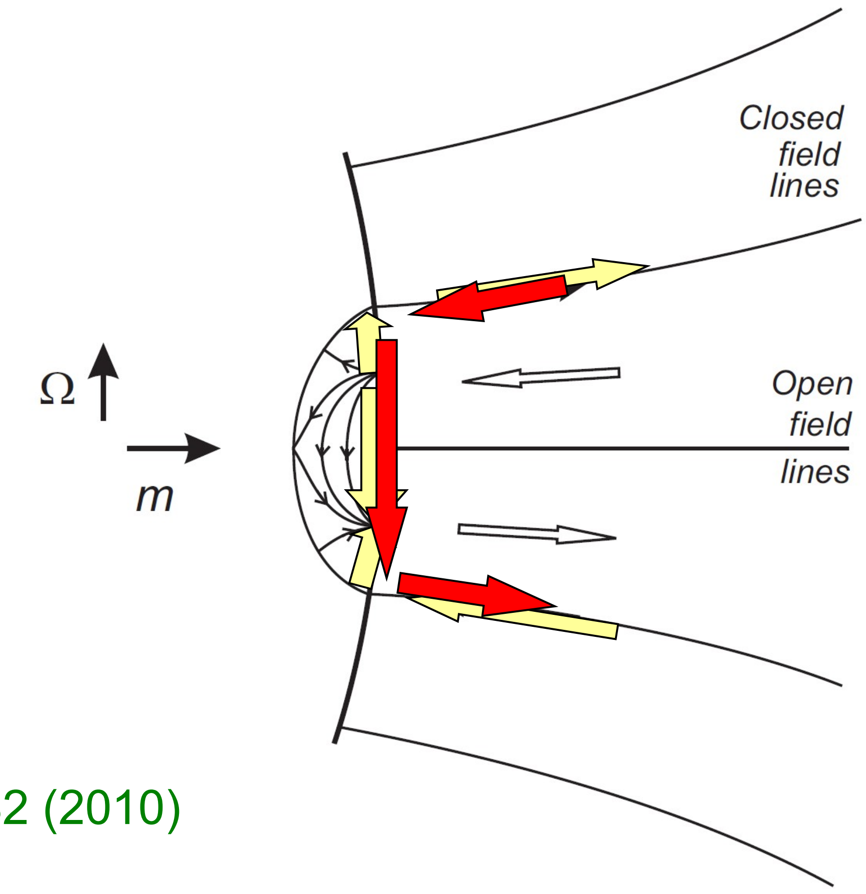
Gruzinov

$$i_{\rm a} \sim \left(\frac{\Omega R}{c}\right)^{-1/2}$$

Separatrix current?

VB, A.V.Gurevich, Ya.N.Istomin JETP **58**, 235 (1983)

$$W_{\rm tot} = \frac{\Omega R^3}{c} \int J_\theta B_n {\rm d}o$$



$$\mathrm{I_{sep} = 3/4\ I_{vol}}$$

X.-N. Bai, A.Spitkovsky ApJ **715**, 1282 (2010)

$$\mathrm{I_{sep} = 20\%\ I_{vol}}$$

Orthogonal rotator – analytically

$$B_r \approx B_0 \frac{R^2}{r^2} \sin\theta \cos(\varphi - \Omega t + \Omega r/c),$$

$$B_\varphi = E_\theta \approx -B_0 \frac{\Omega R^2}{cr} \sin^2\theta \, \cos(\varphi - \Omega t + \Omega r/c).$$

In the wind

$$i_\parallel = -3 \frac{\Omega}{c} \cos\theta$$

Polar cap

$$i_{\rm a}^{\rm A} \approx f_*^{-1/2} \left(\frac{\Omega R}{c}\right)^{-1/2}$$

TOO LOW!

Inclined rotator – MHD

- No monopole Michel-Bogovalov poloidal field
- No magneto-dipole radiation
- Larger energy losses for orthogonal rotator

$$W_{\rm tot}^{\rm (MHD)} \approx \frac{1}{4}\frac{B_0^2\Omega^4R^6}{c^2}(1+\sin^2\chi)$$

- Alignment: inclination angle evolves to 0 deg.

Problem 5.2. Show that the relation similar to (5.24) can be obtained for the conical solutions $\Psi = \Psi(\theta)$, but only at large distances $r \gg R_{\rm L}$ from the compact object. It has the form [Ingraham, 1973, Michel, 1974]

$$4\pi I(\theta) = \Omega_{\rm F}(\theta)\sin\theta\,\frac{{\rm d}\Psi}{{\rm d}\theta}. \qquad (5.25)$$

$$E_\theta = B_\varphi$$

S.Gralla,T.Jacobson, G.Menon, C.Dermer ($B_{\rm p} = 0$)

Pulsar evolution – current losses?

$$
\begin{aligned}
I_{\mathrm{r}}\,\dot{\Omega} &= K_{\parallel}\cos\chi + K_{\perp}\sin\chi, \\
I_{\mathrm{r}}\Omega\,\dot{\chi} &= K_{\perp}\cos\chi - K_{\parallel}\sin\chi,
\end{aligned}
$$

$$
\begin{aligned}
K_{\parallel} &= -c_{\parallel}\frac{B_0^2\Omega^3R^6}{c^3}\,i_{\mathrm{s}}, \\
K_{\perp} &= -c_{\perp}\frac{B_0^2\Omega^3R^6}{c^3}\left(\frac{\Omega R}{c}\right)i_{\mathrm{a}}.
\end{aligned}
$$

$$K_{\perp}^{A} \approx \left(\frac{\Omega R}{c}\right)K_{\parallel}^{A}$$

$$i_{\mathrm{s}} \approx i_{\mathrm{a}} \approx 1$$

BGI

$$
\begin{aligned}
I_{\mathrm{r}}\dot{\Omega} &= K_{\parallel}^{A} + [K_{\perp}^{A} - K_{\parallel}^{A}]\sin^2\chi, \\
I_{\mathrm{r}}\Omega\dot{\chi} &= [K_{\perp}^{A} - K_{\parallel}^{A}]\sin\chi\cos\chi.
\end{aligned}
$$

$$
\begin{aligned}
i_{\mathrm{s}} &= i_{\mathrm{s}}^{A}\cos\chi, \\
i_{\mathrm{a}} &= i_{\mathrm{a}}^{A}\sin\chi.
\end{aligned}
$$

?

$$i_{\mathrm{A}} \sim (\Omega R/c)^{-1}$$

Princeton (MHD)

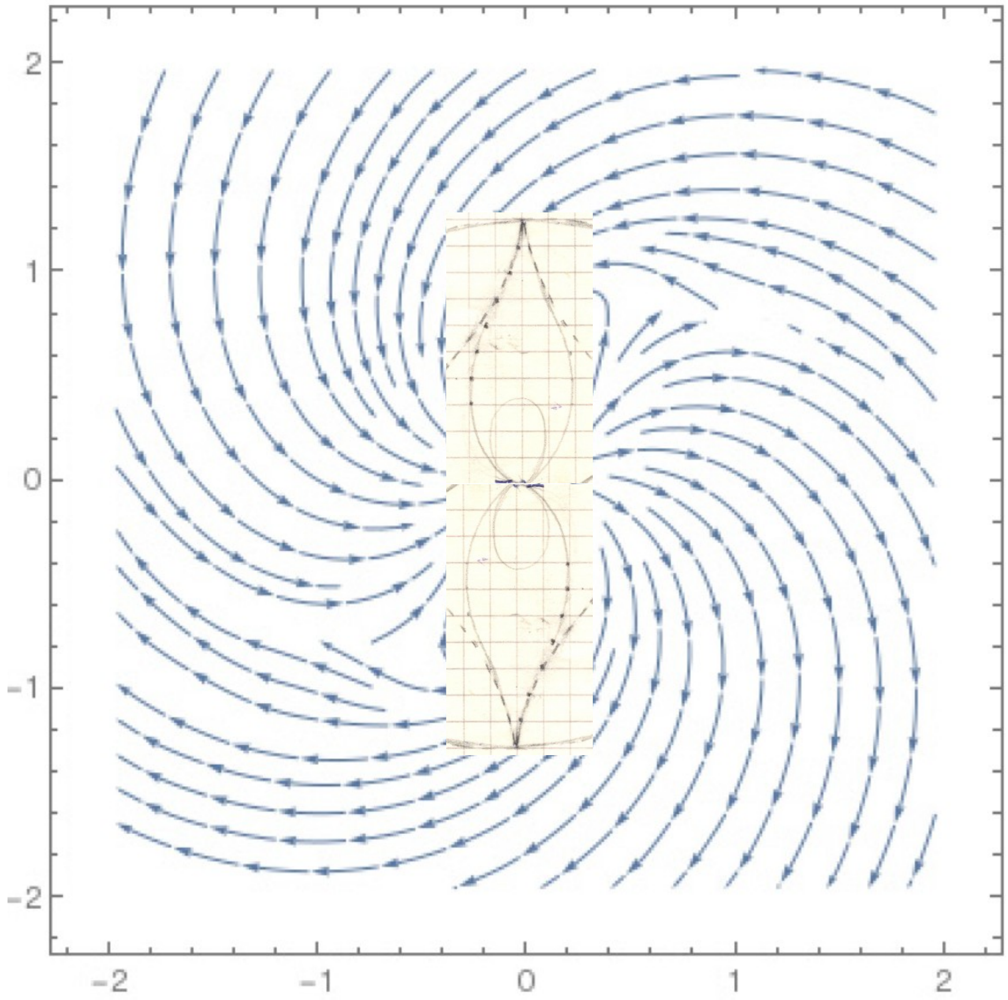
What to do?

Two possibilities:

2. Mismatch (second term)
3. Separatrix currents

Magnetospheric losses

Mismatch



Magnetospheric losses

Torque

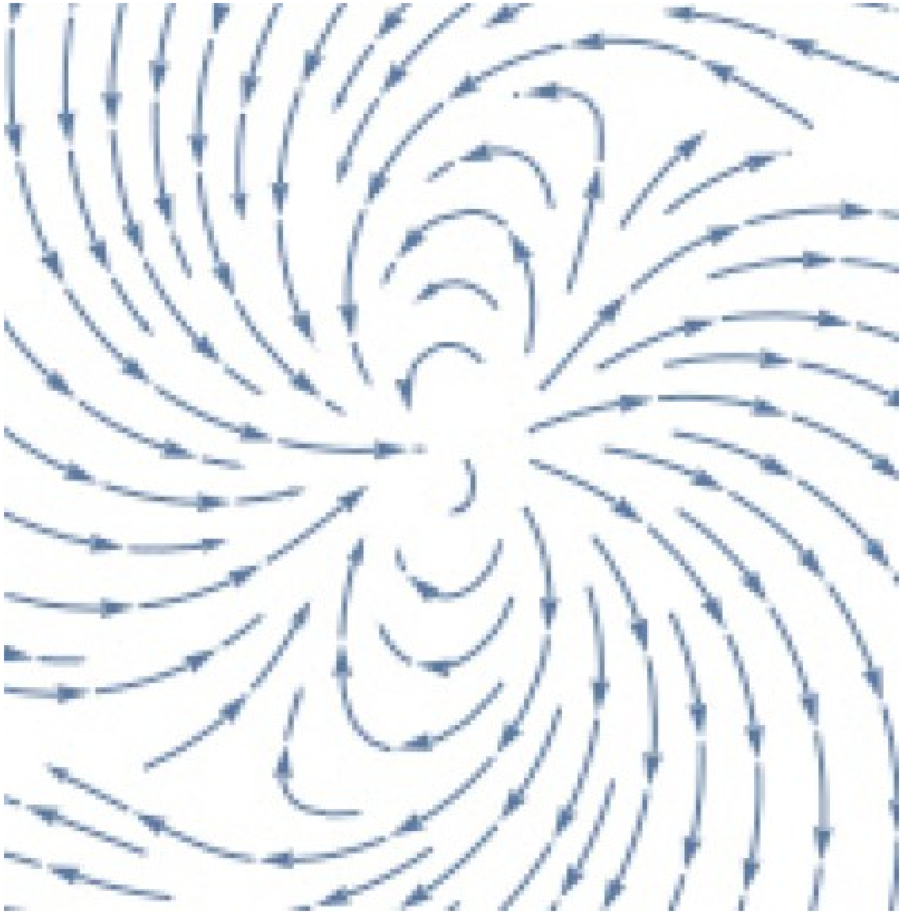
$$I_{\mathrm{r}} \dot{\Omega} = K_{\parallel}^{A} + [K_{\perp}^{A} - K_{\parallel}^{A}] \sin^2 \chi,$$

$$I_{\mathrm{r}} \Omega \dot{\chi} = [K_{\perp}^{A} - K_{\parallel}^{A}] \sin \chi \cos \chi.$$

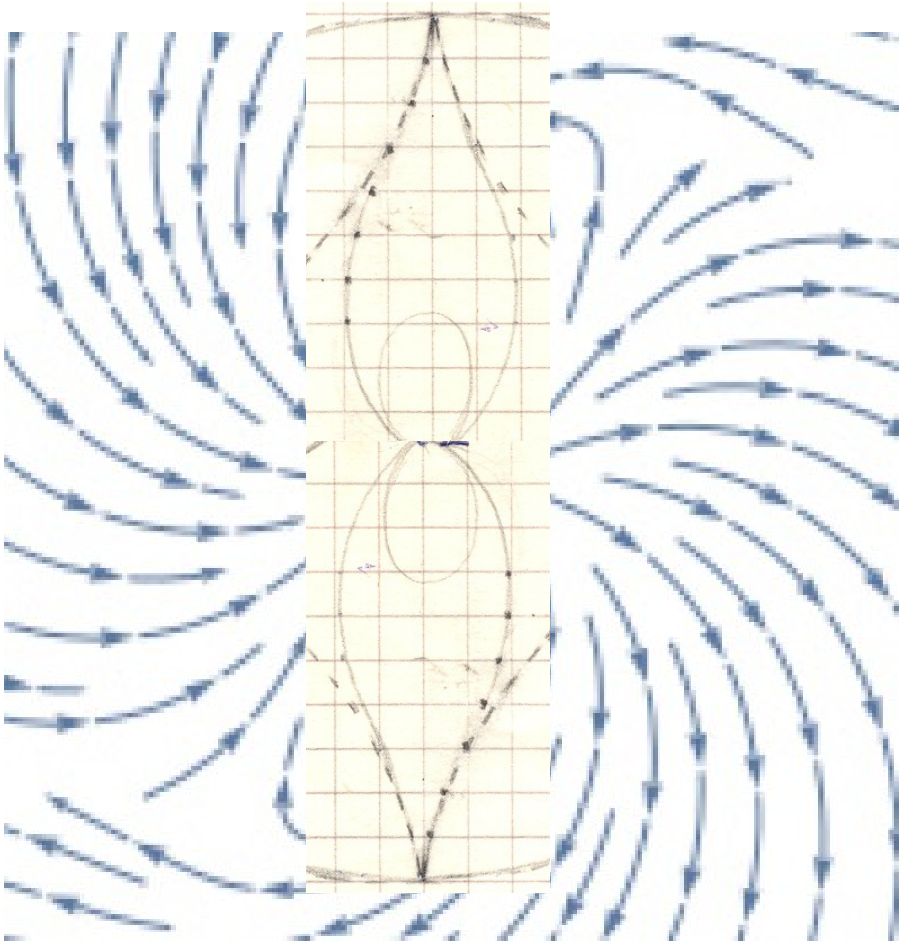
$$K_{\perp}^{\mathrm{mag}} = -A \frac{B_0^2 \Omega^3 R^6}{c^3} i_{\mathrm{a}}$$

$$A \approx 2 \left(\frac{\Omega R}{c} \right)^{1/2}$$

Magnetospheric losses



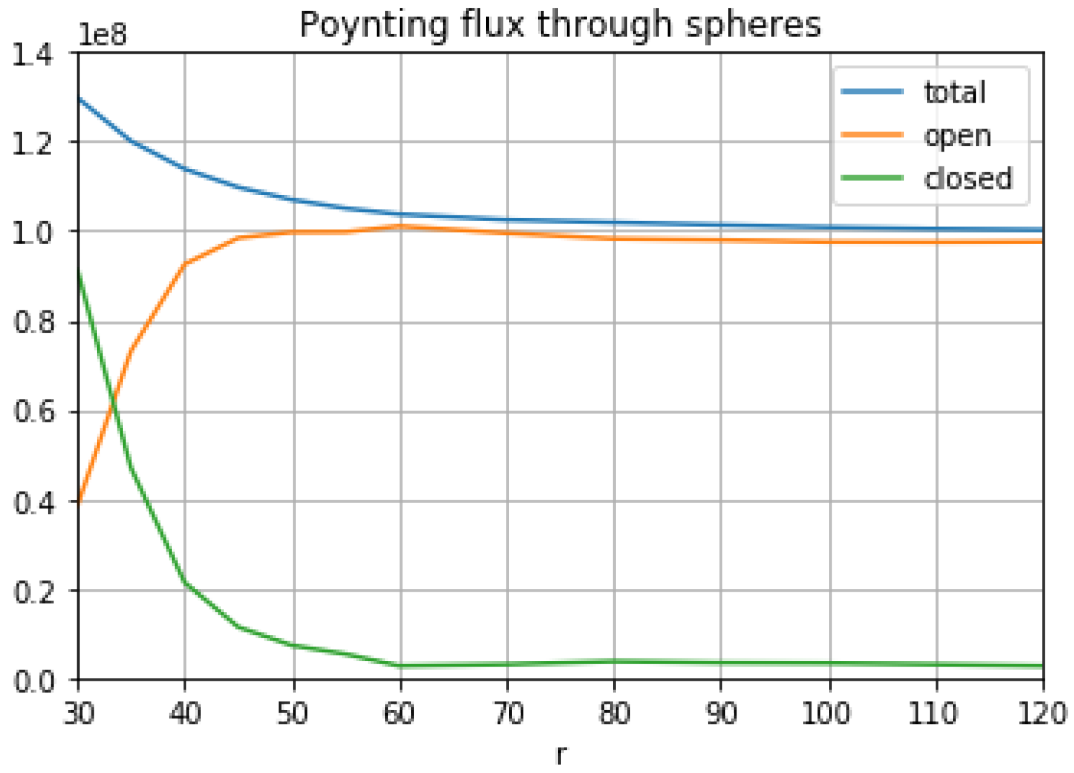
Magnetospheric losses



STEP #VI

An answer

An answer



Poynting flux through spheres
1e8
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
30
40
50
60
70
80
90
100
110
120
r
total
open
closed

Final Conclusion

To explain energy losses of the ‘universal solution’ in addition to direct current losses.

In BGI model they ar to be neglected, as was proposed.