Towards radiopulsar evolution theory

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Under assumption of ideal magnetohydrodynamics (MHD) we investigate the effects of possible non-sphericity of neutron stars on the evolution of the angle between rotational and magnetic axes (inclination angle). For two non-sphericity models we reproduce recent results of observations of Crab nebulae pointing on the increase of inclination angle in the present epoch.

Torques in MHD magnetosphere

Philippov et al. (2014) analyzed the results of time-dependent 3D MHD simulations of pulsar magnetosphere and calculated torques acting on neutron star (coordinate system from Fig. 1):

$$K_x = k_2 K_{\rm aligned} \sin\alpha\cos\alpha \qquad K_z = -K_{\rm aligned}\left(k_0 + k_1 \sin^2\alpha\right)$$

x-component leads to the evolution of inclination angle while z-component causes pulsar spindown. Deriving torque components in basis of magnetic moment vector one can check that the result $\boldsymbol{K}_\parallel = -A\cos\alpha$, $\boldsymbol{K}_\perp = -B\sin\alpha$ (A and B are uniquely connected with k_i) is consistent with expression found in Beskin et al. (1993) for the case of large asymmetric current. It was also shown that coefficients k_i depend on neutron star radius (Fig. 2)

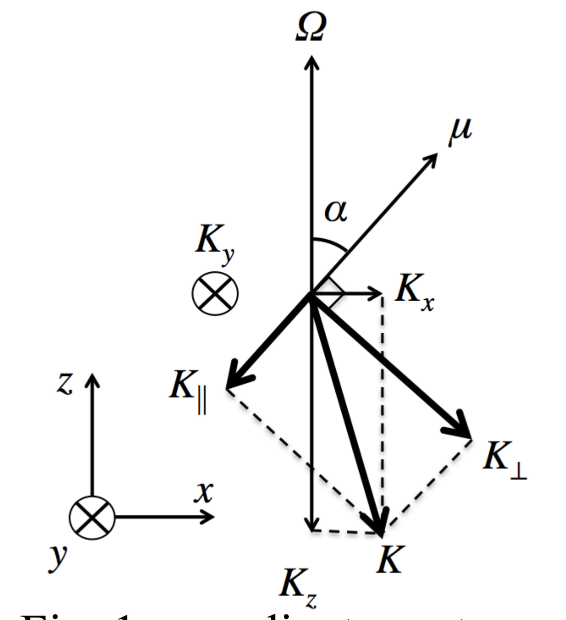


Fig. 1: coordinate system

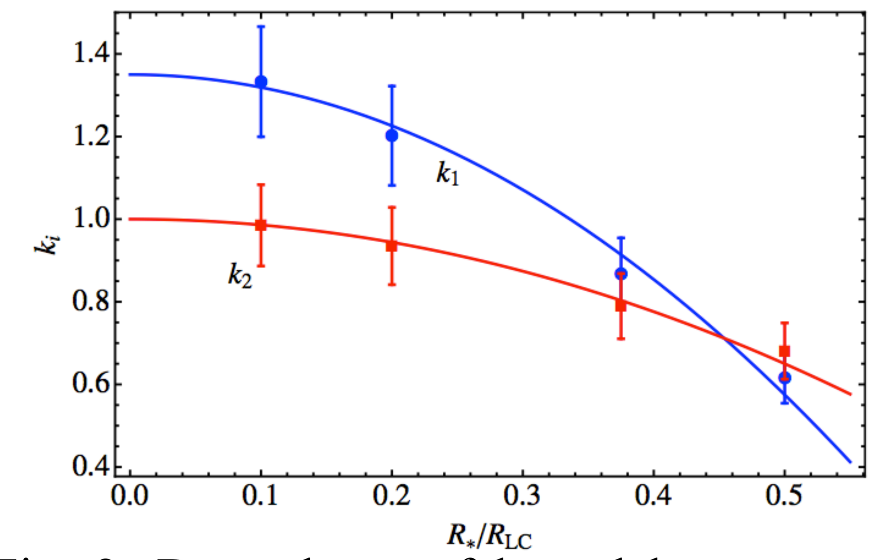


Fig. 2: Dependence of k_1 and k_2 on neutron star radius

Evolution of spherical star: vacuum vs MHD

MHD: $k_0 \approx k_2 \approx 1 \quad k_1 \approx 1.2$

Vacuum: $K_x = \frac{2}{3}K_{\rm aligned}\sin\alpha\cos\alpha \quad K_z = -\frac{2}{3}K_{\rm aligned}\sin^2\alpha$

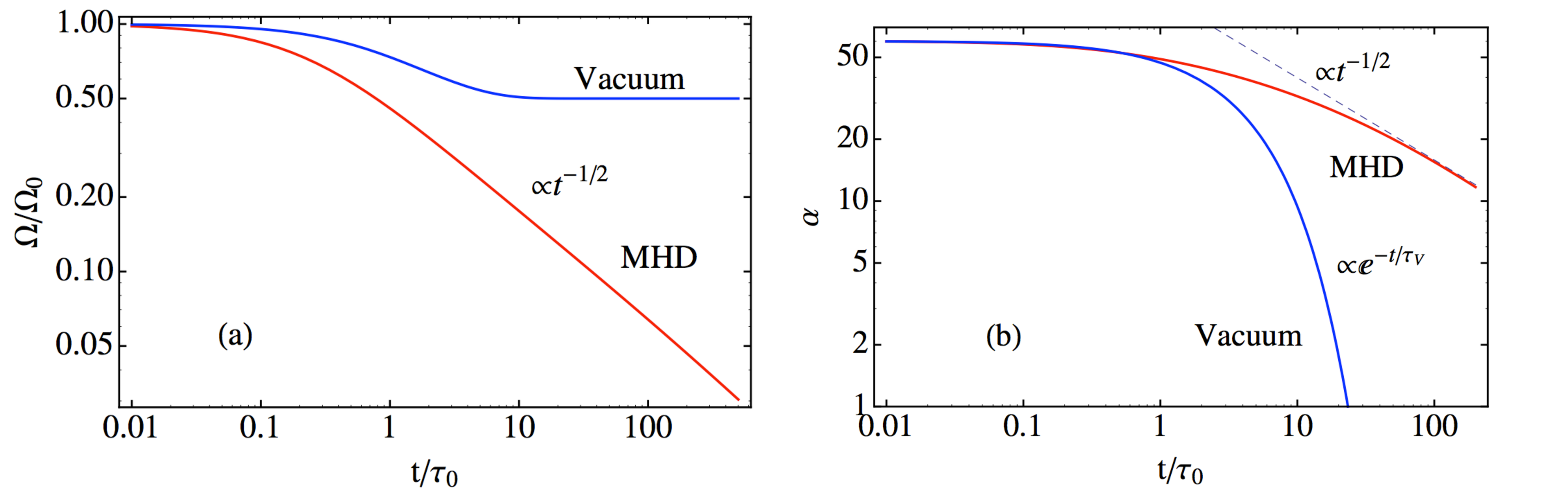


Fig. 3: (a) Evolution of angular velocity Ω for initial inclination angle 60^0 and initial period 10 ms for vacuum and MHD models. Vacuum pulsar evolves towards constant speed, while MHD pulsar continues to loose its rotational energy in the power-law manner. (b) Evolution of the angle between rotational and magnetic axes for the same parameters. Vacuum pulsar moves exponentially fast towards the ground state, for MHD pulsar inclination has power-law behavior.

Evolution of non-spherical pulsar

During analysis of evolution of non-spherical neutron star we use an approach introduced in Melatos et al. (2000). After coordinate transformation we find torques in the system of pulsar's principal axes. In order to trace the evolution of pulsar's rotation we solve the Euler's equation of motion.

Torque component K_y diverges for small radius of neutron star and the approximate expression for this torque is

$$K_y = k_3 \frac{K_{\rm aligned}}{\Omega R_*/c}\sin\alpha\cos\alpha \quad \text{with } k_3\text{=0.1.}$$

We use two mechanisms of pulsar non-sphericity:

- Hydrostatic mechanism [4]. Ellipticity $\epsilon^{\rm cr} = 5\tilde{\mu}\omega^2R^3/4(1+\tilde{\mu})GM$, where $\tilde{\mu} = 38\pi\mu R^4/3GM^2$. This mechanism is connected with rotation of neutron star and symmetry axis has to be close to the rotational axis.
- Magnetic field near poles is larger than equatorial magnetic field. It causes additional difference of moments of inertia for axes, pointing along magnetic moment and perpendicular to it. In this case symmetry axis has to be close to the magnetic moment

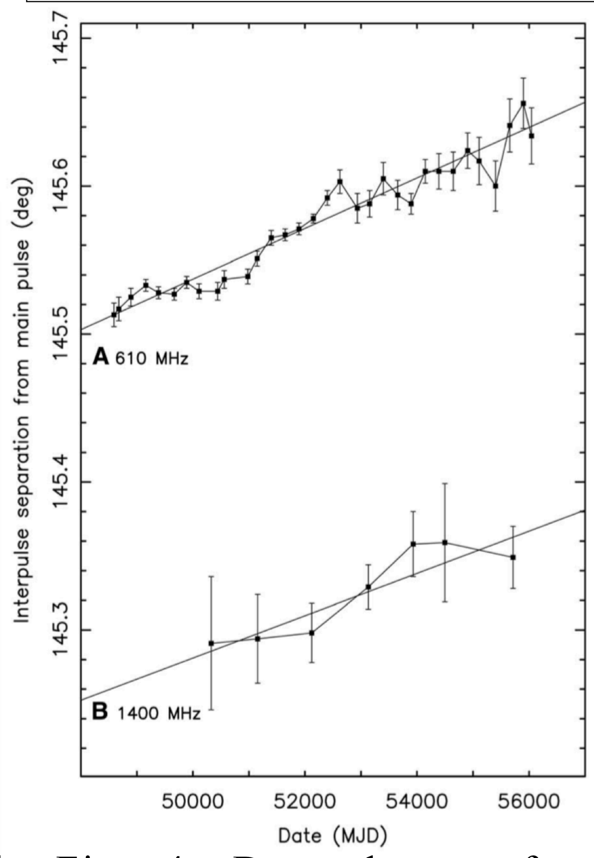


Fig. 4: Dependence of separation of main pulse and interpulse on time [5] for Crab in radio waveband on 610 and 1400 MHz. Separation between peaks linearly grows with time. In models of gamma-ray band emission generation in outer gap and in equatorial current sheet this fact leads to the increase of inclination angle.

Pulsar in Crab nebulae

Lyne et al. (2013) investigated the dependence of separation between main pulse and interpulse on time in radio wave band for pulsar B0531+21 (Crab pulsar). As gamma and radio emission are correlated (and thus produced in the close regions), one can use models for gamma-ray band emission generation to figure out the evolution of inclination angle from evolution of separation between main and secondary pulses in radio band. It was shown that inclination change rate is 0.62^0 per century. Despite the fact that this number is model-dependent, increase of the inclination angle can be explained within the framework of different emission generation models (e.g., outer gap and current sheet). For this pulsar the parameter $n_{\rm br} = \ddot{\Omega}\Omega/\dot{\Omega}^2$ was also measured. It equals 2.5, but for spherical star it has to be more than 3. Precession can easily explain this effect. For both non-sphericity models the following list of parameters of pulsar B0531+21 was reproduced:

- spindown rate
- inclination angle change rate
- $n_{\rm br} = 2.5$

It can be also shown that

- variation period turns out to be around hundred years
- for magnetic non-sphericity model pulsar experience global alignment

Anomalous breaking indices

Not only Crab pulsar has breaking index $n_{\rm br}$ less than 3. In fact, the majority of pulsars have breaking indices different from spherical pulsar estimations. They can be extremely large and both positive and negative. Precession can explain this phenomenon. Figure 5 compares observed dependence of breaking index on pulsar age with evolution curve of a single pulsar. Even though estimation of pulsar parameters of these pulsars requires more careful approach, the general picture claims that precession can explain all of these braking indices.

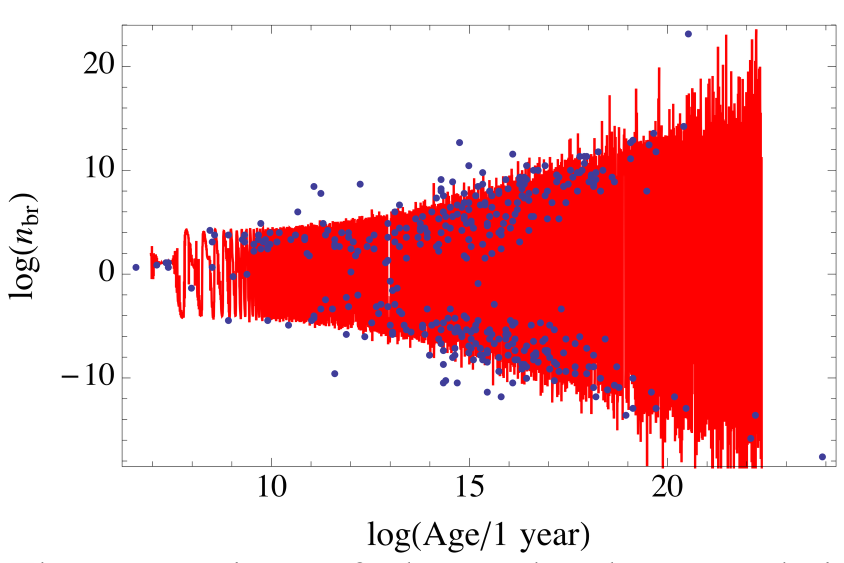


Fig. 5: The comparison of observed pulsar population (blue dots) and evolution curve of a single pulsar with particular geometrical parameters (red line). If $n_{\rm br} < 0$ quantity $-\log(-n_{\rm br})$ is plotted. Because of a very small precession period, red line completely fills a triangle-like area.

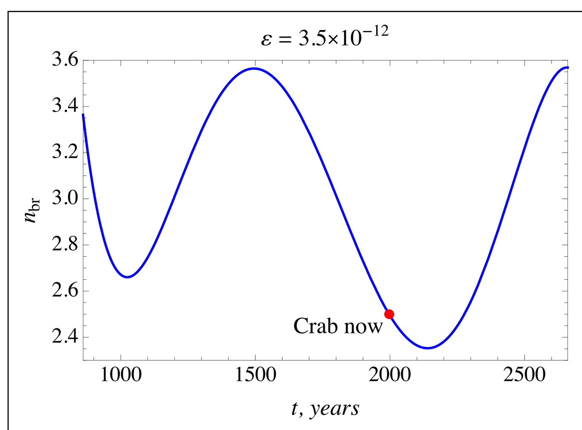
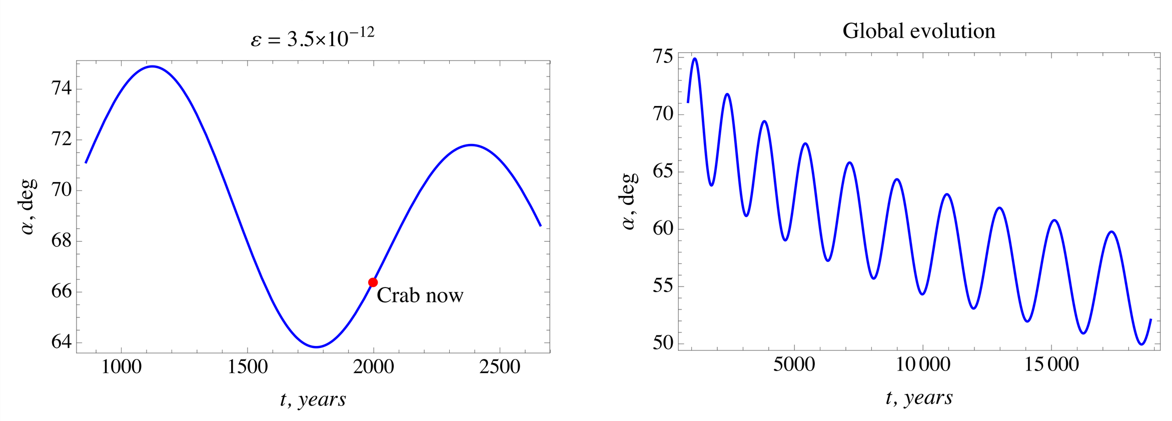


Fig. 6: Inclination angle and $n_{\rm br}$ over time for non-spherical MHD pulsar for magnetic deformation model.



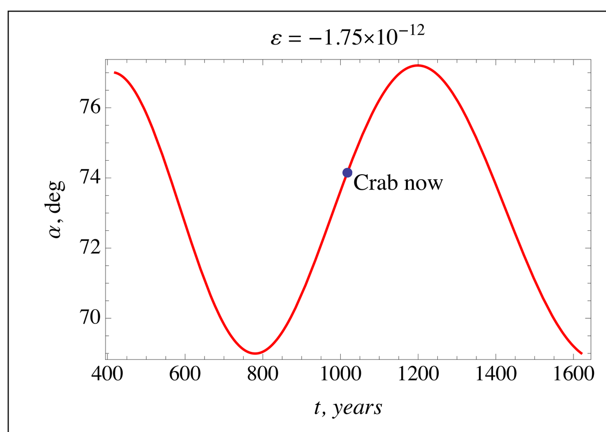
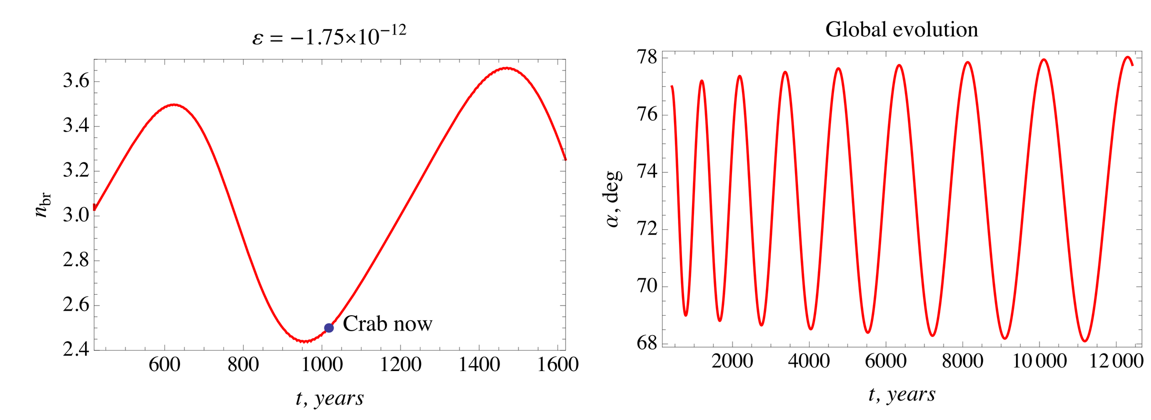


Fig. 7: Inclination angle and $n_{\rm br}$ over time for non-spherical MHD pulsar for hydrostatic deformation model.



Conclusion

- In MHD magnetosphere inclination angle of spherical star evolves much slower than in vacuum model. Pulsar evolves towards axisymmetric state.
- Non-spherical star experiences precession and evolution becomes more complicated. It can be in form of small oscillations on top of the alignment (Fig. 6) or oscillations with no visible change of mean value (Fig. 7).
- Precession allows to explain the behavior of pulsar B0531+21, both inclination angle variation and $n_{\rm br} < 3$.
- Precession is a possible source of anomalous breaking indices.

Literature

[1] A. Philippov, A. Tchekhovskoy, J. Li, 2014, MNRAS, 441, 1879
[2] V. Beskin, A. Gurevich, Y. Istomin, 1993, Physics of pulsar magnetosphere.
[3] A. Melatos, 2000, MNRAS, 313, 217.
[4] P. Goldreich, 1970, ApJ, 160, L11.
[5] A. Lyne et al., 2013, Science, 342, 598.