On the primary beam deceleration in the pulsar wind

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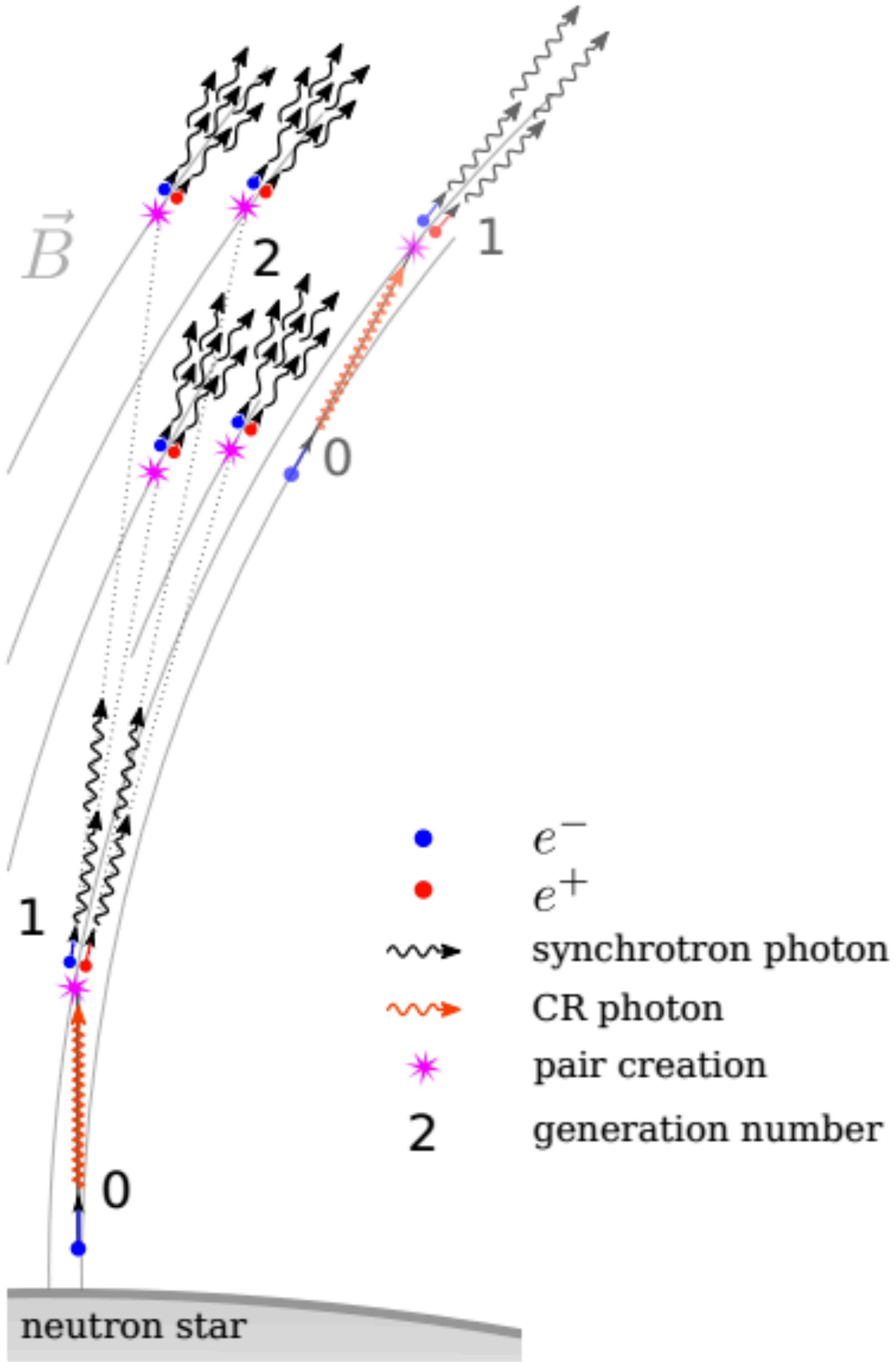
Pair cascade

Sturrock (1971), Ruderman & Sutherland (1975), Arons (1981), Gurevich & Istomin (1985), Medin & Lai (2007), Timokhin & Harding (2015)



- **Pulsar magnetosphere is unstable to the creation of electron-positron pairs**

$$\gamma + B \rightarrow e^+ + e^- + B$$

$$w = \frac{3\sqrt{3}}{16\sqrt{2}} \frac{e^3 B \sin\theta}{\hbar m_e c^3} \exp\left(-\frac{8}{3} \frac{B_{cr}}{B\sin\theta} \frac{m_e c^2}{\mathcal{E}_{ph}}\right)$$

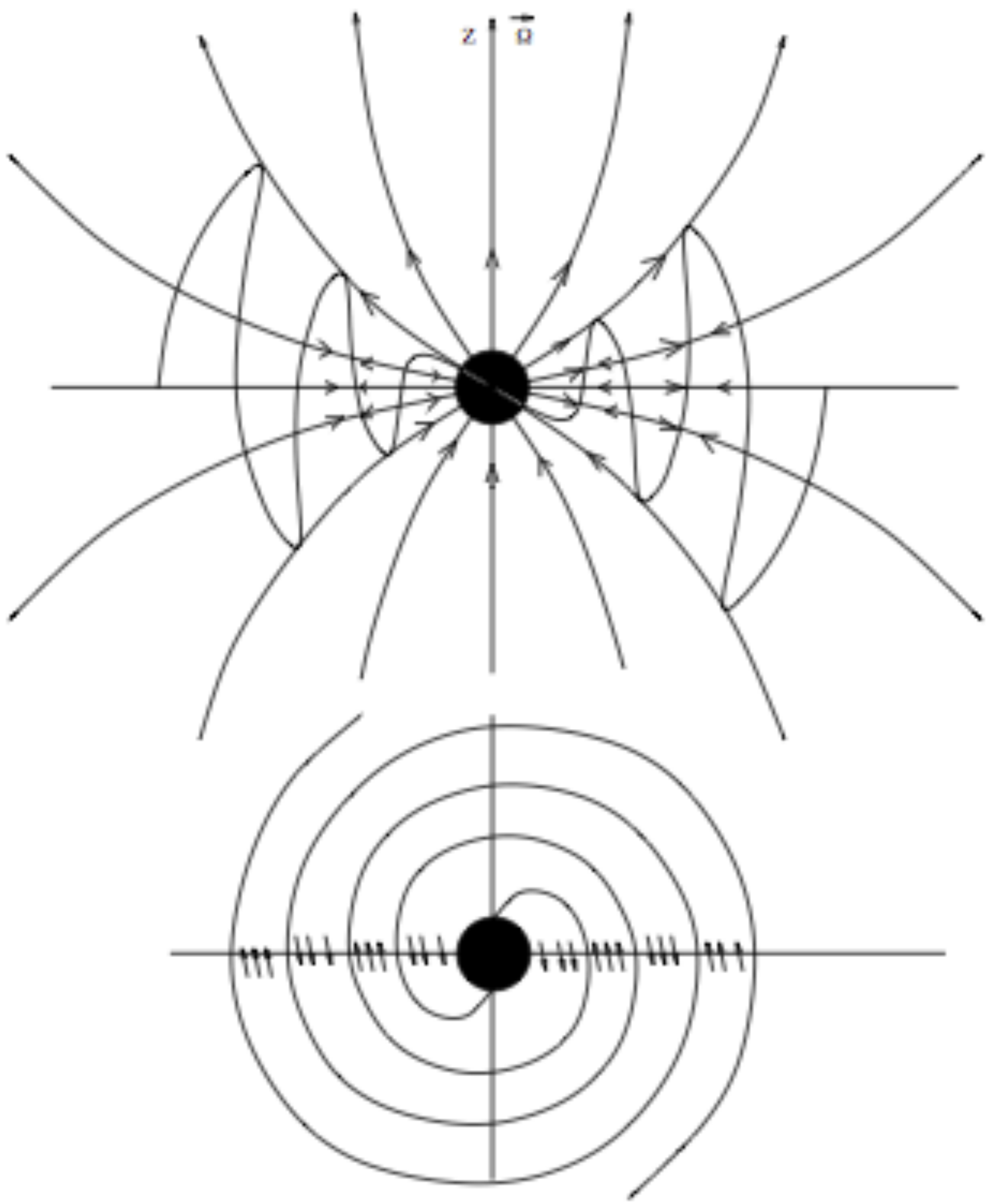
- **High multiplicity**

$$n^{\pm} \sim (10^3\text{–}10^5) n_{GJ}$$

Magnetosphere consists of high-energy primary particles and low-energy secondary particles with high concentration

Magnetospheric structure: analytics

Michel (1973), Bogovalov (1999)



$$B_{\rm p} = B_{\rm L}\frac{R_{\rm L}^2}{r^2}\,\Theta(\Phi),$$

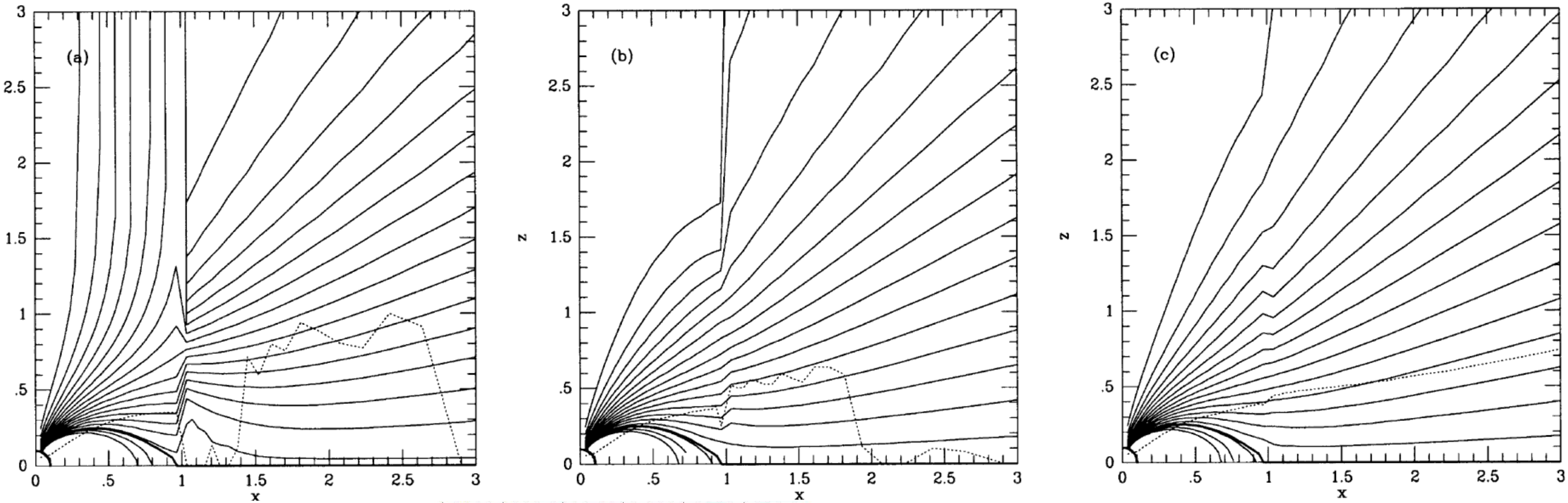
$$B_\varphi = E_\theta = -B_{\rm L}\frac{R_{\rm L}}{r}\sin\theta\,\Theta(\Phi)$$

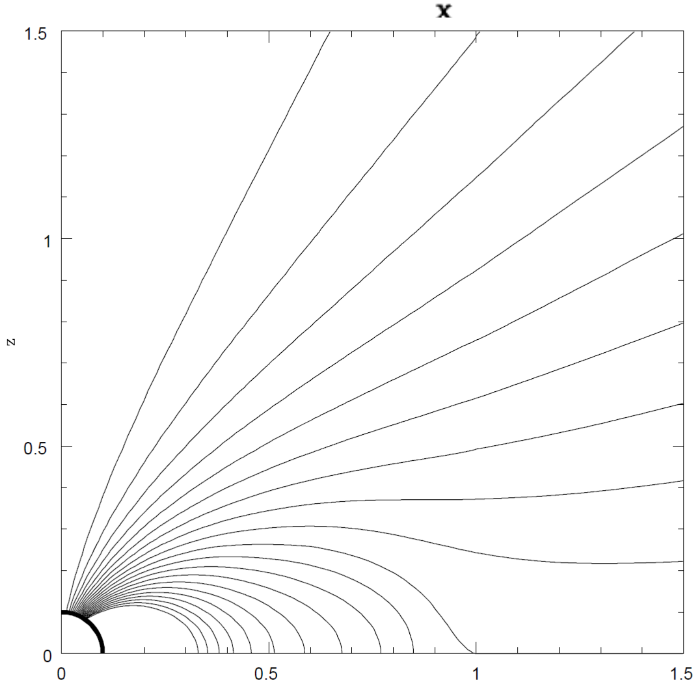
$$\Phi = \cos\theta\cos\chi - \sin\theta\sin\chi\cos(\Omega r/c + \varphi - \Omega t)$$

Fast primary particles inevitably intersect the magnetospheric current sheet!

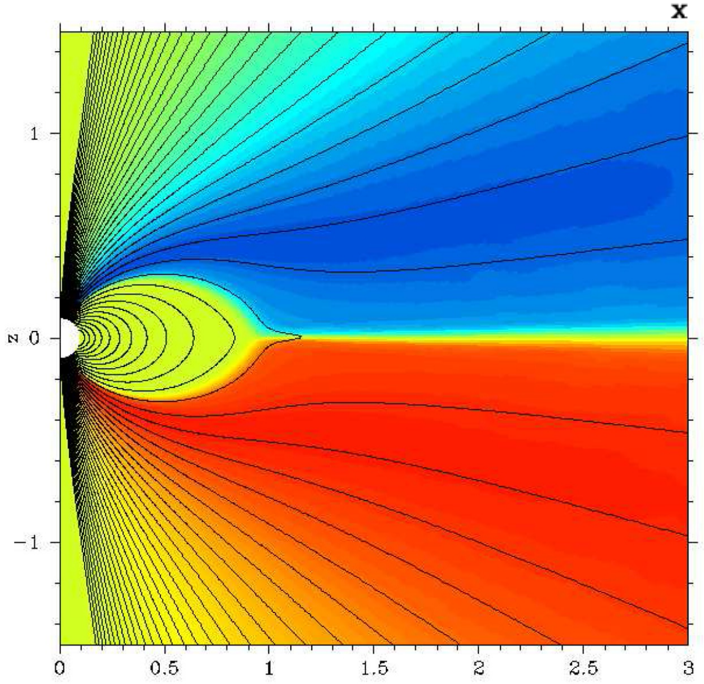
Magnetospheric structure: numerics

I. Contopoulos, D. Kazanas, Ch. Fendt (1999)

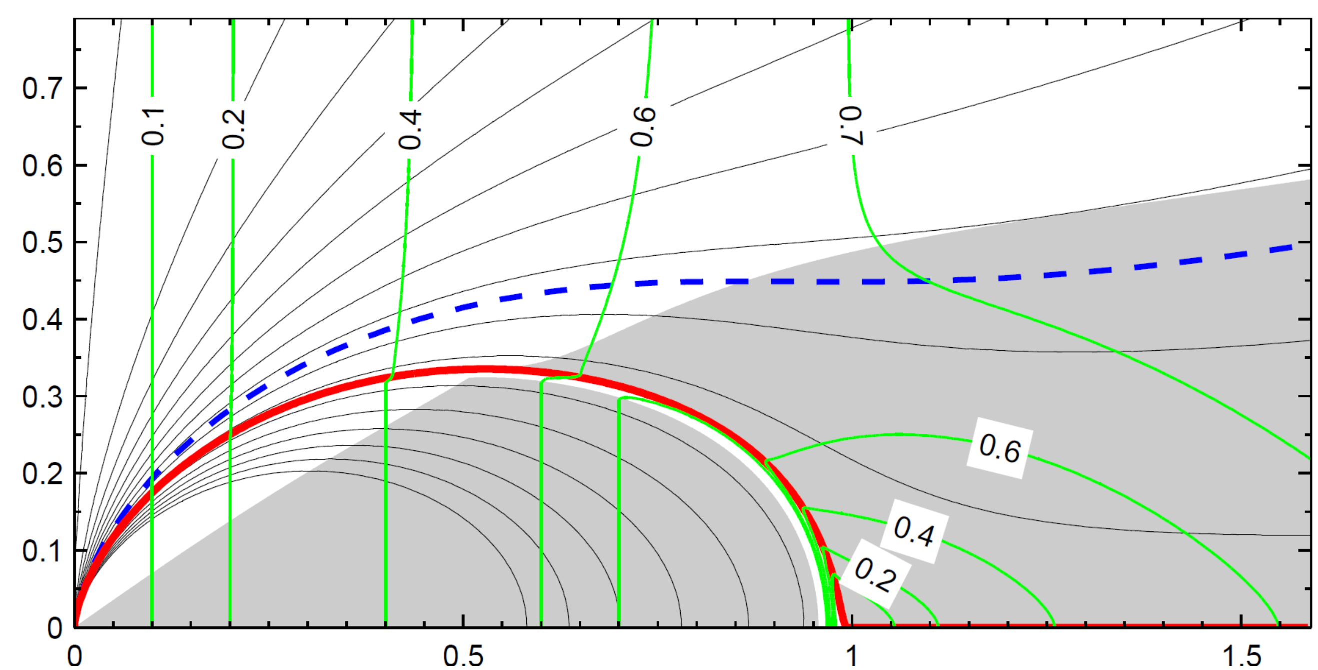




Gruzinov (2005)



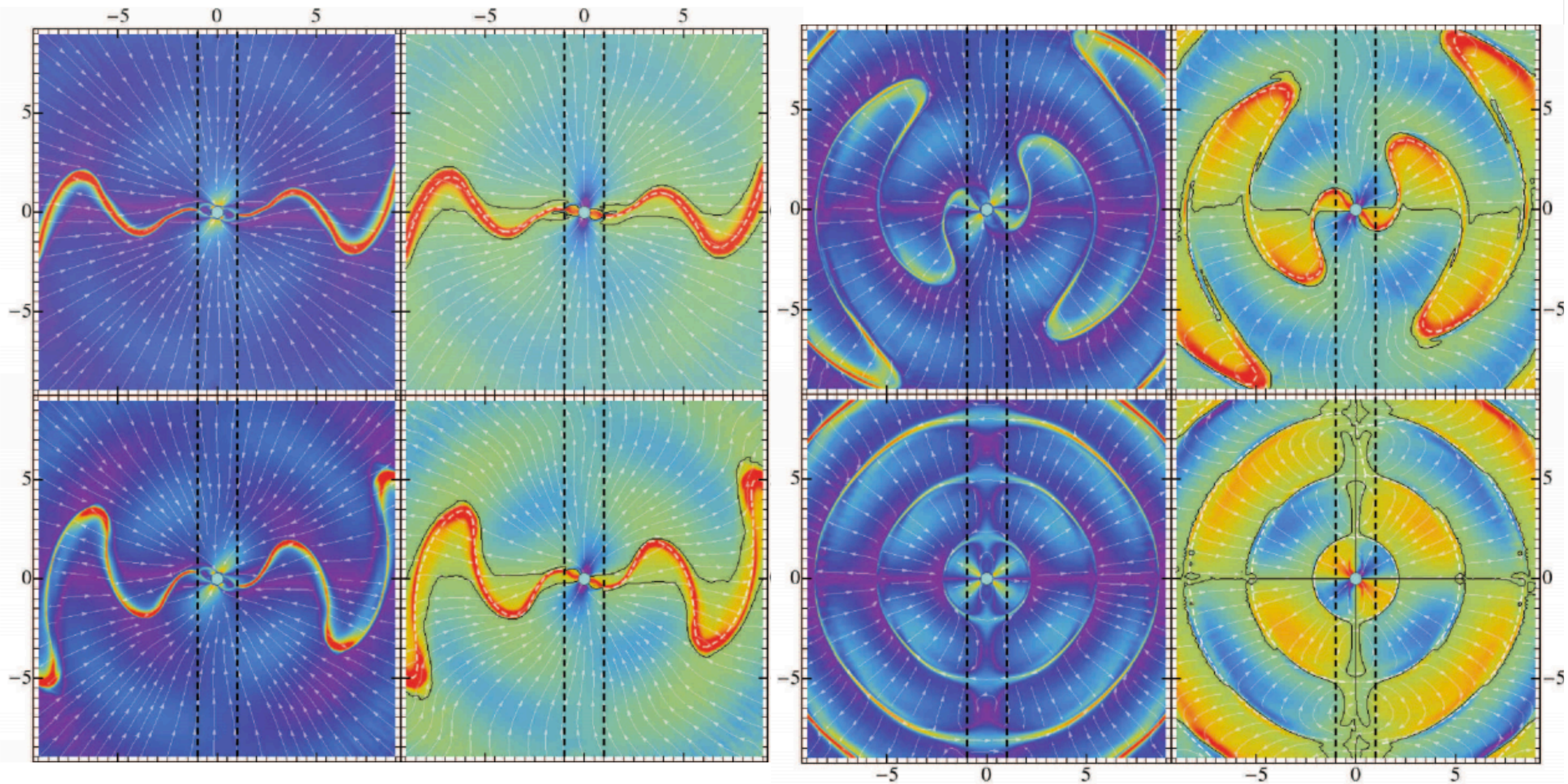
Komissarov (2005)



Timokhin (2005)

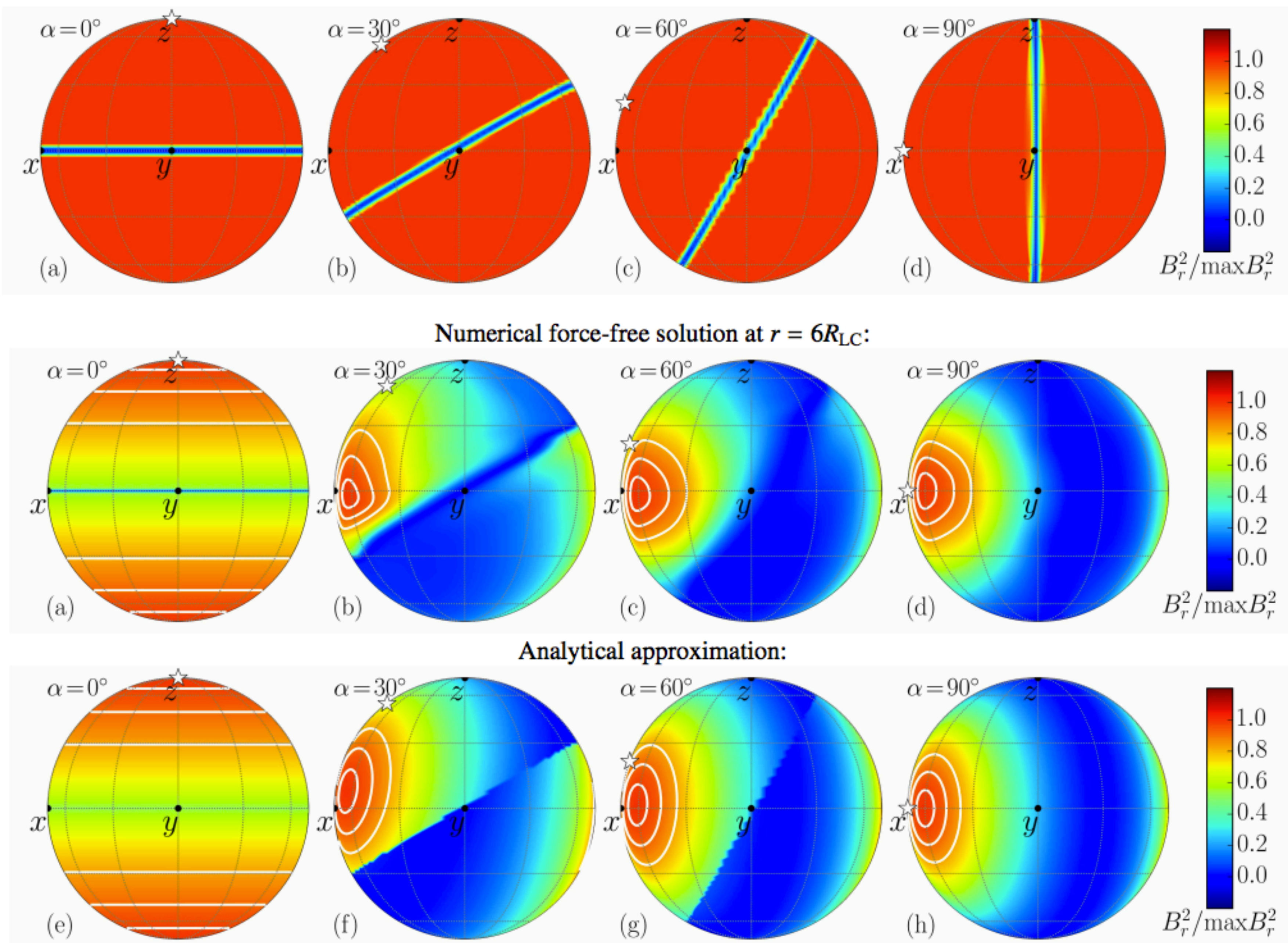
Magnetospheric structure: numerics

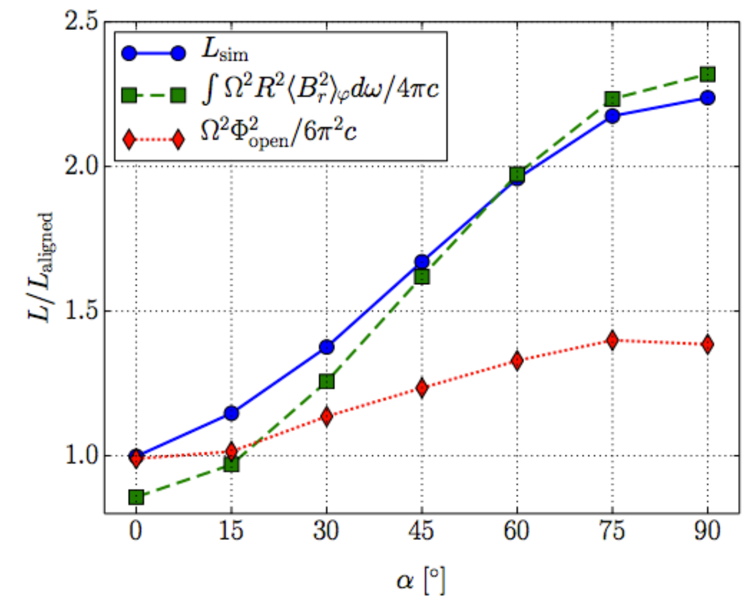
C. Kalapotharakos, I. Contopoulos, D. Kazanas (2011)



Magnetospheric structure: numerics (MHD)

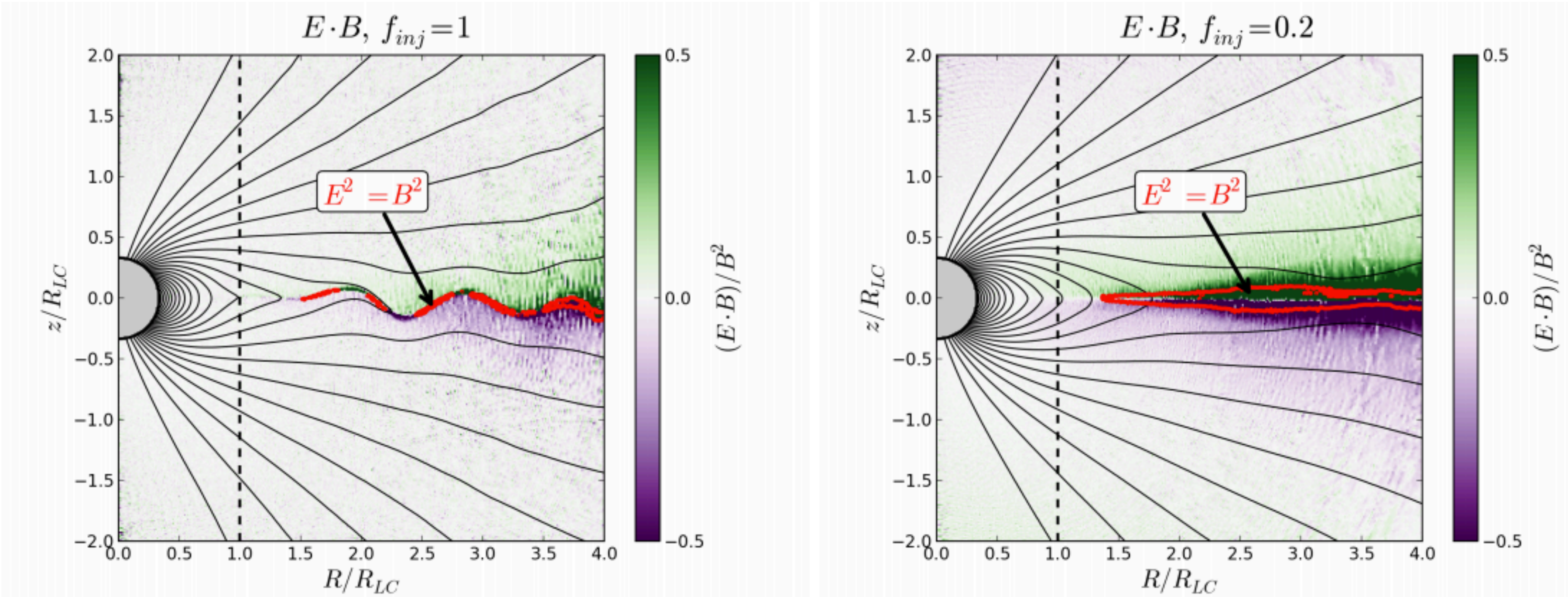
Tchekhovskoy A., Philippov A., Spitkovsky A. (2015)



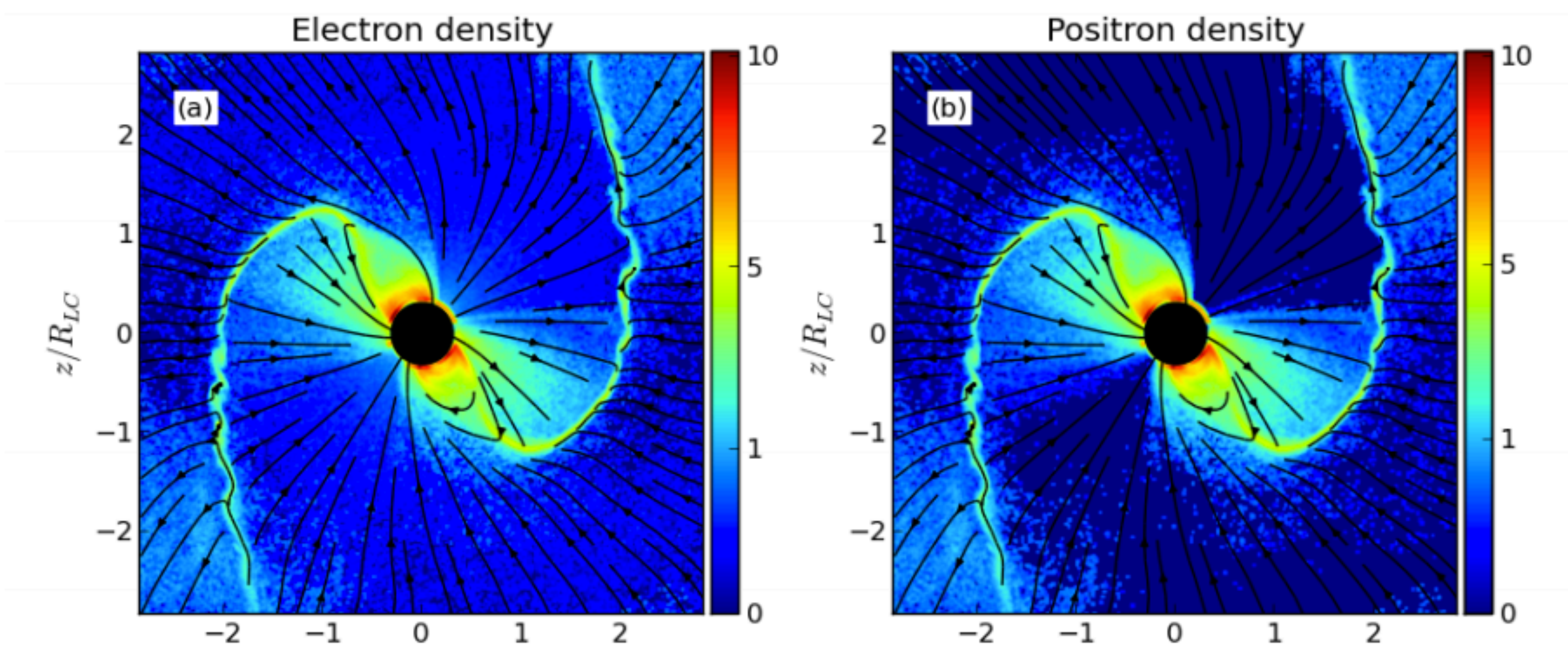


Magnetospheric structure: numerics (PIC)

Cerutti B., Philippov A., Spitkovsky A. (2015)



Philippov A., Spitkovsky A., Cerutti B. (2015)



Problem

- How the Lorenz-factor of the beam evolves during its motion through the secondary plasma?
- Is the secondary plasma accelerated by the beam?
- Is the beam important for the current sheet studies?

Method: three-fluid MHD

- **Exact force-free solution (Michel 1973)**

$$B_r = B_{\rm L}\frac{R_{\rm L}^2}{r^2}, \qquad n^{\pm} = \lambda\frac{\Omega B_{\rm L}}{2\pi c e}\frac{R_{\rm L}^2}{r^2},$$

$$E_\theta = B_\varphi = -B_{\rm L}\frac{R_{\rm L}}{r}\sin\theta, \qquad n^{\rm b} = \frac{\Omega B_{\rm L}}{2\pi c e}\frac{R_{\rm L}^2}{r^2}\cos\theta.$$

- **Flux functions**

$$\mathbf{E} = -\nabla\Phi_{\rm e}(r,\theta),$$

$$\mathbf{B}_{\rm p} = \frac{\nabla\Psi\times\mathbf{e}_\varphi}{2\pi r\sin\theta}.$$

- **Perturbations**

$$\begin{aligned}
n^{\pm} &= \frac{\Omega B_{\rm L}}{2\pi c e}\frac{R_{\rm L}^2}{r^2}\left[\lambda+\eta^{\pm}(r,\theta)\right],\\
n^{\rm b} &= \frac{\Omega B_{\rm L}}{2\pi c e}\frac{R_{\rm L}^2}{r^2}\left[\cos\theta+\eta^{\rm b}(r,\theta)\right],\\
\Phi_{\rm e}(r,\theta) &= \frac{\Omega R_{\rm L}^2 B_{\rm L}}{c}\left[-\cos\theta+\delta(r,\theta)\right],\\
\Psi(r,\theta) &= 2\pi B_{\rm L}R_{\rm L}^2\left[1-\cos\theta+\varepsilon f(r,\theta)\right],\\
v_r^{\pm,\rm b} &= c\left[1-\xi_r^{\pm,\rm b}(r,\theta)\right],\\
v_{\theta,\varphi}^{\pm,\rm b} &= c\xi_{\theta,\varphi}^{\pm,\rm b}(r,\theta),
\end{aligned}$$

$$\begin{aligned}
B_r &= B_{\rm L}\frac{R_{\rm L}^2}{r^2}\left(1+\frac{\varepsilon}{\sin\theta}\frac{\partial f}{\partial\theta}\right),\\
B_\theta &= -\varepsilon B_{\rm L}\frac{R_{\rm L}^2}{r\sin\theta}\frac{\partial f}{\partial r},\\
B_\varphi &= B_{\rm L}\frac{\Omega R_{\rm L}}{c}\frac{R_{\rm L}}{r}\left[-\sin\theta-\zeta(r,\theta)\right],\\
E_r &= -B_{\rm L}\frac{\Omega R_{\rm L}^2}{c}\frac{\partial\delta}{\partial r},\\
E_\theta &= B_{\rm L}\frac{\Omega R_{\rm L}^2}{cr}\left(-\sin\theta-\frac{\partial\delta}{\partial\theta}\right).
\end{aligned}$$

Maxwell equations (stationary) + equations of motion

$$\frac{\partial}{\partial r}\left(\xi_\theta^\pm \gamma^\pm\right) + \frac{\xi_\theta^\pm \gamma^\pm}{r} = \pm 4\lambda\sigma\left(-\frac{1}{r}\frac{\partial \delta}{\partial \theta} + \frac{\zeta}{r} - \frac{\sin\theta}{r}\xi_r^\pm + \frac{c}{\Omega r^2}\xi_\varphi^\pm\right),$$

$$\frac{\partial}{\partial r}\left(\xi_\theta^{\mathrm{b}} \gamma^{\mathrm{b}}\right) + \frac{\xi_\theta^{\mathrm{b}} \gamma^{\mathrm{b}}}{r} = -4\lambda\sigma\left(-\frac{1}{r}\frac{\partial \delta}{\partial \theta} + \frac{\zeta}{r} - \frac{\sin\theta}{r}\xi_r^{\mathrm{b}} + \frac{c}{\Omega r^2}\xi_\varphi^{\mathrm{b}}\right),$$

$$\frac{\partial}{\partial r}\left(\gamma^\pm\right) = \pm 4\lambda\sigma\left(-\frac{\partial \delta}{\partial r} - \frac{\sin\theta}{r}\xi_\theta^\pm\right),$$

$$\frac{\partial}{\partial r}\left(\gamma^{\mathrm{b}}\right) = -4\lambda\sigma\left(-\frac{\partial \delta}{\partial r} - \frac{\sin\theta}{r}\xi_\theta^{\mathrm{b}}\right),$$

$$\frac{\partial}{\partial r}\left(\xi_\varphi^\pm \gamma^\pm\right) + \frac{\xi_\varphi^\pm \gamma^\pm}{r} = \mp 4\lambda\sigma\left(\varepsilon\frac{c}{\Omega r\sin\theta}\frac{\partial f}{\partial r} + \frac{c}{\Omega r^2}\xi_\theta^\pm\right),$$

$$\frac{\partial}{\partial r}\left(\xi_\varphi^{\mathrm{b}} \gamma^{\mathrm{b}}\right) + \frac{\xi_\varphi^{\mathrm{b}} \gamma^{\mathrm{b}}}{r} = 4\lambda\sigma\left(\varepsilon\frac{c}{\Omega r\sin\theta}\frac{\partial f}{\partial r} + \frac{c}{\Omega r^2}\xi_\theta^{\mathrm{b}}\right).$$

$$2(\Delta\eta + \eta^{\mathrm{b}}) + \frac{\partial}{\partial r}\left(r^2\frac{\partial \delta}{\partial r}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\delta}{\partial\theta}\right) = 0,$$

$$\frac{1}{2\sin\theta}\frac{\partial}{\partial\theta}(\zeta\sin\theta) = \lambda\Delta\xi_r - \xi_r^{\mathrm{b}}\cos\theta - \Delta\eta - \eta^{\mathrm{b}},$$

$$\frac{\partial\zeta}{\partial r} = \frac{2}{r}\left(\lambda\Delta\xi_\theta - \xi_\theta^{\mathrm{b}}\cos\theta\right),$$

$$\frac{\varepsilon}{\sin\theta}\frac{\partial^2 f}{\partial r^2} + \frac{\varepsilon}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial f}{\partial\theta}\right) = 2\frac{\Omega}{rc}\left(\cos\theta\,\xi_\varphi^{\mathrm{b}} - \lambda\Delta\xi_\varphi\right).$$

$$\begin{aligned}
\delta(R_{\mathrm{L}},\theta) &= 0, \\
\varepsilon f(R_{\mathrm{L}},\theta) &= 0, \\
\eta^+(R_{\mathrm{L}},\theta) - \eta^-(R_{\mathrm{L}},\theta) &= 0, \\
\eta^{\mathrm{b}}(R_{\mathrm{L}},\theta) &= 0,
\end{aligned}
\qquad
\begin{aligned}
\xi_\theta^\pm(R_{\mathrm{L}},\theta) &= 0, \\
\xi_\theta^{\mathrm{b}}(R_{\mathrm{L}},\theta) &= 0, \\
\xi_\varphi^{\mathrm{b}}(R_{\mathrm{L}},\theta) &= 0, \\
\xi_\varphi^\pm(R_{\mathrm{L}},\theta) &= 0, \\
\gamma^\pm(R_{\mathrm{L}},\theta) &= \gamma_{\mathrm{in}}, \\
\gamma^{\mathrm{b}}(R_{\mathrm{L}},\theta) &= \gamma_{\mathrm{in}}^{\mathrm{b}},
\end{aligned}$$

13 equations, 15 boundary conditions

General properties

- **Integrals of motion**

$$\zeta - \frac{2}{\tan\theta}\delta + \frac{\lambda(\gamma^+ + \gamma^-) + (\gamma^{\rm b} - \gamma^{\rm b}_{\rm in})\cos\theta}{2\lambda\sigma\sin\theta} = \frac{1}{\sigma\sin\theta}\gamma_{\rm in} + \frac{l(\theta)}{\sin\theta},$$

Conservation of total energy flux along the magnetic field line

$$\delta = \varepsilon f \mp \frac{1}{4\lambda\sigma}\gamma^{\pm}\left(1 - \frac{\Omega r\sin\theta}{c}\xi^{\pm}_{\varphi}\right) \pm \frac{1}{4\lambda\sigma}\gamma_{\rm in},$$

$$\delta = \varepsilon f + \frac{1}{4\lambda\sigma}\gamma^{\rm b}\left(1 - \frac{\Omega r\sin\theta}{c}\xi^{\rm b}_{\varphi}\right) - \frac{1}{4\lambda\sigma}\gamma^{\rm b}_{\rm in}.$$

Conservation of z-component of angular momentum for all types of particles

- $\lambda\sigma$ **>> 1**

$\xi^+ = \xi^-$ and $\gamma^+ = \gamma^- = \gamma$

$$2\gamma^3 - 2\sigma\left[K + \frac{1}{2x^2} + \frac{\gamma_{\rm in}}{\sigma}\right]\gamma^2 + \sigma\sin^2\theta + \sigma\frac{c^2\gamma_{\rm in}^2}{\Omega^2 r^2} = 0$$

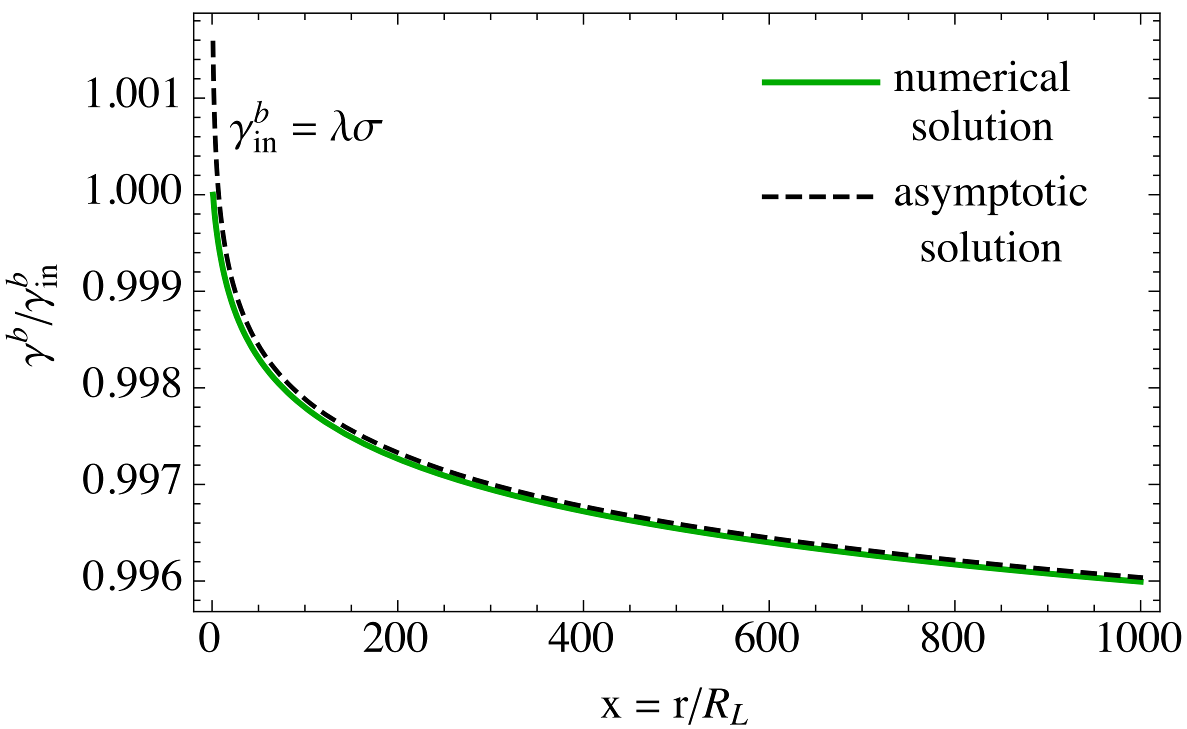
$$K(r,\theta) = 2\cos\theta\delta - \sin\theta\frac{\partial\delta}{\partial\theta} - \frac{\gamma^{\rm b} - \gamma^{\rm b}_{\rm in}}{2\sigma\lambda}\cos\theta + \frac{l(\theta)}{\sin\theta}.$$

Beam deceleration

- **General equation**

$$\frac{\partial}{\partial x}\left(x^2\frac{\partial p}{\partial x}\right) = \Lambda^2\left(\frac{1}{(\gamma_{\rm in}^{\rm b})^2}\frac{1}{p+1} - \frac{1}{\gamma^2} - \frac{1}{x^2}\frac{p}{p+1} + \frac{\Delta}{x^2}\right)$$

$$p = \frac{(\gamma^{\rm b})^2 - (\gamma_{\rm in}^{\rm b})^2}{(\gamma_{\rm in}^{\rm b})^2},\ \Lambda = \frac{4\lambda\sigma}{\gamma_{\rm in}^{\rm b}},\ \Delta = \frac{\gamma^2 - \gamma_{\rm in}^2}{\gamma^2}$$



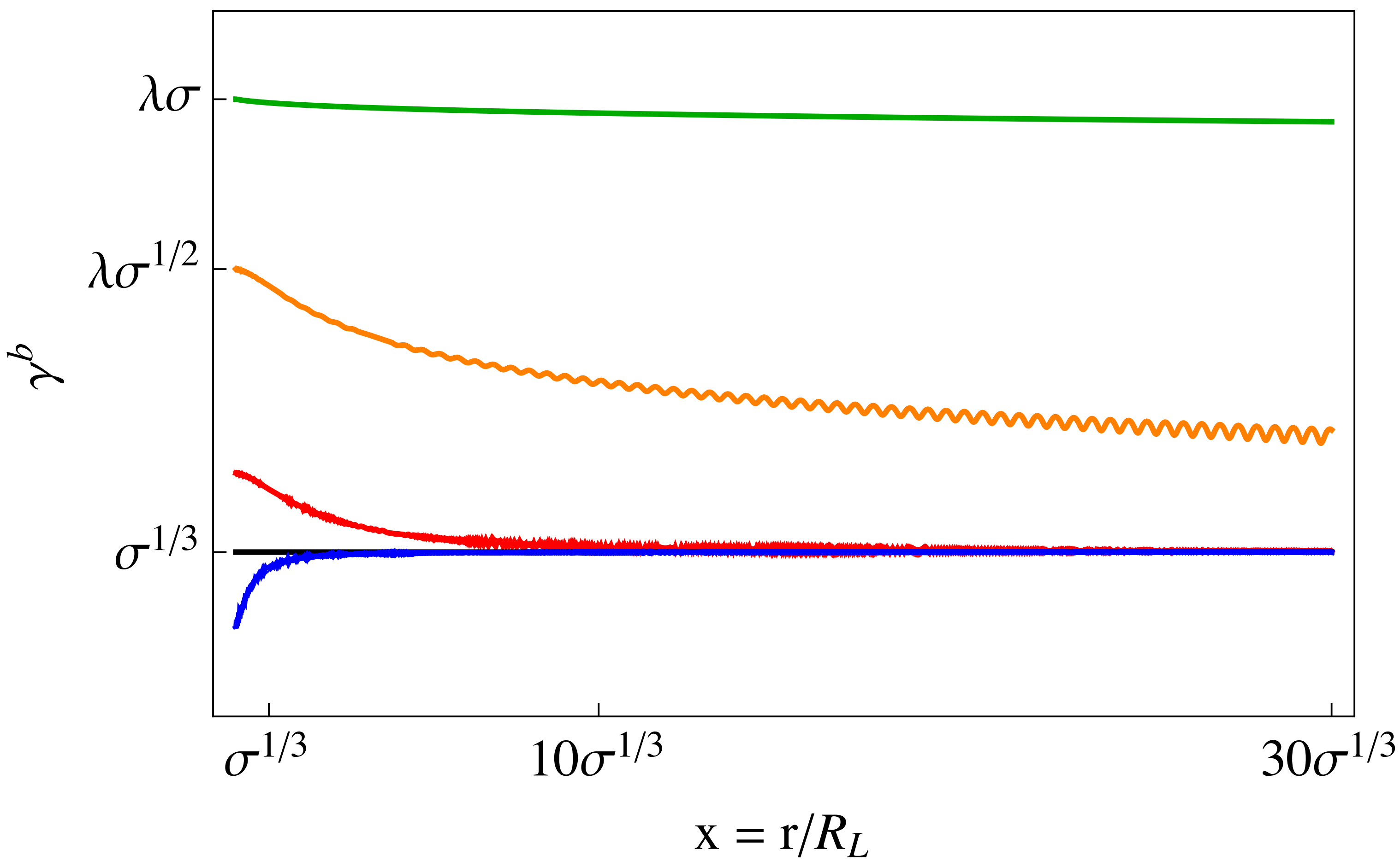
- **Exact solution for very energetic beam**

$$p(x) = \Lambda^2\int_1^x {\rm d}x' \sin\left(\frac{\Lambda}{x} - \frac{\Lambda}{x'}\right)\left[\frac{1}{\gamma^2(x')} + \frac{\Delta(x')}{(x')^2}\right]$$

$$p(x) \simeq -\left(\frac{\Lambda}{\gamma_{\rm out}}\right)^2\left[\ln(x/\Lambda) - \Gamma\right]$$

- **Different regimes**

$$\begin{aligned} \gamma_{\rm out}^{\rm b} &= \gamma_{\rm out}; & \gamma_{\rm in}^{\rm b} < \lambda\sigma^{1/2}, \\ \gamma_{\rm out} < \gamma_{\rm out}^{\rm b} &< \gamma_{\rm in}^{\rm b}; & \lambda\sigma^{1/2} < \gamma_{\rm in}^{\rm b} < \lambda\sigma, \\ \gamma_{\rm out}^{\rm b} &= \gamma_{\rm in}^{\rm b}; & \gamma_{\rm in}^{\rm b} > \lambda\sigma, \end{aligned}$$



Plasma acceleration

- **General equation**

$$2\gamma^3 - 2\sigma\left[K + \frac{1}{2x^2} + \frac{\gamma_{\rm in}}{\sigma}\right]\gamma^2 + \sigma\sin^2\theta + \sigma\frac{c^2\gamma_{\rm in}^2}{\Omega^2 r^2} = 0$$

$$K(r,\theta) = 2\cos\theta\delta - \sin\theta\frac{\partial\delta}{\partial\theta} - \frac{\gamma^{\rm b} - \gamma_{\rm in}^{\rm b}}{2\sigma\lambda}\cos\theta + \frac{l(\theta)}{\sin\theta}.$$

- **Conditions at fast magnetosonic surface**

Fast rotator $(\gamma_{\rm in} < \sigma^{1/3})$

$$\delta_{\rm F} \approx \sigma^{-2/3},$$
$$r_{\rm F} \approx \sigma^{1/3}\sin^{-1/3}\theta\, R_{\rm L},$$
$$\gamma_{\rm F} = \sigma^{1/3}\sin^{2/3}\theta,$$

Beskin & Rafikov (2000)

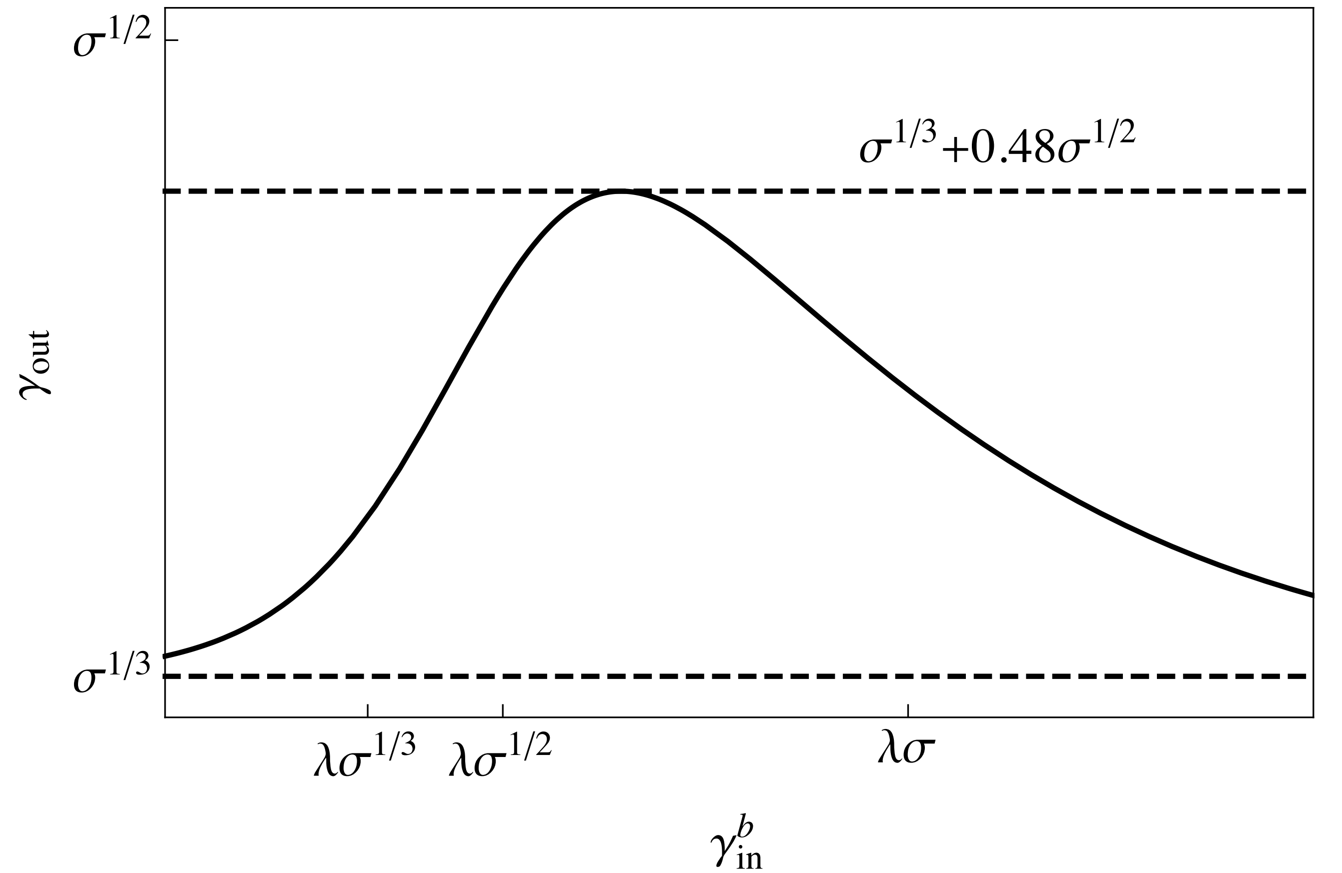
Slow rotator $(\gamma_{\rm in} > \sigma^{1/3})$

$$\delta \approx \gamma_{\rm in}\sigma^{-1},$$
$$r_{\rm F} \approx \sigma^{1/2}\gamma_{\rm in}^{-1/2} R_{\rm L},$$
$$\gamma_{\rm F} \approx \gamma_{\rm in}.$$

The same as for one-fluid MHD

- **After the fast magnetosonic surface**

$$\gamma_{\rm out} = \max(\sigma^{1/3}, \gamma_{\rm in}) - \frac{\gamma_{\rm out}^{\rm b} - \gamma_{\rm in}^{\rm b}}{2\lambda}$$



Results

- In most pulsars deceleration of the beam is insignificant
- Among 2003 sources only 65 have relatively small initial gamma-factors of the beam leading to deceleration
- Beam may accelerate secondary plasma, but only by a factor of $\sigma^{1/6}$, which is usually less then 10
- The beam deceleration occurs on a scale of the fast magnetosonic surface, so even for the fastest pulsars, primary particles will eventually intersect the current sheet.