Radio pulsars (thirty years after)

V.S.Beskin, A.V.Gurevich, Ya.N.Istomin

(R.Rafikov, A.Tchekhovskoy, A.Philippov, A.Zheltoukhov, E.Nokhrina, H.Hakobyan, L.Arzamasskiy, V.Prokofev)

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Radio pulsars

30 years – first pulsar publication

25 years – theory of the pulsar radio emission

20 years – book

Three points

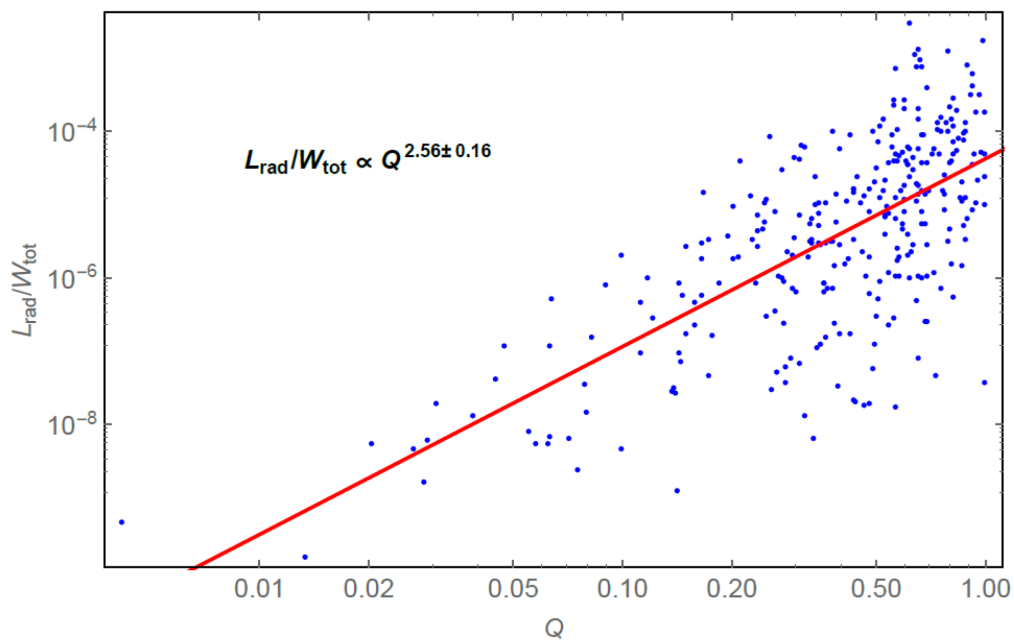
- Current losses
- Asymptotic behavior
- Anomalous torque

One more prediction

$$Q = 2\, P^{11/10}\, \dot{P}_{-15}^{-4/10} \quad (\text{RS} - \text{style gap})$$

- $H/R_0 \sim Q\,, \quad r_{in}/R_0 \sim Q^{7/9} \qquad \text{for } Q < 1$
- $W_{part}/W_{tot} \sim Q^2$

$a = L_{rad}/W_{tot}$

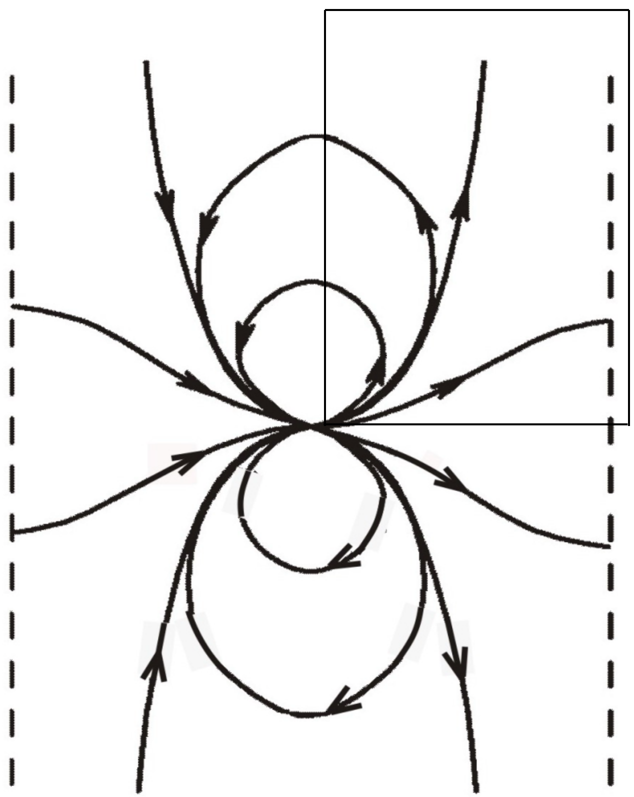


Very briefly

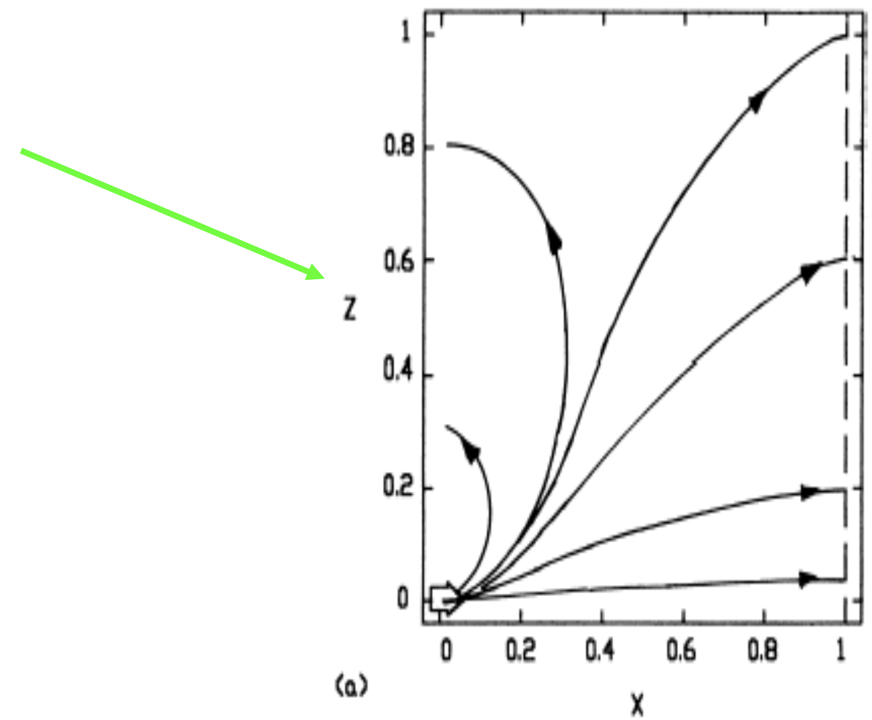
- Modern numerical simulations confirm the current loss mechanism of the radio pulsar braking.
- Current sheet in the pulsar wind asymptotical infinity (we do not beleive in) is essentially time-dependent.
- Anomalous torque acting on the body depends on its internal structure (as it depends on the angular momentum of the electromagnetic field inside the body).

Current losses

Orthogonal rotator, $I = 0$



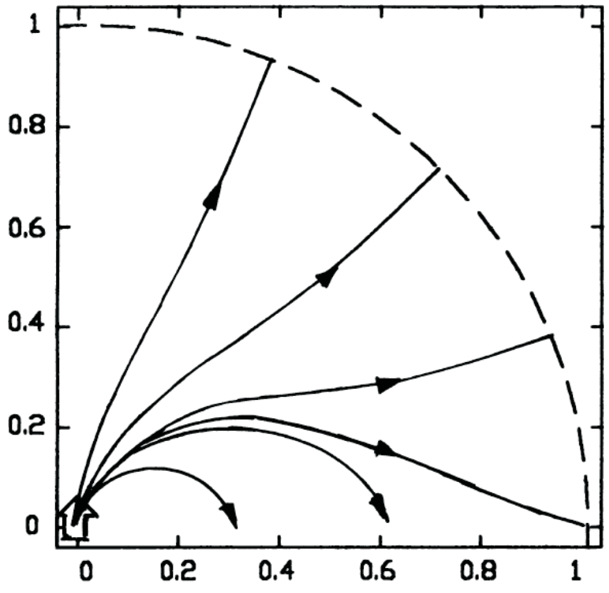
$\chi = 90^{\circ}$

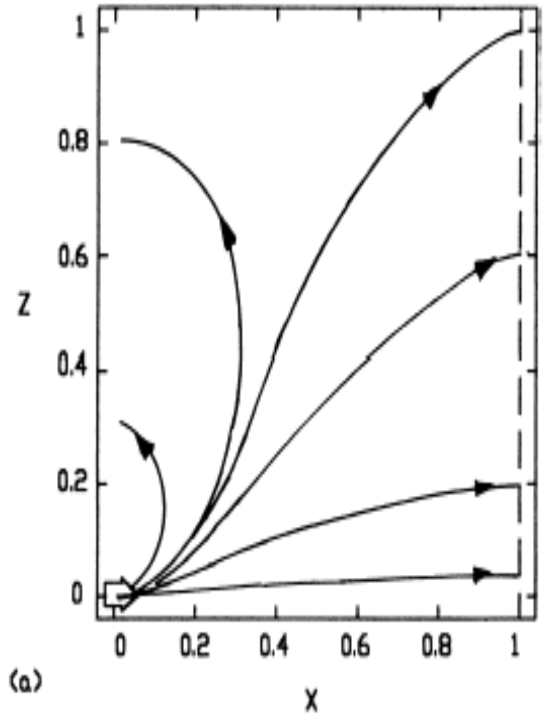


VB, A.V.Gurevich, Ya.N.Istomin, JETP, **58**, 235 (1983)

L.Mestel, P.Panagi, S.Shibata, MNRAS, **309**, 388 (1999)

Orthogonal rotator, $I = 0$

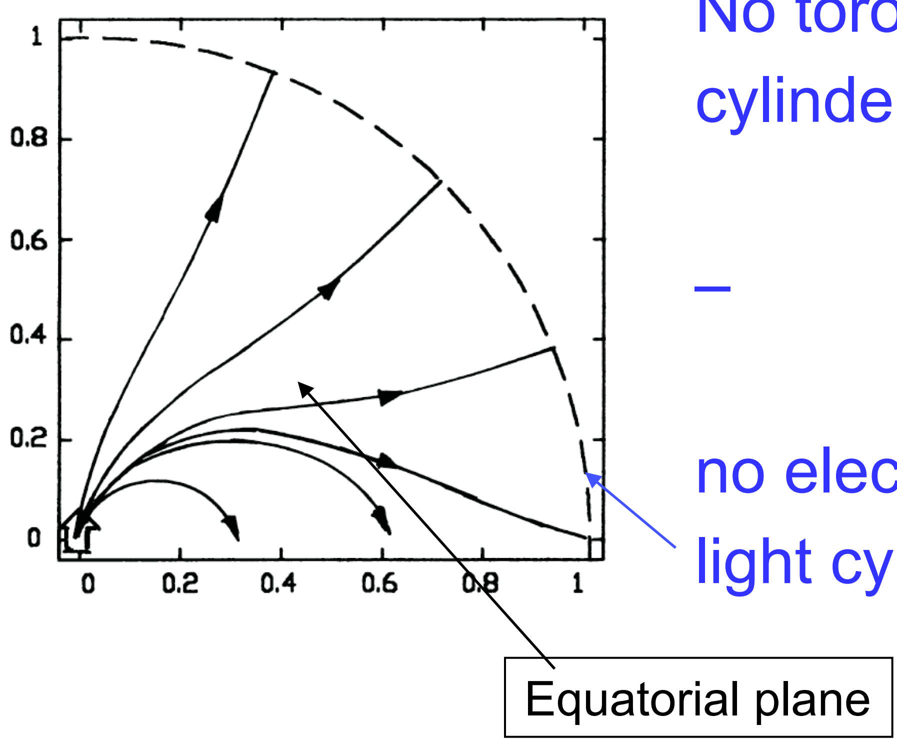




L.Mestel, P.Panagi, S.Shibata, MNRAS, **309**, 388 (1999)

NO magnetodipole radiation

VB, A.V.Gurevich, Ya.N.Istomin, JETP **58**, 235 (1983)
L.Mestel, P.Panagi, S.Shibata, MNRAS, **309**, 388 (1999)

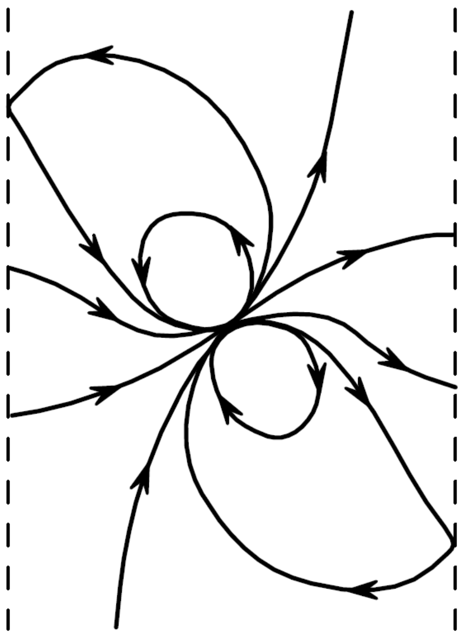


No toroidal magnetic field at the light cylinder for zero longitudinal current

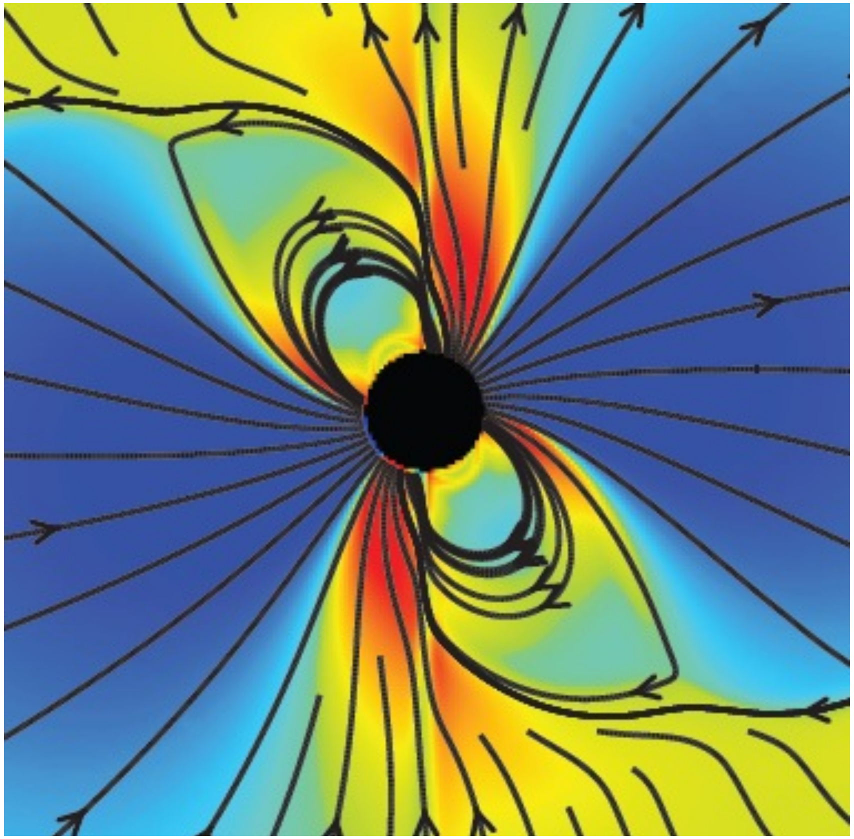
– $$B_\varphi \propto (1 - x_r^2)^2$$

no electromagnetic flux through the light cylinder (dashed line).

Incline rotator

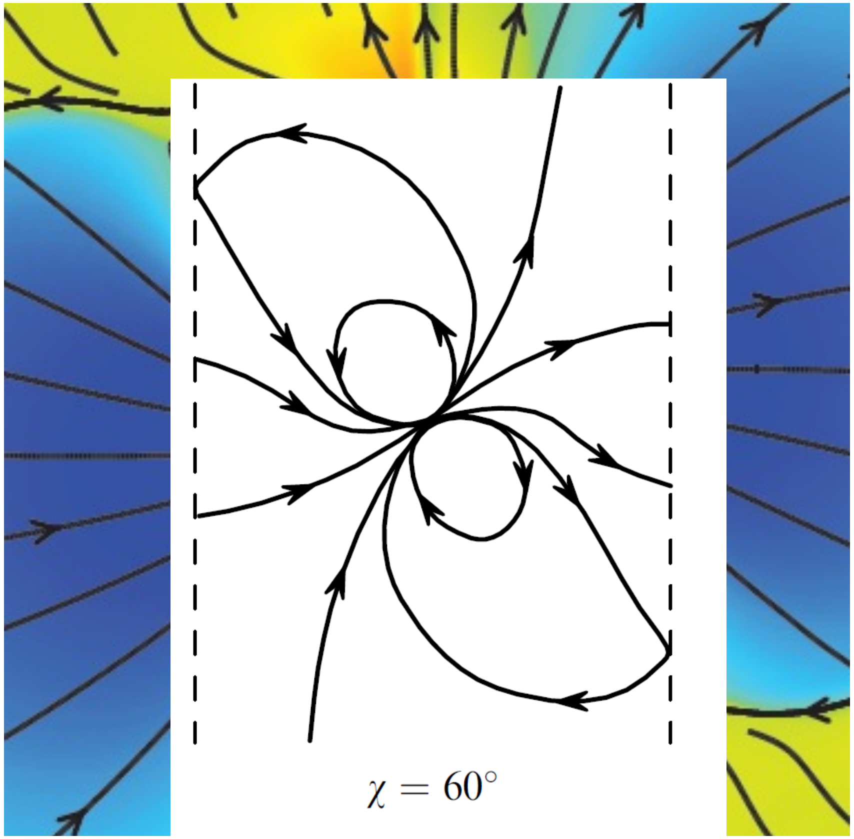


$\chi = 60^\circ$



A.Tchekhovskoy,
A.Spitkovsky, J.Li,
arxiv.org/pdf/1211.2803.pdf

Incline rotator



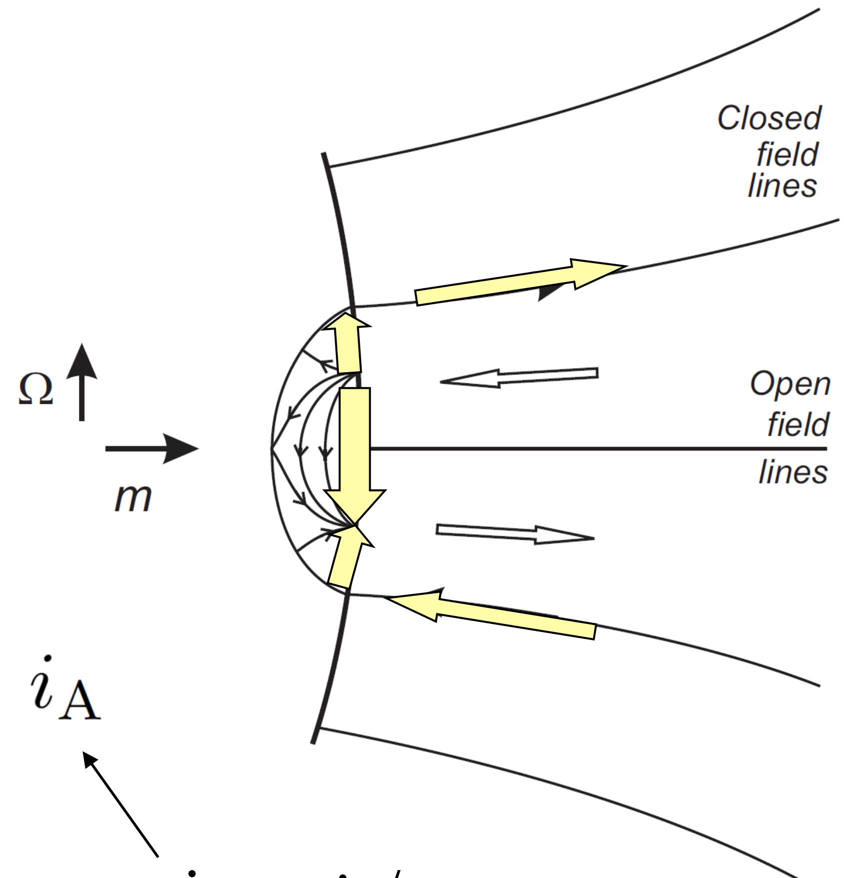
A.Tchekhovskoy,
A.Spitkovsky, J.Li,
arxiv.org/pdf/1211.2803.pdf

Orthogonal rotator

VB, A.V.Gurevich, Ya.N.Istomin, JETP **58**, 235 (1983)

$$j_{\mathrm{GJ}} \approx \frac{\Omega B}{2\pi} \cos \mathbf{q}_{\mathrm{m}}$$

$$\mathbf{K} = \frac{1}{c} \int [\mathbf{r} \times [\mathbf{J}_{\mathrm{s}} \times \mathbf{B}]] \mathrm{d}S$$



$$W_{\mathrm{tot}}^{(90)} = c_{\perp} \frac{B_0^2 \Omega^4 R^6}{c^3} \left(\frac{\Omega R}{c} \right) i_{\mathrm{A}}$$

$$i_{\mathrm{A}} = j_{\parallel} / \rho_{\mathrm{GJ}} c$$

$$W_{\mathrm{tot}} \approx \frac{B_0^2 \Omega^4 R^6}{c^3} \cos^2 \chi$$

$i_{\mathrm{A}} = j_{\parallel} / \rho_{\mathrm{GJ}} c$

BGI – main results

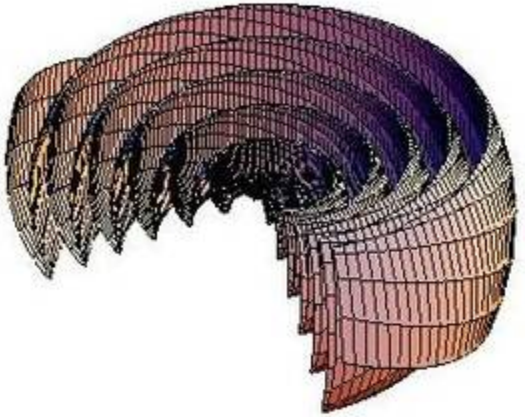
- No magnetodipole radiation
- **All** energy losses are to be connected with current losses

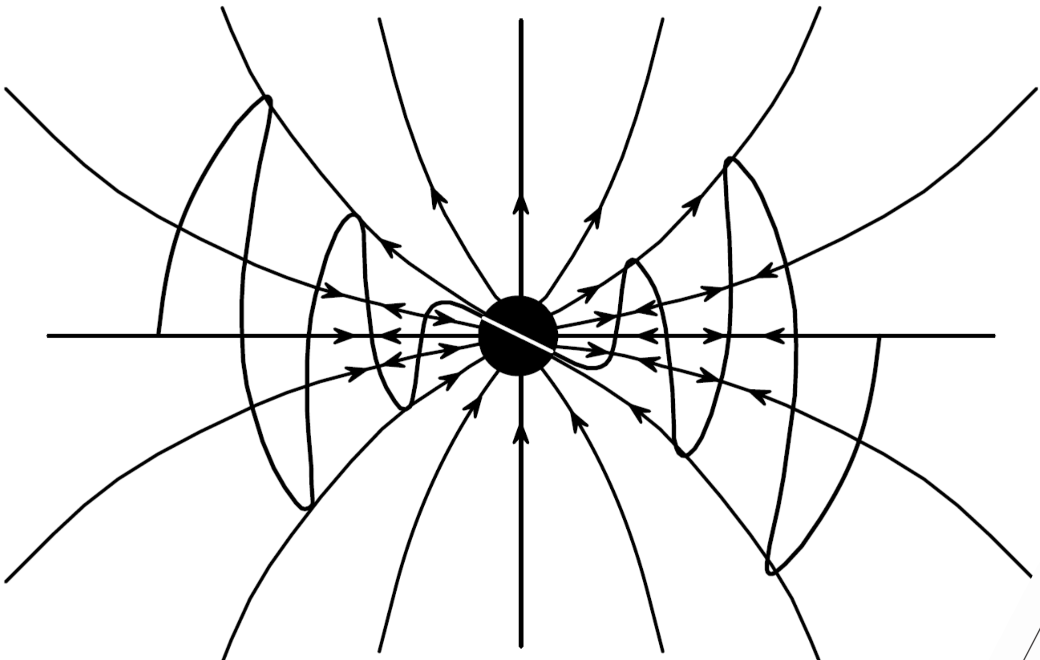
$$W_{\rm tot} \approx \frac{B_0^2 \Omega^4 R^6}{c^3} \cos^2\chi$$

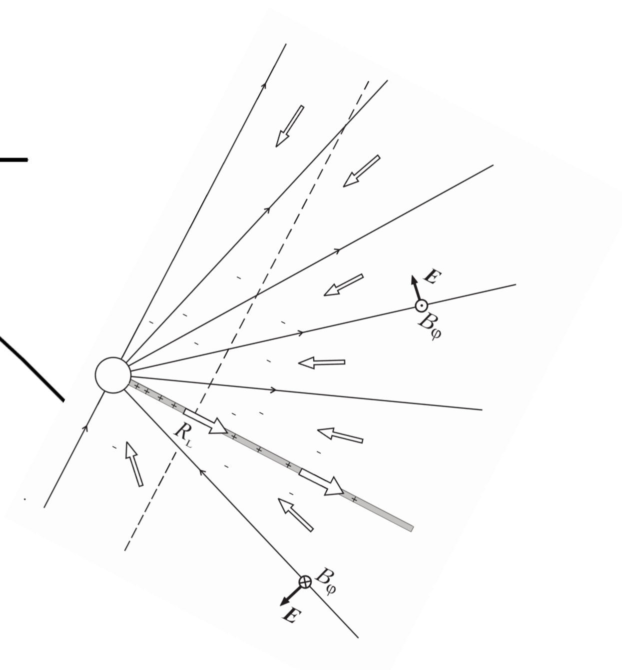
- Smaller energy losses for orthogonal rotator
- Inclination angle evolves to 90 deg.
- Existence of the back electric current along the separatrix

Orthogonal rotator – inclined split monopole

S.V.Bogovalov, A&A, **349**, 1017 (1999)

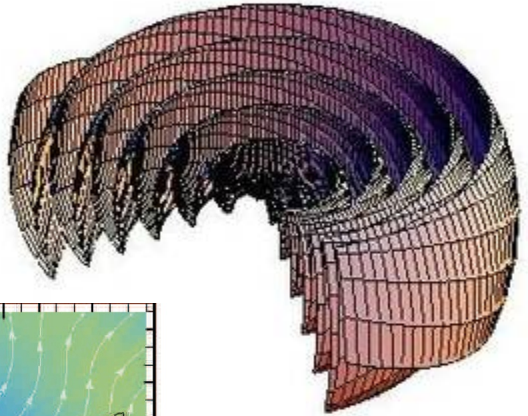


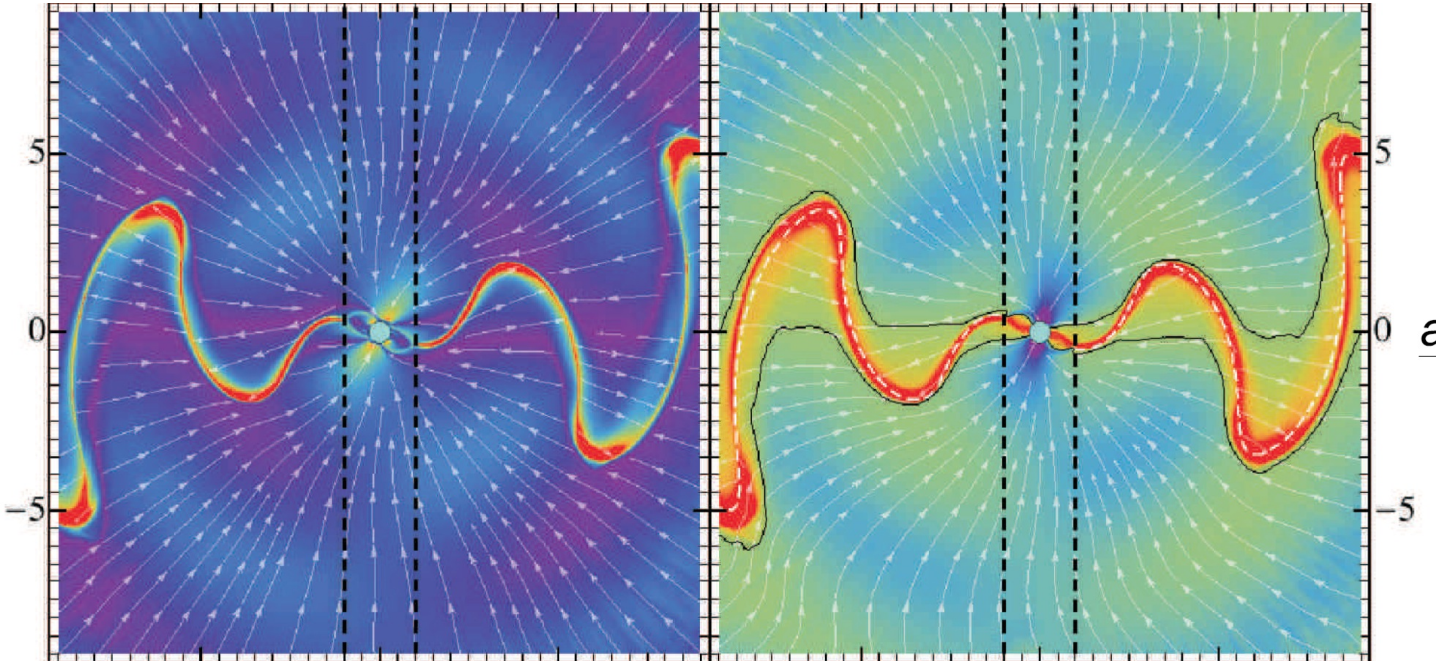




Orthogonal rotator – numerical

I.Contopoulos et al, (2012)





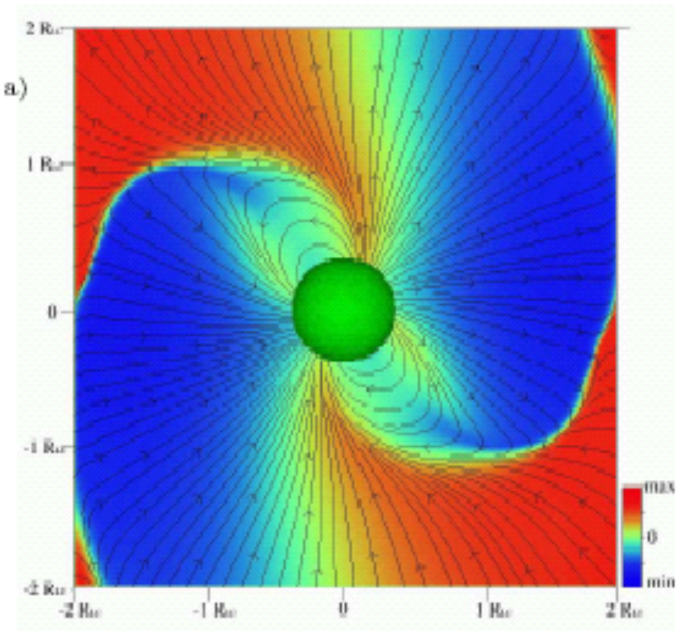
$$\Phi = \sin\alpha \sin\theta \sin(\varphi - \Omega t + \Omega r/c) + \cos\theta \cos\alpha$$

Split monopole – main results

- No magnetodipole radiation
- Energy losses do not depend on the inclination angle
- Existence of the current sheet separating magnetic fluxes
- No information about the inclination angle evolution

Incline rotator – numerical

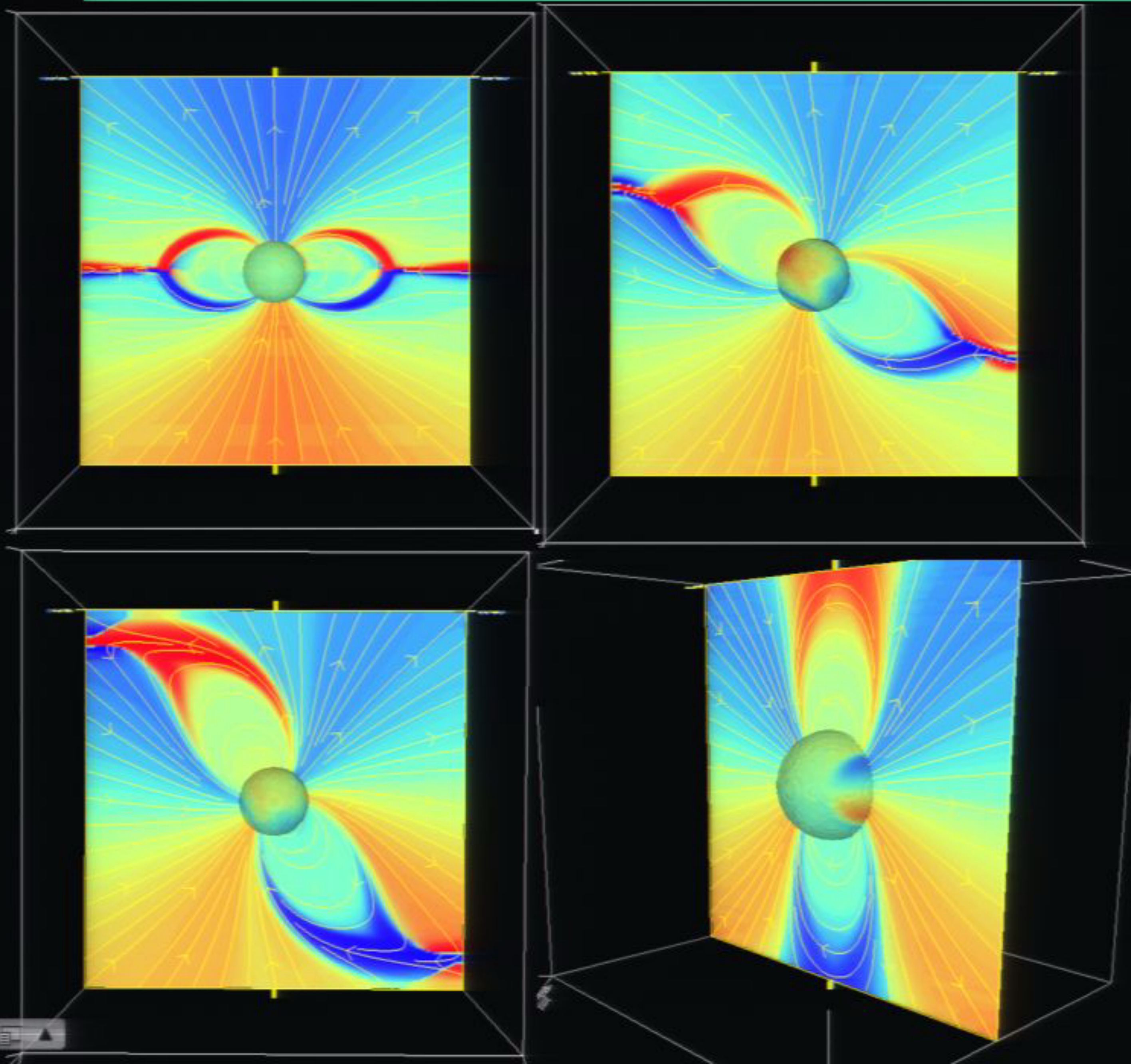
A.Spitkovsky, ApJ Lett., **648**, L51 (2006)



$$W_{\rm tot} \approx \frac{1}{4}\,\frac{B_0^2\Omega^4 R^6}{c^3}\,(1+\sin^2\chi)$$

- No magnetodipole radiation
- Back electric current along the separatrix
- Smaller energy losses for axisymmetric rotator

Magnetospheric currents

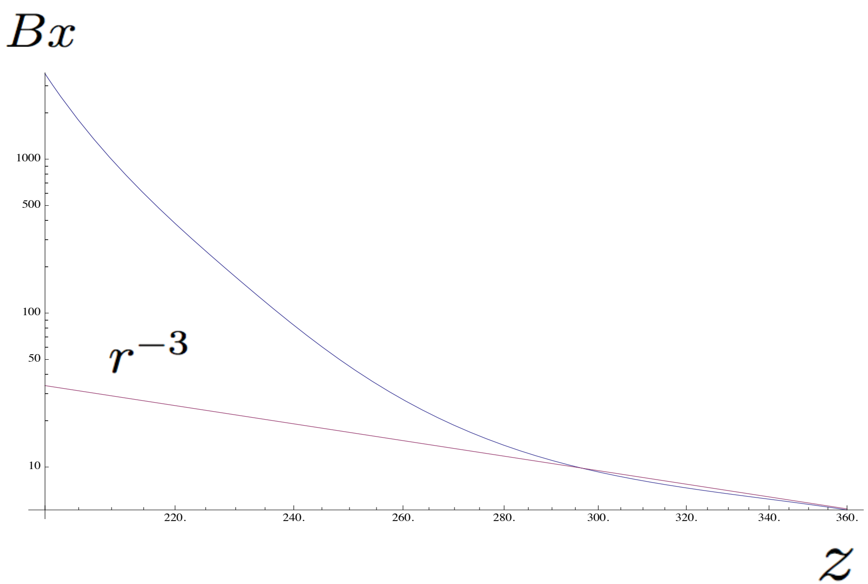


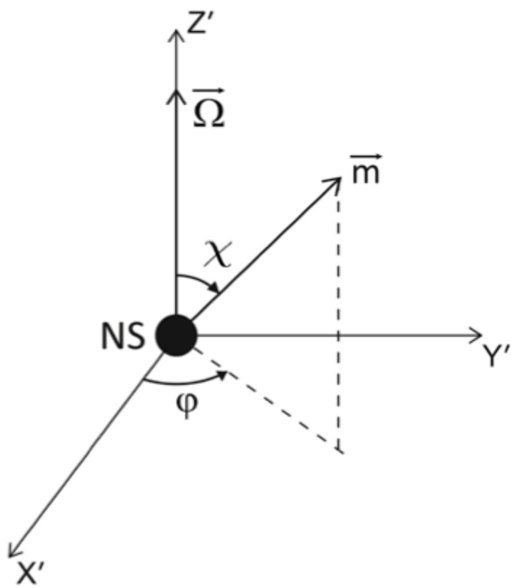
Oppositely flowing currents can occupy the same open flux tube. Does this have any obervational implications?

There is always a null-current field line in the open zone.

Spitkovsky solution, $\chi = 60^{o}$

No magnetodipole radiation





In vacuum $B_x = \frac{\ddot{d}}{cr}$

VB, YA.N.Istomin, A.A.Philippov, Phys. Uspekhi, **56**, 164 (2013)

Current losses again

$$K_{\parallel} = -\frac{B_0^2\Omega^3R^6}{c^3}\left[c_{\parallel}i_{\rm S} + \mu_{\parallel}\left(\frac{\Omega R}{c}\right)^{1/2}i_{\rm A}\right],$$

$$K_{\perp} = -\frac{B_0^2\Omega^3R^6}{c^3}\left[\mu_{\perp}\left(\frac{\Omega R}{c}\right)^{1/2}i_{\rm S} + c_{\perp}\left(\frac{\Omega R}{c}\right)i_{\rm A}\right]$$

$$J_r\frac{{\rm d}\Omega}{{\rm d}t} = K_{\parallel}\cos\chi + K_{\perp}\sin\chi,$$

$$J_r\Omega\frac{{\rm d}\chi}{{\rm d}t} = K_{\perp}\cos\chi - K_{\parallel}\sin\chi.$$

VB, A.V.Gurevich, Ya.N.Istomin, JETP **58**, 235 (1983)

Current losses again

$$K_{\parallel} = -\frac{B_0^2\Omega^3R^6}{c^3}\left[c_{\parallel}i_{\rm S} + \mu_{\parallel}\left(\frac{\Omega R}{c}\right)^{1/2} i_{\rm A}\right],$$

$$K_{\perp} = -\frac{B_0^2\Omega^3R^6}{c^3}\left[\mu_{\perp}\left(\frac{\Omega R}{c}\right)^{1/2} i_{\rm S} + c_{\perp}\left(\frac{\Omega R}{c}\right) i_{\rm A}\right]$$

$$J_r\frac{{\rm d}\Omega}{{\rm d}t} = K_{\parallel}^{(A)} + \left[K_{\perp}^{(A)} - K_{\parallel}^{(A)}\right]\sin^2\chi,$$

$$J_r\Omega\frac{{\rm d}\chi}{{\rm d}t} = \left[K_{\perp}^{(A)} - K_{\parallel}^{(A)}\right]\sin\chi\cos\chi.$$

A.Philippov, A.Tchekhovskoy, J.Li

$$\boxed{K_{\perp}^{(A)} = i_{\rm A}\left(\frac{\Omega R}{c}\right)K_{\parallel}^{(A)}}$$

Current losses again

$$J_r \frac{\mathrm{d}\Omega}{\mathrm{d}t} = K_{\parallel}^{(A)} + \left[K_{\perp}^{(A)} - K_{\parallel}^{(A)}\right] \sin^2\chi,$$

$$J_r \Omega \frac{\mathrm{d}\chi}{\mathrm{d}t} = \left[K_{\perp}^{(A)} - K_{\parallel}^{(A)}\right] \sin\chi \cos\chi.$$

- One-to-one correspondence

  χ evolves to 90 deg. if $W_{tot}(0) > W_{tot}(90)$

  χ evolves to 0 deg. if $W_{tot}(0) < W_{tot}(90)$

$$K_{\perp}^{(A)} = i_{\mathrm{A}} \left(\frac{\Omega R}{c}\right) K_{\parallel}^{(A)}$$

- For BGI $i_A \sim 1$
- For Michel-Bogovalov $\left[K_{\perp}^{(A)} - K_{\parallel}^{(A)}\right] = 0$
- For Spitkovsky et al low the asymmetrical current is to be (much) larger than GJ one

$$i_A > (\Omega R/c)^{-1}$$

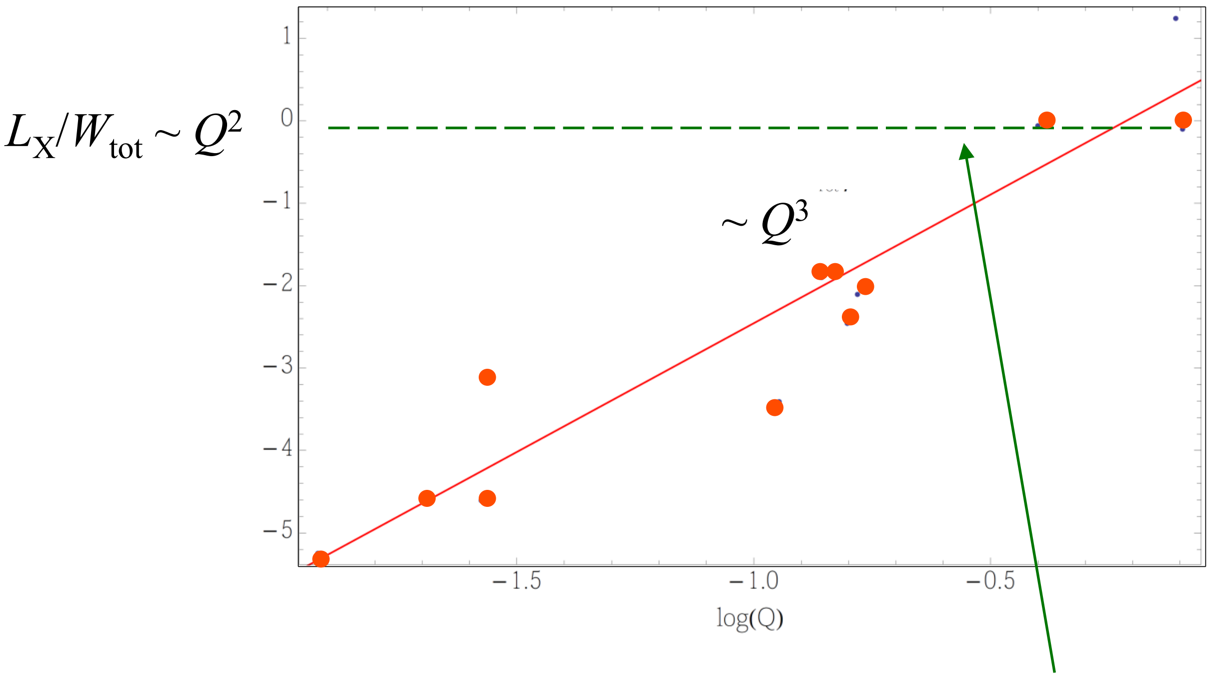
Main problems

- For orthogonal rotator the longitudinal current is to be 10000 (!!!) times larger than local GJ one.
- Heating problem.
- Magnetic field disturbance.

Heating problem

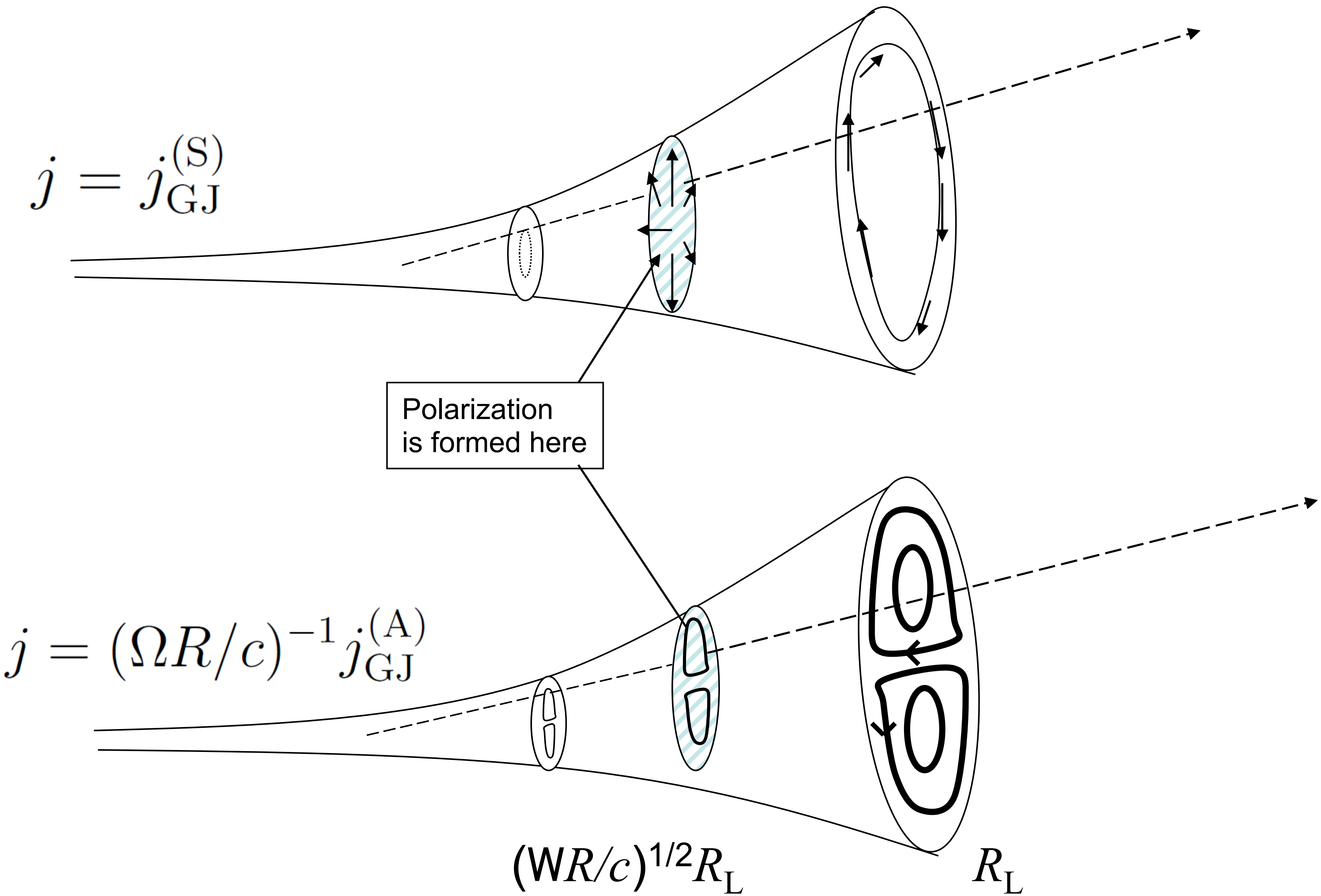
- $Q = 2\, P^{11/10}\, \dot{P}_{-15}^{-4/10}$

- $H/R_0 \sim Q\,, \quad r_{in}/R_0 \sim Q^{7/9}$



Istomin & Sobyanin (2010), Timokhin (2010), Timokhin & Arons (2013)

Magnetic field disturbance



A problem

Can the pair creation process generate large enough longitudinal current to avoid the formation of the light surface?

The main point of view – YES.

Our point of view – NO.

If not, the light surface $|\boldsymbol{E}| = |\boldsymbol{B}|$ is to exist.

Prediction

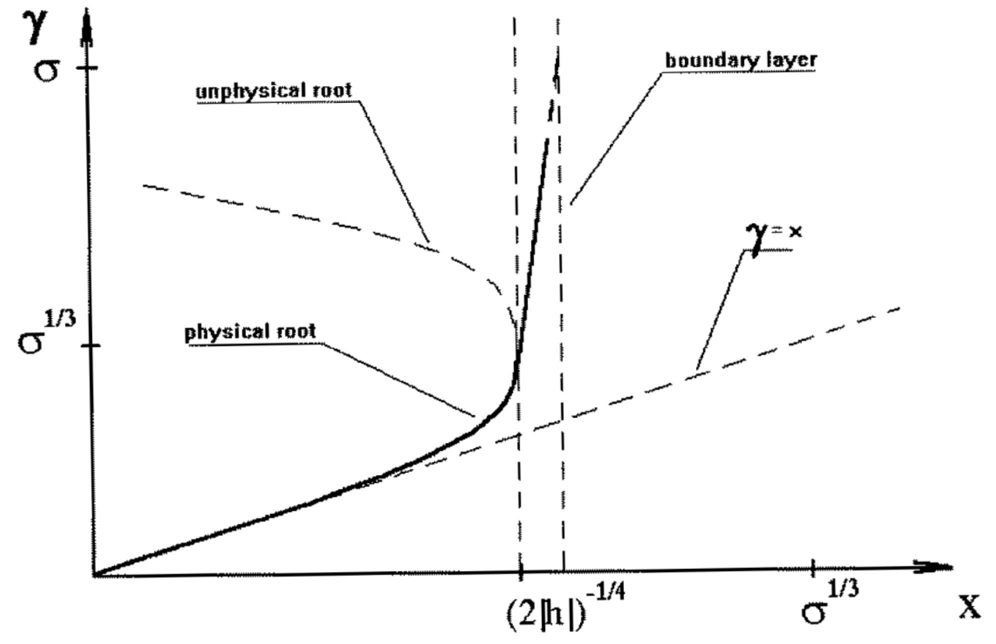
VB, A.V.Gurevich & Ya.N.Istomin, JETP, **85,** 235 (1983)
VB, R.R.Rafikov, MNRAS, **313**, 433 (2000)

- Narrow sheet $\Delta r \sim R_L/\lambda$
- Effective particle acceleration up to $\gamma \sim \sigma$ (10^6 for Crab)

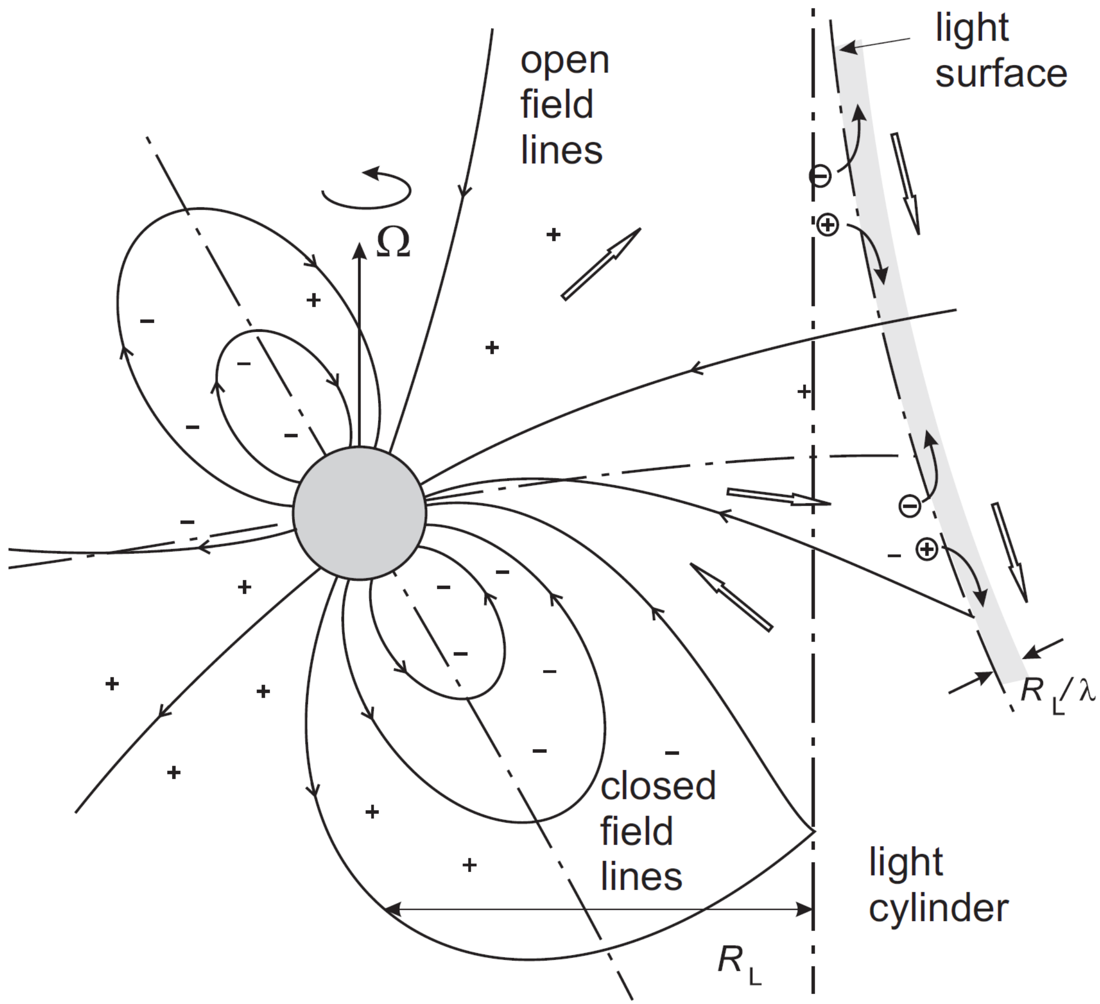
$$\lambda = \frac{n^{(\mathrm{lab})}}{n_{\mathrm{GJ}}}$$

$$\sigma = \frac{\Omega^2 \Psi_{\mathrm{tot}}}{8\pi^2 c^2 \mu \eta}$$

maximum bulk Lorentz-factor



Prediction



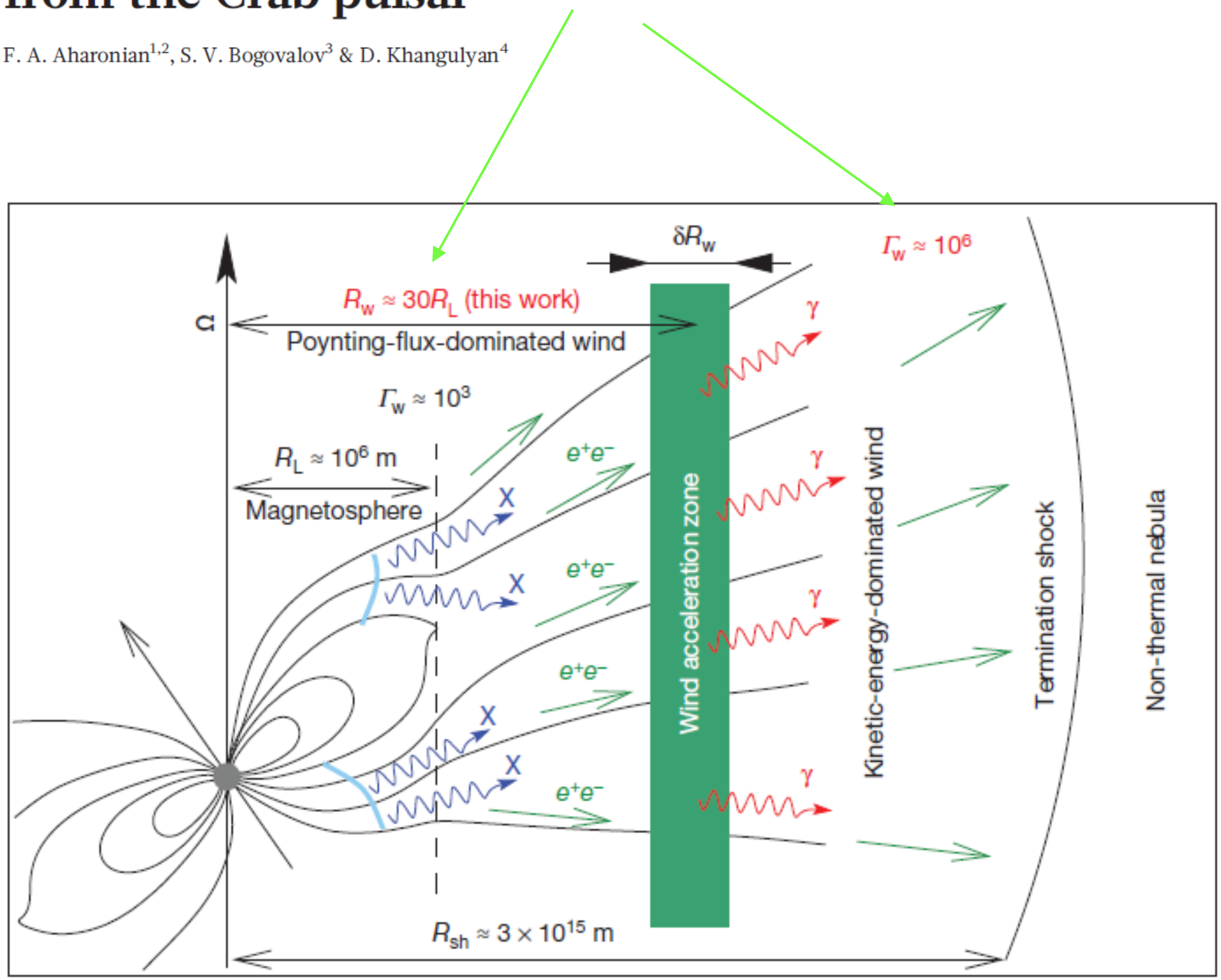
open
field
lines
light
surface
Ω
closed
field
lines
light
cylinder
R_L
R_L/λ

LETTER

doi:10.1038/nature10793

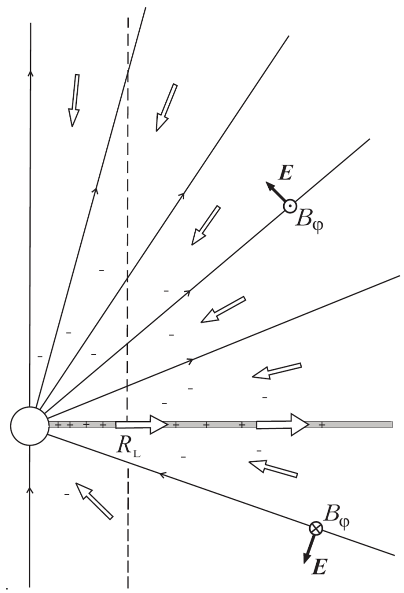
Abrupt acceleration of a 'cold' ultrarelativistic wind from the Crab pulsar

F. A. Aharonian[1,2], S. V. Bogovalov[3] & D. Khangulyan[4]

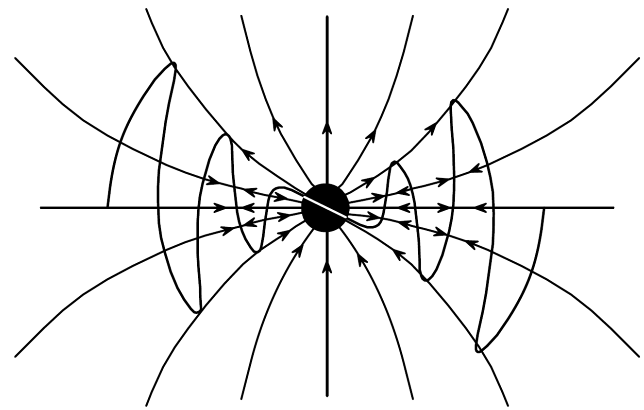


Asymptotic behaviour

Force-free magnetosphere: simple analytical solutions



F.C.Michel (1973)

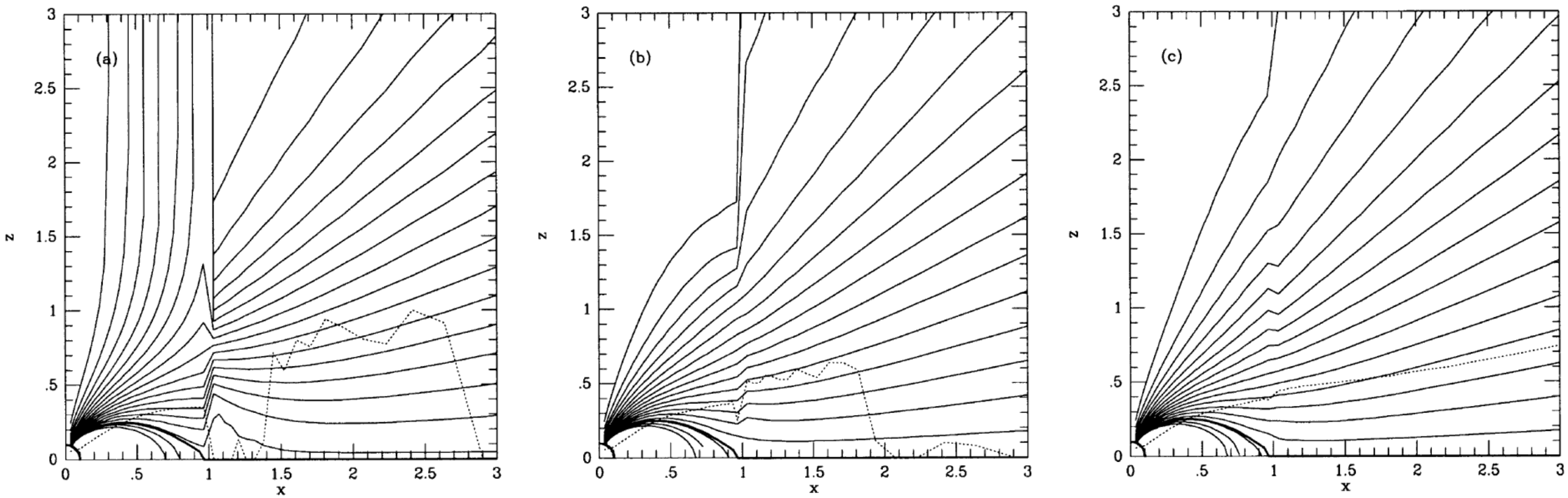


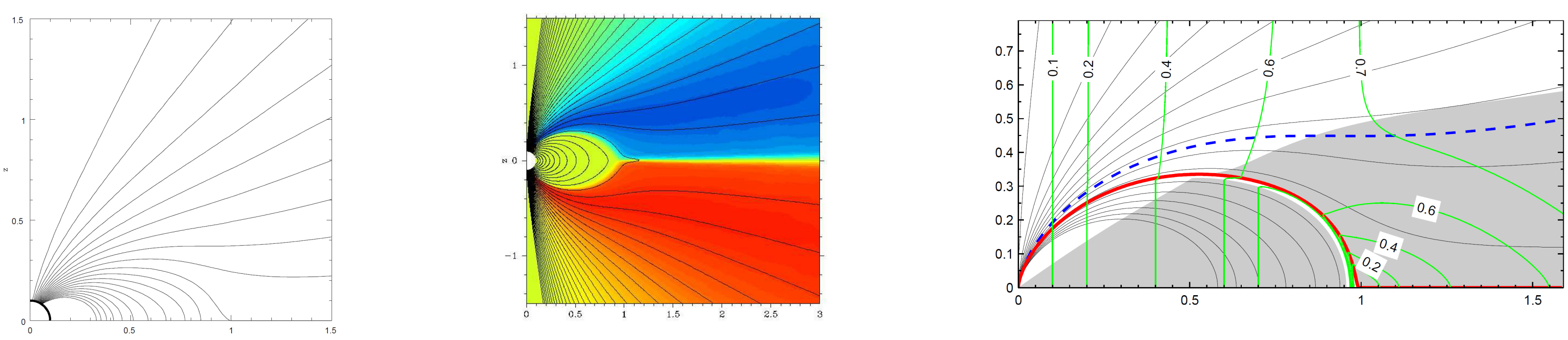
S.V.Bogovalov (1999)

$$B_{\hat{\varphi}} = E_{\hat{\theta}} = -B_0 \left(\frac{\Omega R}{c}\right) \frac{R}{r} \sin\theta$$

$$S \propto \sin^2\theta$$

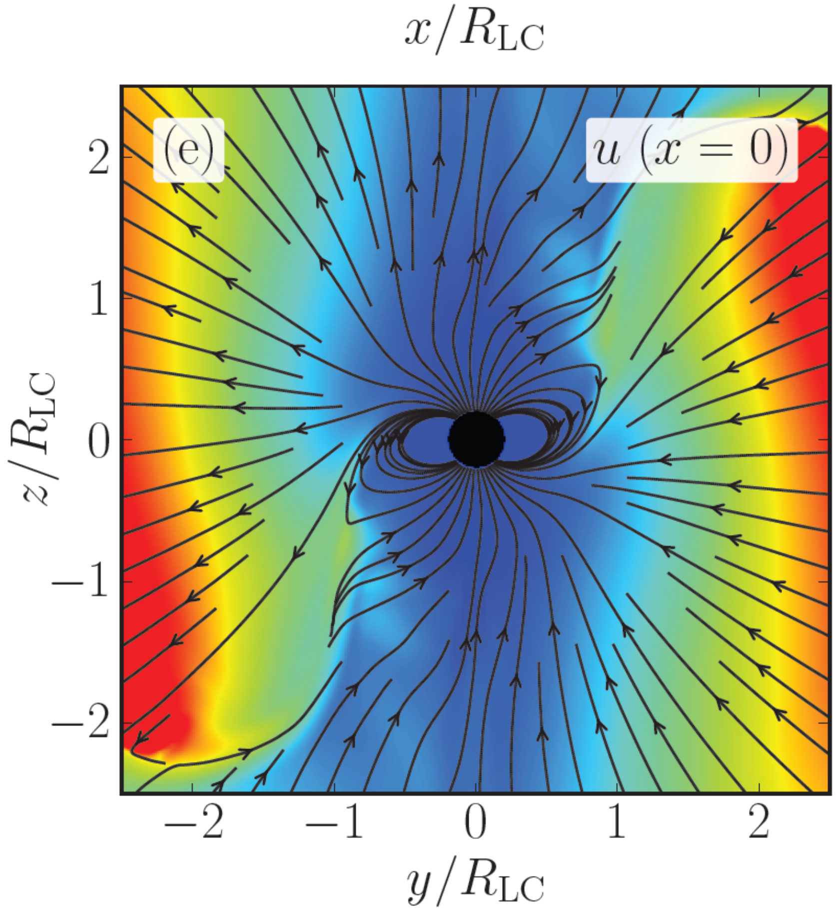
I.Contopoulos, D.Kazanas & Ch.Fendt, ApJ, **511**, 351 (1999)





Gruzinov (2005), Komissarov (2005), A.Timokhin (2005)

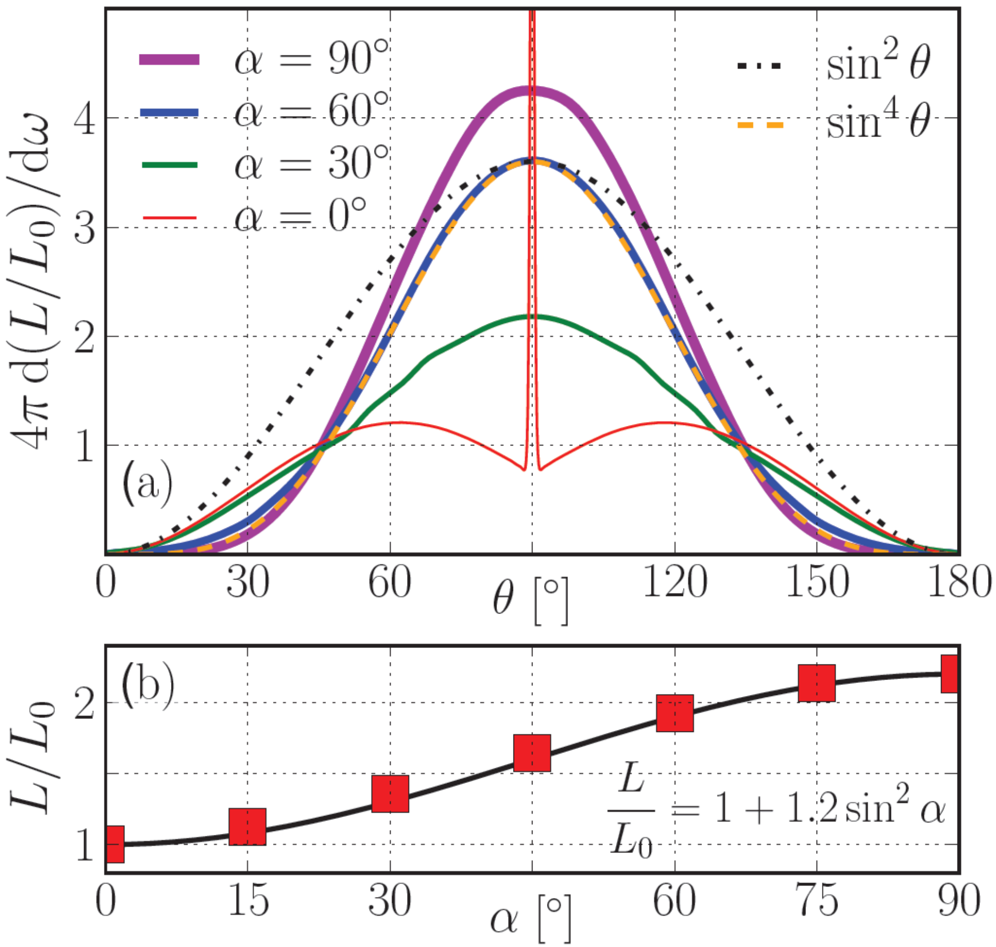
Orthogonal rotator – numerical



A.Tchekhovskoy,
A.Spitkovsky, J.Li,
arxiv.org/pdf/1211.2803.pdf

MHD

Orthogonal rotator – numerical



A.Tchekhovskoy,
A.Spitkovsky, J.Li,
arxiv.org/pdf/1211.2803.pdf

MHD

For larrge enough inclination angle

$B_r \sim \sin\theta$

$E_\theta,\ B_\varphi \sim \sin^2\theta$

Numerical – main results

- No magnetodipole radiation
- Larger energy losses for orthogonal rotator
- No monopole Michel-Bogovalov poloidal field
- Inclination angle evolves to 0 deg.

Problem 5.2. Show that the relation similar to (5.24) can be obtained for the conical solutions $\Psi = \Psi(\theta)$, but only at large distances $r \gg R_{\mathrm{L}}$ from the compact object. It has the form [Ingraham, 1973, Michel, 1974]

$$4\pi I(\theta) = \Omega_{\mathrm{F}}(\theta) \sin\theta \, \frac{\mathrm{d}\Psi}{\mathrm{d}\theta}. \tag{5.25}$$

$E_\theta = B_\varphi$

S.Gralla, T.Jacobson, G.Menon, C.Dermer ($B_{\mathrm{p}} = 0$)

Orthogonal rotator – numerical

- The shape of the current sheet

$$\Phi = \sin\alpha\sin\theta\sin(\varphi - \Omega t + \Omega r/c) + \cos\theta\cos\alpha$$

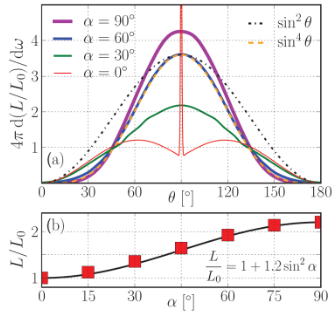
doesn't depend on the azimuthal structure.

$B_r \sim \sin\theta$
$E_\theta, B_\varphi \sim \sin^2\theta$

- The general condition is to be fulfilled

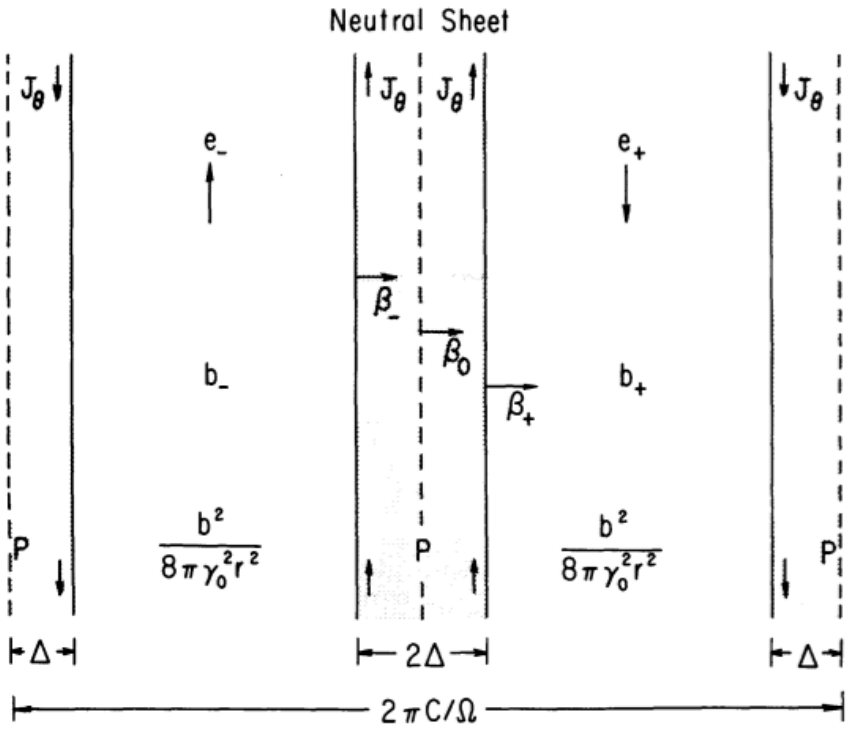
$$W_{\rm tot}(\theta) = \sin^2\theta B_r^2(\theta)$$

(as φ– average)



Internal structure

F.Coroniti, ApJ, **349**, 538 (1990), F.C.Michel (1994)



$$n \propto 1/r^2 \Rightarrow \Delta \propto r$$

Not a self-consistent solution

Co-moving reference frame

M.Lyutikov, Phys. Rev. D, **83**, 124035 (2011)

$$\partial_t^2 \Omega = \partial_r^2 \Omega$$

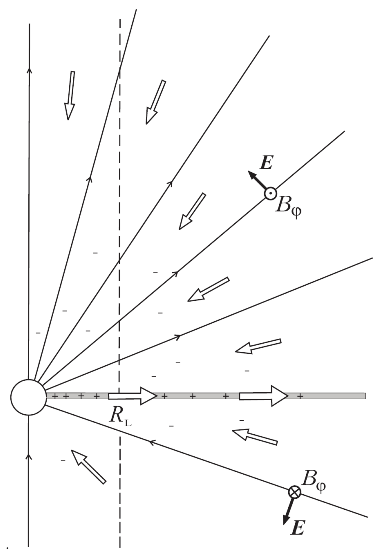
$$B_{\hat{\varphi}} = E_{\hat{\theta}} = -B_0 \left(\frac{\Omega R}{c} \right) \frac{R}{r} \sin\theta$$

$$\Omega = \Omega(r \pm ct)$$

Exact force-free solution <u>inside</u> a sheet

For $B_p = 0$ describes internal region

Fields in the laboratory frame



$$B_\varphi = \frac{1}{\beta} \frac{B_L R_L}{r} \sin\theta \tanh\left(\frac{r - \beta ct}{\Delta} \right)$$

$$E_\theta = \frac{B_L R_L}{r} \sin\theta \tanh\left(\frac{r - \beta ct}{\Delta} \right),$$

Force-free limit corresponds to $\beta = 1$

$\nabla \times \mathbf{E} = -(1/c)\partial \mathbf{B}/\partial t$ is valid for any b

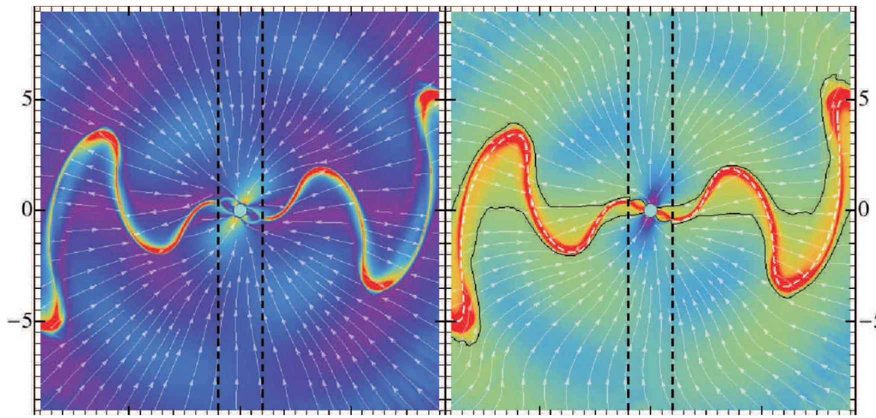
Boost

Due to the finite speed of the sheet it's possible to provide boost into comoving frame. Fields in this frame:

- orthogonal

$$B_y = \frac{B_L R_L \sin^m \theta}{ct\beta^2\gamma^2} \tanh\left(\frac{x}{\gamma\Delta}\right)$$

$$E_x = \frac{z B_L R_L \sin^m \theta}{c^2 t^2 \beta^2\gamma^2} \tanh\left(\frac{x}{\gamma\Delta}\right)$$



- aligned

$$B_y = \frac{B_{\rm L} R_{\rm L}}{\gamma^2\beta^2} \frac{1}{ct} f\left(\frac{z}{\beta ct}\right),$$

$$E_x = \frac{B_{\rm L} R_{\rm L}}{\gamma^2\beta^2} \frac{z}{c^2 t^2} f\left(\frac{z}{\beta ct}\right)$$

Maxwell equation $\nabla \times \mathbf{E} = -(1/c)\partial \mathbf{B}/\partial t$ is satisfied inside and outside the sheet.

Co-moving reference frame

In a first approximation the concentration decreases as $1/t^2$, therefore $f(x, t)$ should vary as $1/t$.

$$f(x,t) = x/kct$$

The z-component of the electric field will necessarily appear and will accelerate particles in current sheet.

Sheet parameters:

$$k = 1/4\lambda\gamma$$

$$E_z^{\max} = \frac{kB_{\rm s}}{2ct\gamma^2\beta^2}$$

$$\Delta = R_{\rm s}\sigma^{1/3}/4\lambda\gamma$$

Particle acceleration (estimate):

$$\left.\frac{\mathrm{d}\gamma}{\mathrm{d}t/t_0}\right|_{t=t_0} = \frac{\sigma}{2\gamma}$$

$$\sigma = \gamma^3$$

Internal structure

Quasiadiabatic invariant (Zeleniy et al., 2013)

$$L \ll r_{\mathrm{p}} \Rightarrow I_z = \int p_z dz \approx \mathrm{const}$$

$$I_z = \frac{1}{\pi} p(t) \sqrt{\frac{cp(t)}{eB(t)} L(t)} \;\; \psi(s) \qquad s = p_x / p$$

$$\psi(s) = \int_{-\sqrt{s+1}} \sqrt{1-(s-\xi^2)^2} d\xi$$

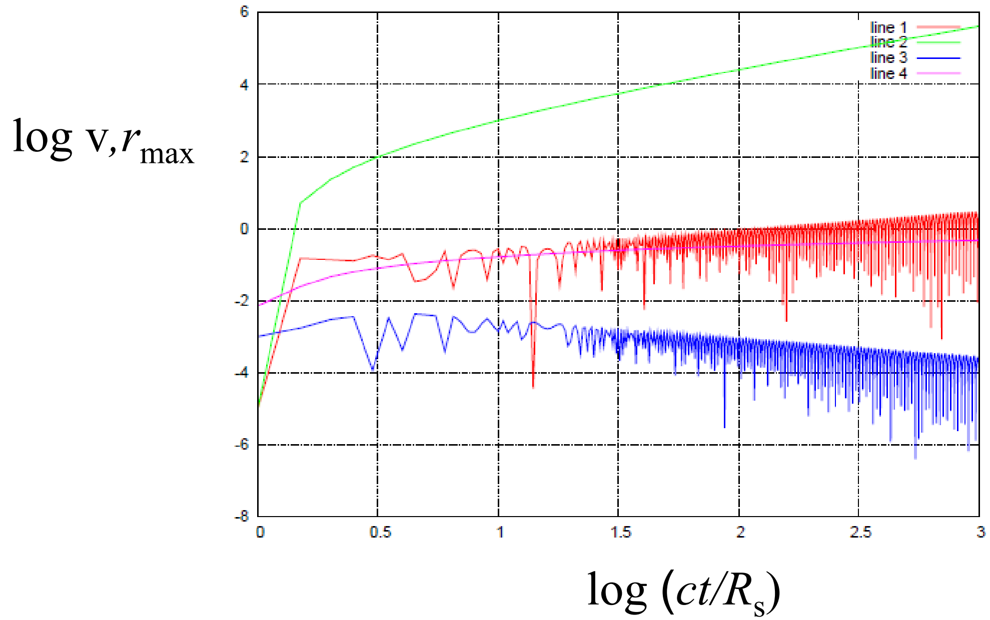
Internal structure

$B(t) \sim 1/t$

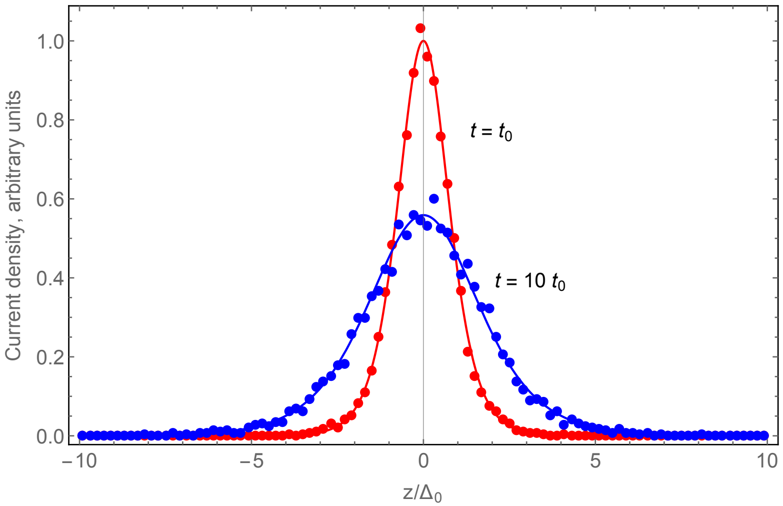
$L(t) \sim t$

$\mathrm{v}(t) \sim t^{-1/4}$

$r_{\max} \sim t^{1/4}$



Simple solution

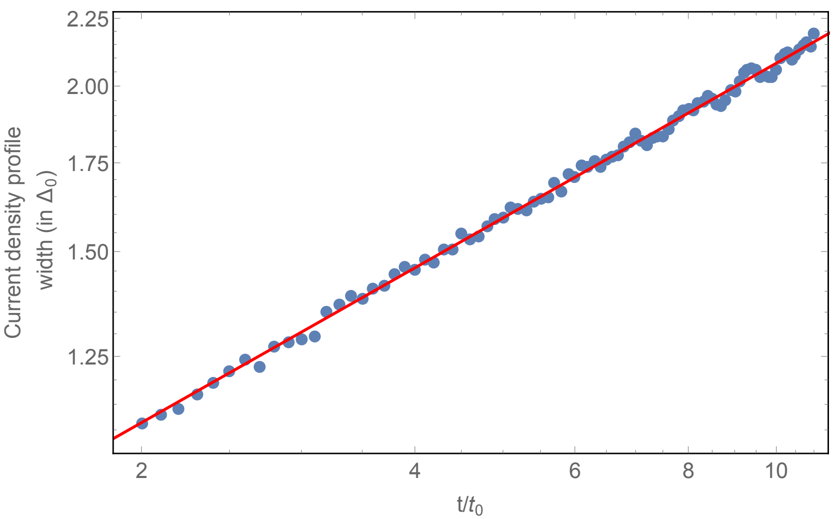


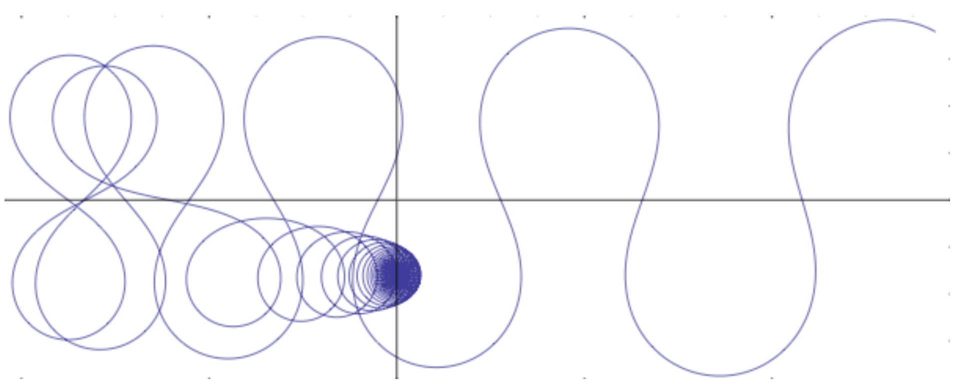
In special a case of $p_x = 0$ one can obtain a simple self-consistent solution:

$$L \sim r_{\max} \sim t^{1/2}$$

$$n \sim t^{-1}$$

$$p \sim t^{-1/2}$$

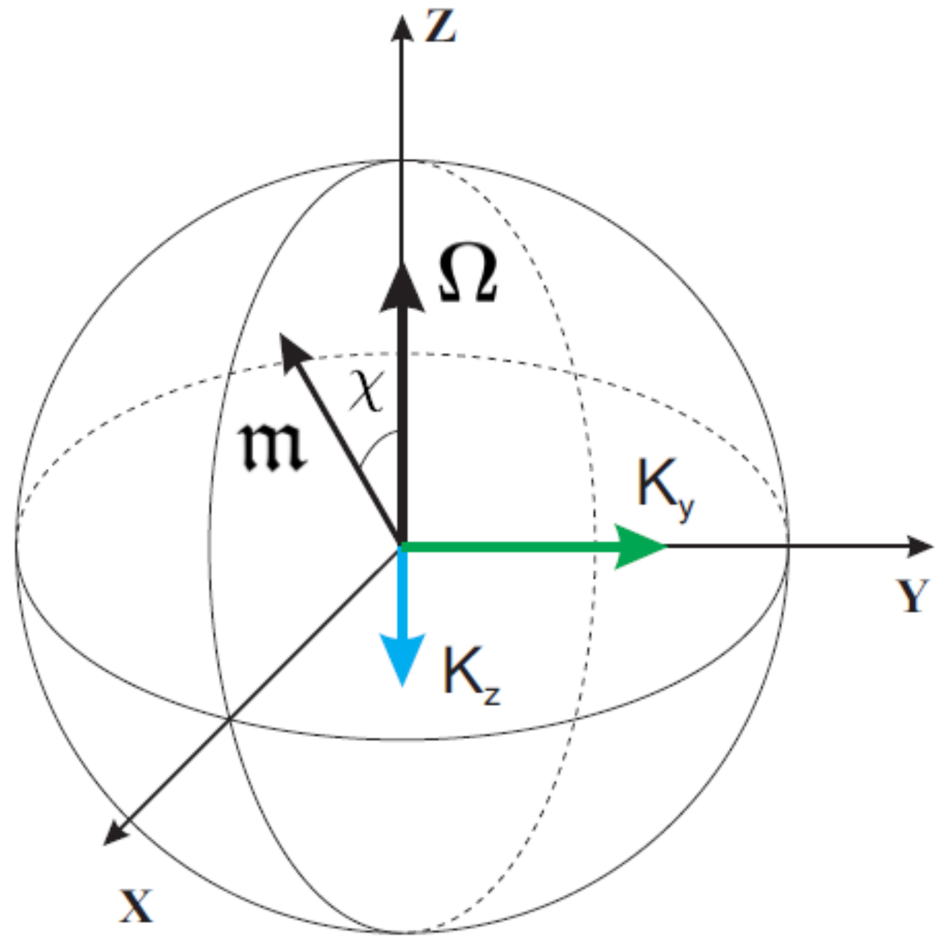




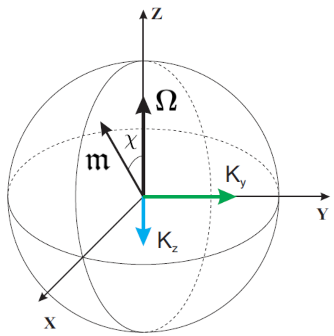
Results

- The shape of the current sheet does not depend on the asymptotic field behavior.
- In the comoving reference frame the current sheet is essentially time-dependent.
- The particle motion inside the sheet was described using quasi-adiabatic invariant.
- Our approach allows one to obtain simple self-consistent solution.

Anomalous torque



Anomalous torque



$$K_{z'} = \frac{2}{3}\frac{\mathfrak{m}^2}{R^3}\left(\frac{\Omega R}{c}\right)^3 \sin^2\chi$$

$$K_{x'} = \frac{2}{3}\frac{\mathfrak{m}^2}{R^3}\left(\frac{\Omega R}{c}\right)^3 \sin\chi\cos\chi$$

$$\boxed{K_{y'} = \xi\frac{\mathfrak{m}^2}{R^3}\left(\frac{\Omega R}{c}\right)^2 \sin\chi\cos\chi}$$

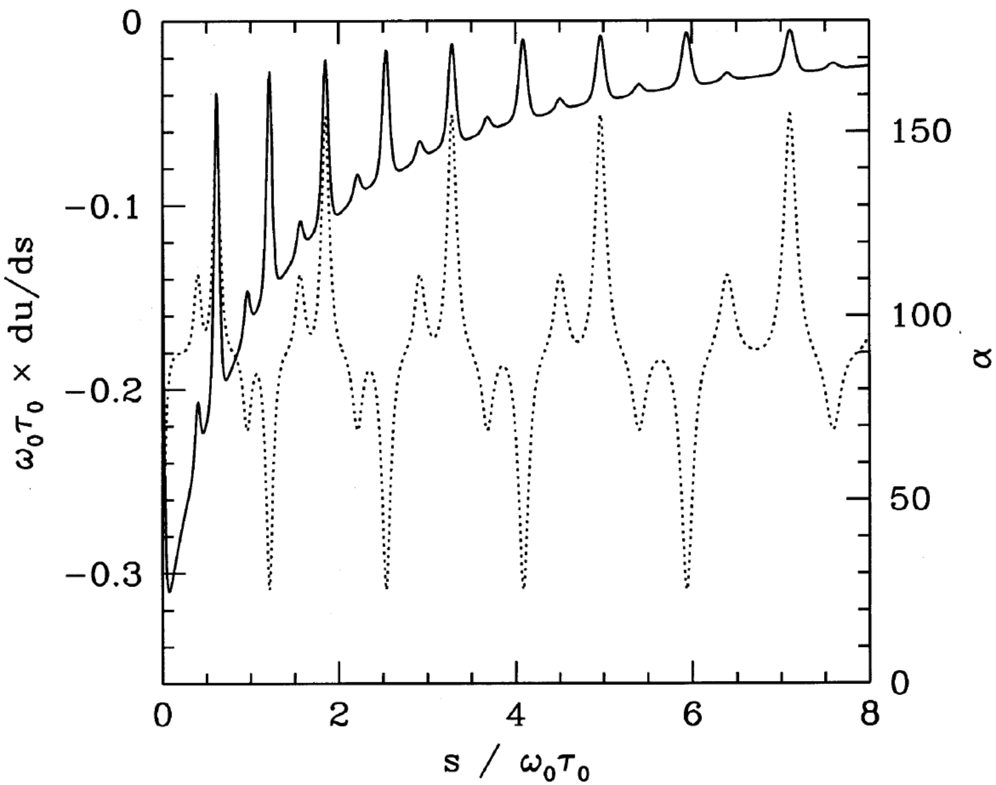
$\xi = 1$ (Davis & Goldstein 1970, Goldreich 1970)

$\xi = 1/5$ (Good&Ng 1985)

$\xi = 0$ (Michel 1991, Istomin 2005)

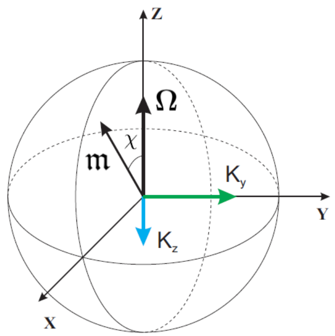
$\xi = 3/5$ (Melatos 2000)

Evolution



A.Melatos, MNRAS, **313**, 217 (2000),
D.Barsukov & A.Tsygan, MNRAS, **409**, 1077 (2010),
G.Beskin, A.Biryukov, S.Karpov, MNRAS,**420**, 103 (2012)

Anomalous torque



Main assumptions

- Rotation in vacuum
- Freezing-in condition inside the body

$$\mathbf{E} + \beta_{\mathrm{R}} \times \mathbf{B} = 0$$

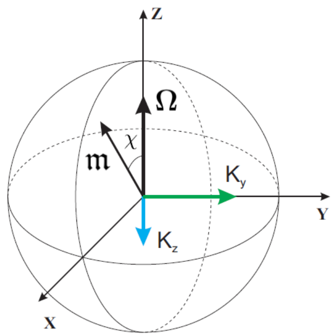
$$\beta_{\mathrm{R}} = \mathbf{\Omega} \times \mathbf{r}/c$$

- Corotation currents $\mathbf{j} = c\rho_{\mathrm{e}}\beta_{\mathrm{R}}$ only (hence, zero volume force)

$$\mathrm{d}\mathbf{F} = \rho_{\mathrm{e}}\mathbf{E}\,\mathrm{d}V + [\mathbf{j} \times \mathbf{B}]/c\,\mathrm{d}V$$

- Spherical body

Anomalous torque



Basic equation

$$\frac{\mathrm{d}\,\mathbf{L}_{\mathrm{field}}}{\mathrm{d}t} + \mathbf{K}^M + \int [\mathbf{r}\times\mathbf{F}]\mathrm{d}V = 0$$

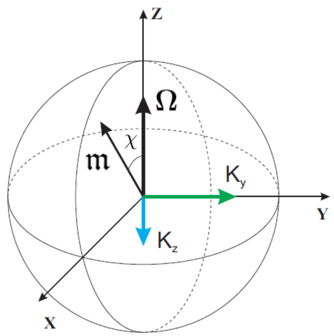
Electromagnetic angular momentum

$$\mathbf{L}_{\mathrm{field}} = \int \frac{[\mathbf{r}\times[\mathbf{E}\times\mathbf{B}]]}{4\pi c}\mathrm{d}V$$

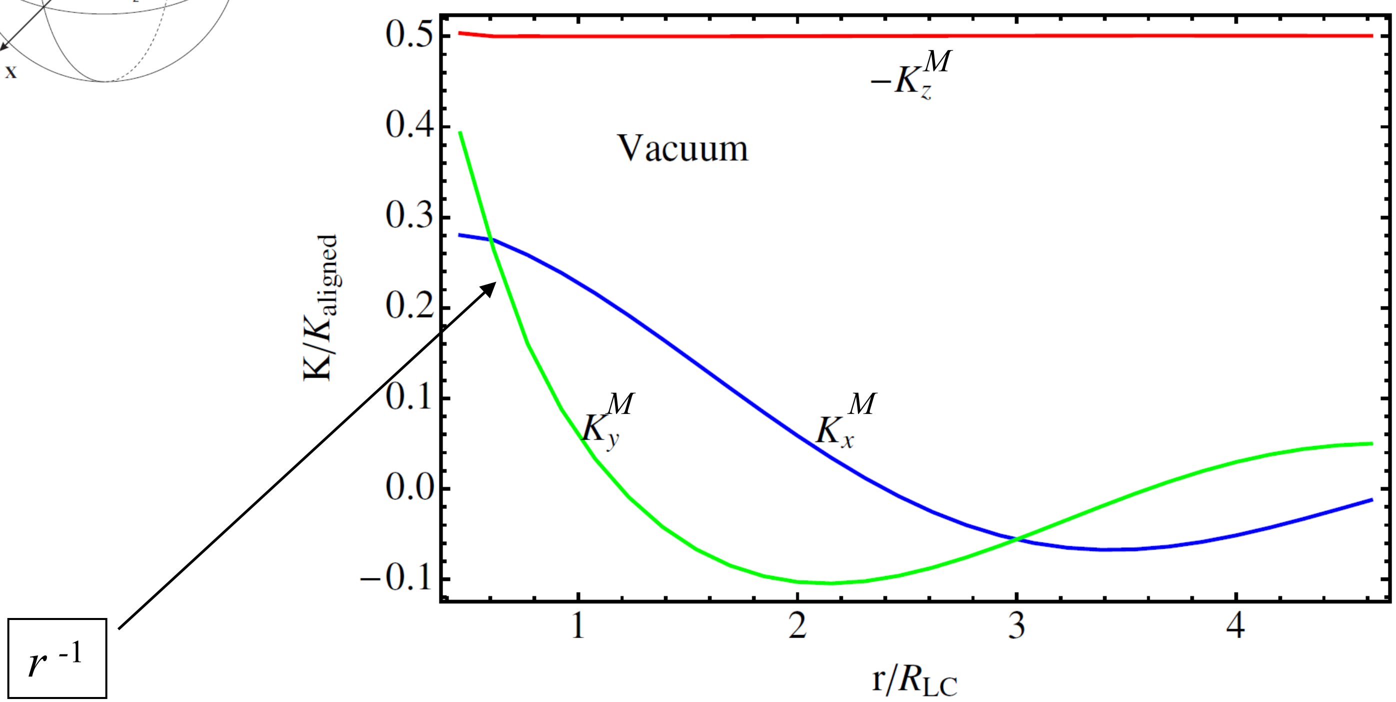
Flux $K_i^M = -\int \varepsilon_{ijk} r_j T_{kl} \mathrm{d}S_l$ (spherical surface)

$$\mathbf{K}^M_{y'} = \frac{R^3}{4\pi}\int \Big([\mathbf{n}\times\mathbf{B}]_{y'}(\mathbf{Bn}) + [\mathbf{n}\times\mathbf{E}]_{y'}(\mathbf{En}) \Big)\mathrm{d}o$$

Anomalous torque

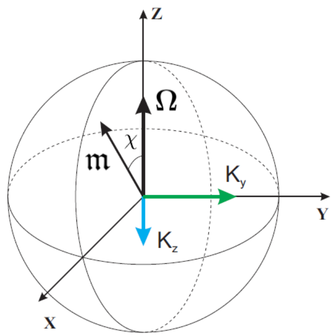


A.Philippov, A.Tchekhovskoy, J.Li, http://arxiv.org/abs/1311.1513



$$\frac{\mathrm{d}\,\mathbf{L}_{\rm field}}{\mathrm{d}t} + \mathbf{K}^M + \int [\mathbf{r}\times\mathbf{F}]\mathrm{d}V = 0$$

Anomalous torque



Torque, acting on the body

$$\frac{\mathrm{d}\,\mathbf{L}_{\mathrm{mat}}}{\mathrm{d}t} = \int [\mathbf{r} \times \mathbf{F}]\mathrm{d}V$$

$$\mathrm{d}\mathbf{F} = \sigma_{\mathrm{e}}\mathbf{E}\,\mathrm{d}S + [\mathbf{I}_{\mathrm{S}} \times \mathbf{B}]/c\,\mathrm{d}S$$

$$\boxed{\mathbf{K} = \int \mathbf{r} \times \mathrm{d}\mathbf{F} = \frac{R^3}{4\pi}\int \Big([\mathbf{n} \times \{\mathbf{B}\}]\,(\mathbf{B}\mathbf{n}) + [\mathbf{n} \times \mathbf{E}]\,(\{\mathbf{E}\}\,\mathbf{n}) \Big)\mathrm{d}o}$$

- for spherical body
- for highly conducting interior
- for corotation currents only

Anomalous torque

Calculations

- small parameter $\varepsilon = \dfrac{\Omega R}{c}$
- quasi-stationary $\varphi - \Omega t$

$$\nabla \times (\mathbf{E} + \beta_{\rm R} \times \mathbf{B}) = 0, \qquad \beta_{\rm R} = \mathbf{\Omega} \times \mathbf{r}/c$$

$$\nabla \times (\mathbf{B} - \beta_{\rm R} \times \mathbf{E}) = \frac{4\pi}{c}\mathbf{j} - 4\pi\rho_e\beta_{\rm R} \qquad \mathbf{j} = c\rho_{\rm e}\beta_{\rm R}$$

$$\boxed{\begin{aligned} \mathbf{E} + \beta_{\rm R} \times \mathbf{B} &= -\nabla\psi, \\ \mathbf{B} - \beta_{\rm R} \times \mathbf{E} &= \nabla h \end{aligned}}$$

$$\longrightarrow \psi^{(1)} \longrightarrow E^{(1)}$$

$$\longrightarrow h^{(2)} \longrightarrow B^{(2)}$$

Anomalous torque

$$
\begin{aligned}
B_r^{\perp} &= \frac{|\mathfrak{m}|}{r^3}\sin\theta\,\mathrm{Re}\,\left(2 - 2i\frac{\Omega r}{c}\right)\exp\left(i\frac{\Omega r}{c} + i\varphi - i\Omega t\right), \\
B_\theta^{\perp} &= \frac{|\mathfrak{m}|}{r^3}\cos\theta\,\mathrm{Re}\,\left(-1 + i\frac{\Omega r}{c} + \frac{\Omega^2 r^2}{c^2}\right)\exp\left(i\frac{\Omega r}{c} + i\varphi - i\Omega t\right), \\
B_\varphi^{\perp} &= \frac{|\mathfrak{m}|}{r^3}\,\mathrm{Re}\,\left(-i - \frac{\Omega r}{c} + i\frac{\Omega^2 r^2}{c^2}\right)\exp\left(i\frac{\Omega r}{c} + i\varphi - i\Omega t\right), \\
E_r^{\perp} &= 0, \\
E_\theta^{\perp} &= \frac{|\mathfrak{m}|\Omega}{r^2 c}\,\mathrm{Re}\,\left(-1 + i\frac{\Omega r}{c}\right)\exp\left(i\frac{\Omega r}{c} + i\varphi - i\Omega t\right), \\
E_\varphi^{\perp} &= \frac{|\mathfrak{m}|\Omega}{r^2 c}\cos\theta\,\mathrm{Re}\,\left(-i - \frac{\Omega r}{c}\right)\exp\left(i\frac{\Omega r}{c} + i\varphi - i\Omega t\right).
\end{aligned}
$$

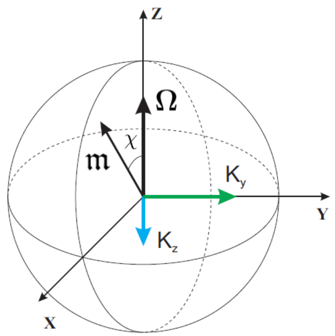
Anomalous torque

$$\begin{aligned} \mathbf{E} + \beta_{\rm R} \times \mathbf{B} &= -\nabla\psi, \\ \mathbf{B} - \beta_{\rm R} \times \mathbf{E} &= \nabla h \end{aligned}$$

Calculations

- Freezing-in condition $\psi^{(\rm In)} = 0$
- Hence, no freedom in $E^{(1)}$
- Exactly corresponds to vacuum solution
- Freedom in $B^{(2)}$(free functions $h^{(2)}$)
- Result DOES NOT depend on this freedom

Anomalous torque



Torque, acting on the body

$$\mathbf{K} = \int \mathbf{r} \times \mathrm{d}\mathbf{F} = \frac{R^3}{4\pi} \int \Big([\mathbf{n} \times \{\mathbf{B}\}] \, (\mathbf{B}\mathbf{n}) + [\mathbf{n} \times \mathbf{E}] \, (\{\mathbf{E}\} \, \mathbf{n}) \Big) \mathrm{d}o$$

$$\boxed{K = K^M(R+0) - K^M(R-0)}$$

$$\xi = \frac{3}{5}$$

$$\mathbf{K}^M = - \frac{\mathrm{d}\,\mathbf{L}_{\mathrm{field}}}{\mathrm{d}t}$$

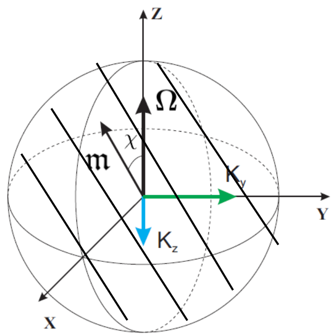
$$\mathbf{L}_{\mathrm{field}} = \int \frac{[\mathbf{r} \times [\mathbf{E} \times \mathbf{B}]]}{4\pi c} \mathrm{d}V$$

- Does not depend on the internal structure and freedom in $h^{(2)}$
- Obtained by A.Melatos (2000) for Deutsch solution

Depends on the internal structure

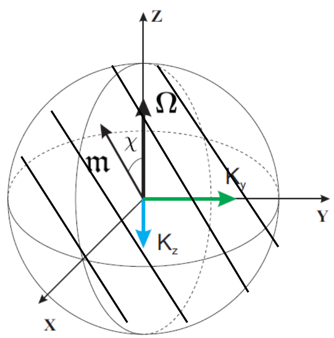
Anomalous torque

Three cases



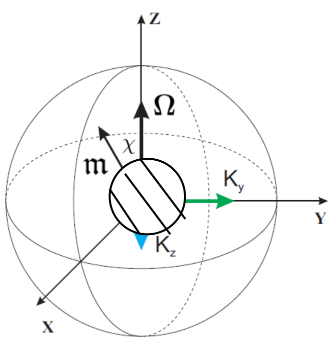
- Homogeneously magnetized sphere

$$K_{y'} = \frac{1}{3}\frac{\mathfrak{m}^2}{R^3}\left(\frac{\Omega R}{c}\right)^2 \sin\chi\cos\chi$$



- Homogeneously magnetized hollow envelope

$$K_{y'} = \frac{31}{45}\frac{\mathfrak{m}^2}{R^3}\left(\frac{\Omega R}{c}\right)^2 \sin\chi\cos\chi$$



- Homogeneously magnetized core $R_{\text{in}} < R$

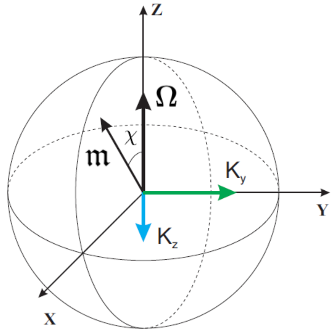
$$K_{y'} = \left(\frac{8}{15} - \frac{1}{5}\frac{R}{R_{\text{in}}}\right)\frac{\mathfrak{m}^2}{R^3}\left(\frac{\Omega R}{c}\right)^2 \sin\chi\cos\chi$$

Comparison with previous results

$\xi = 1$ (Davis & Goldstein 1970, Goldreich 1970)
$\xi = 1/5$ (Good&Ng 1985)
$\xi = 0$ (Michel 1991, Istomin 2005)
$\xi = 3/5$ (Melatos 2000)

$\xi = 1$ – electric current only
$\xi = 0$ – outer sphere only
$\xi = 3/5$ – stress for $r = R+0$

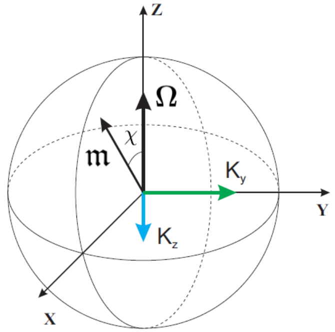
Anomalous torque



Main conclusion

Anomalous torque acting on the body depends on the internal structure of the electromagnetic field inside the body:

Anomalous torque



More conclusions

- Nonzero angular momentum L_{field} of the electromagnetic field exist only in Ω^2 order.
- In $\boldsymbol{\Omega}^3$ order it vanishes. It implies no problem in calculating the braking torque.
- Hence, energy losses can be determined via the flux $K_i = -\int \varepsilon_{ijk} r_j T_{kl} \mathrm{d}S_l$