On determination of the pulsar image size in the picture surface

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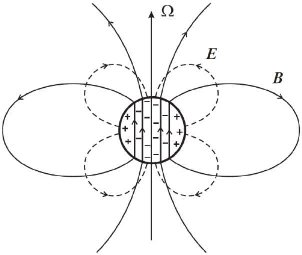
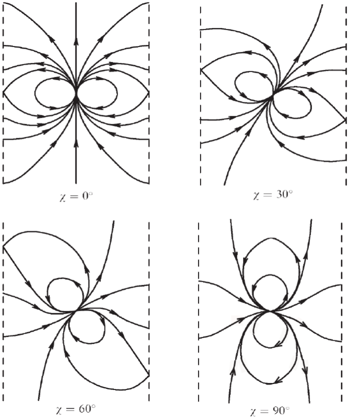
Dipole magnetic field $B \sim 10^{12}\ Gs$;
Quadrupol electric field $E_{||} \sim \frac{\Omega R}{c} B_0$ /generated by the rotation/.

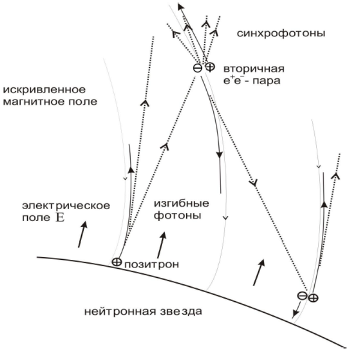


The last open fieldline defines the polar cap:
$R_{\mathrm{pc}} = (\frac{\Omega}{c} R)^{1/2} R.$



Secondary plasma generation *[QED: E.M. Lifshitz, et al.]*:

$$\gamma + B \to e^+ + e^- + B;$$
$$w(B, \theta, E_{\rm ph}) \propto \sin\theta;$$
$$\omega_{\rm cur} \sim (c/R_{\rm cur})\gamma^3.$$

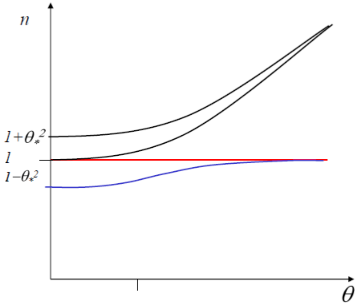


Dielectric tensor for relativistic plasma with background magnetic field (z-axis) *[J.H. Taylor & R.N. Manchester (1977)]*:

$$\epsilon_{ij} = \mathrm{diag}\{1, 1, 1 - \left\langle \frac{\omega_{\mathrm{pl}}^2}{\gamma^3 \omega^2} \right\rangle\};$$

$$\theta_* = \left\langle \left(\frac{\omega_{\mathrm{pl}}^2}{\gamma^3 \omega^2} \right)^{1/4} \right\rangle.$$

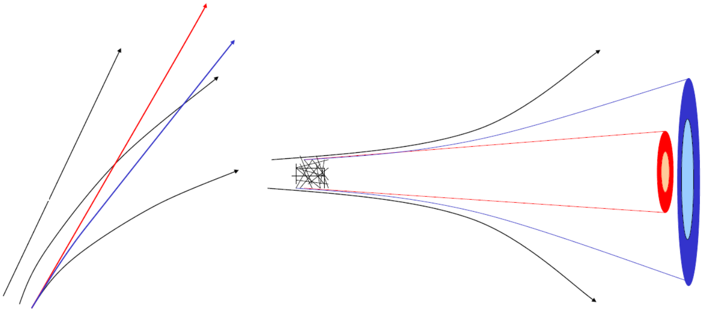
$$\begin{aligned} n_1 &= 1, \\ n_2 &= 1 + \frac{\theta^2}{4} - \left(\frac{\omega_p^2}{\omega^2} < \frac{1}{\gamma^3} > + \frac{\theta^4}{16} \right)^{1/2} \\ n_3 &= 1 + \frac{\theta^2}{4} + \left(\frac{\omega_p^2}{\omega^2} < \frac{1}{\gamma^3} > + \frac{\theta^4}{16} \right)^{1/2} \\ n_4 &= \frac{1}{\cos\theta}. \end{aligned}$$



Ray trajectories for different modes (core-conal model):

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} = c\frac{\partial}{\partial \mathbf{k}}\left(\frac{k}{n}\right)$$

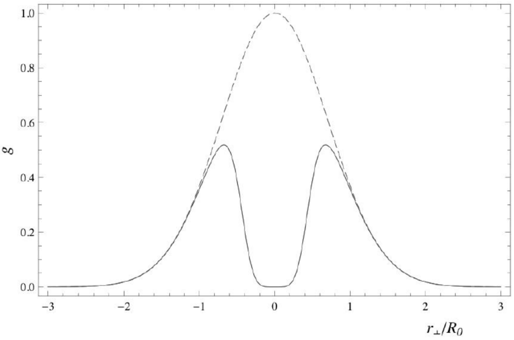
$$\frac{\mathrm{d}\mathbf{k}}{\mathrm{d}t} = -c\frac{\partial}{\partial \mathbf{r}}\left(\frac{k}{n}\right)$$



The density of generated e-p plasma near the polar cap *[V.S. Beskin, A.A. Philippov (2012)]*:

$$n_{\rm e} = \lambda g(r_\perp) n_{\rm GJ};$$

$$g(r_\perp) = \left[1 + \left(\frac{f_0 R_0}{r_\perp}\right)^5\right]^{-1} \exp\left(-r_\perp^2/R_0^2\right).$$

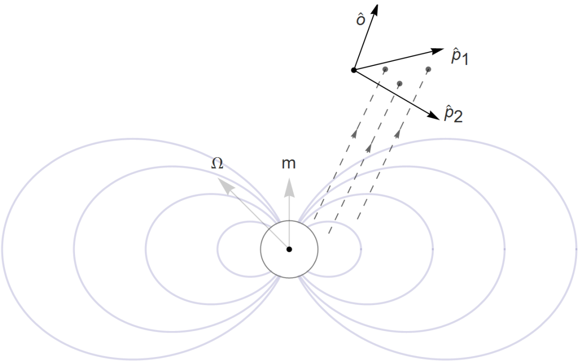


Intensity from heigth dependence: $h(r) = \exp\left[-A\frac{(r-R_m)^2}{R_m^2}\right]$.
Curvature photon intensity: $d(\theta) = \exp\left(-\gamma^2\theta^2\right)$.
Four parameters are: A, γ, χ, R_m.

Finally: $I = \int\limits_{\mathbf{r}(t)} g(r_\perp)h(r)d(\theta)\mathrm{d}r$.
$\mathbf{r}(t)$ from geometrical optics equations (with $n = n_1 \equiv 1$ for X-mode and $n = n_2$ for O-mode).

The image is being integrated on the $\{\mathbf{p}_1, \mathbf{p}_2\}$ plane.

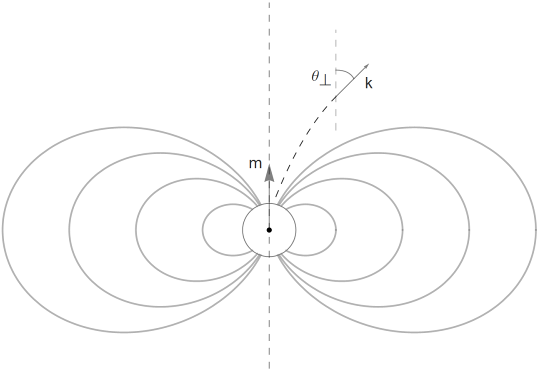
$$I(a_1, a_2, \varphi_0) = \int_{\mathbf{r}(t)} g(r_\perp) h(r) d(\theta) \mathrm{d}r$$



For O-mode, taken $g(r_\perp) \equiv 1$ *[V.S. Beskin, et al. (1993)]*:

$$\theta_\perp(\infty) = \left(\frac{\Omega R}{c}\right)^{0.36} \left(\frac{1}{\omega^2}\left\langle\frac{\omega_\mathrm{p}^2}{\gamma^3}\right\rangle\right)^{0.07} f_\mathrm{em}^{0.36}\left(\frac{r_\mathrm{em}}{R}\right)^{0.15} \propto \nu^{-0.14}$$

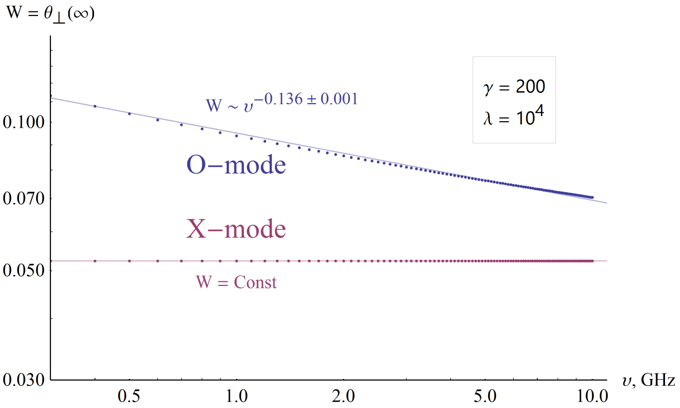
$$f = \frac{c}{\Omega R}\left(\frac{l}{R}\right)^{-1}\sin^2\theta_\mathrm{m} \sim 1$$

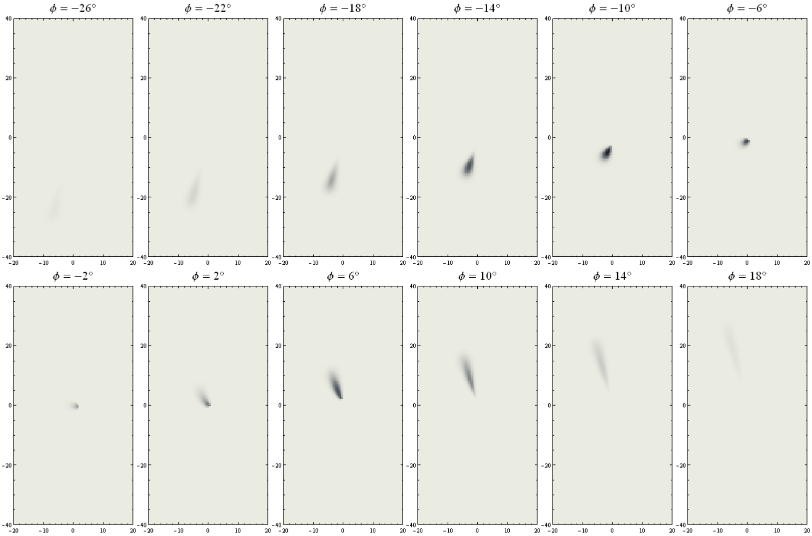


Taken $g(r_\perp) \equiv 1$:

O-mode: $\theta_\perp(\infty) \propto \nu^{-0.14}$

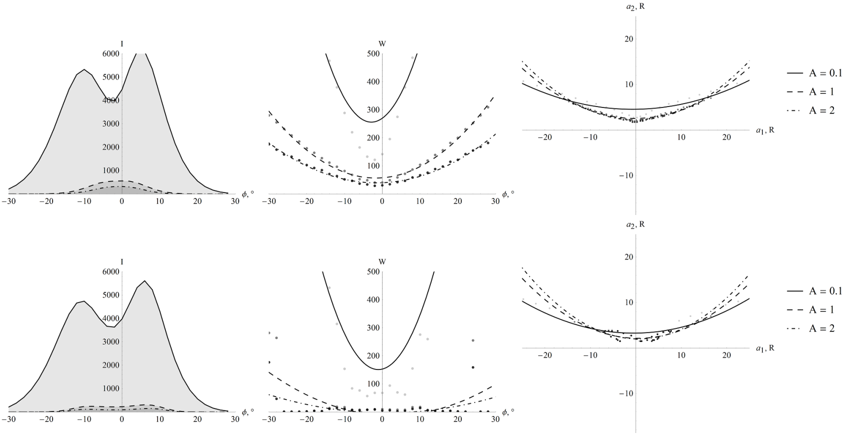
X-mode: $\theta_\perp(\infty) = Const$



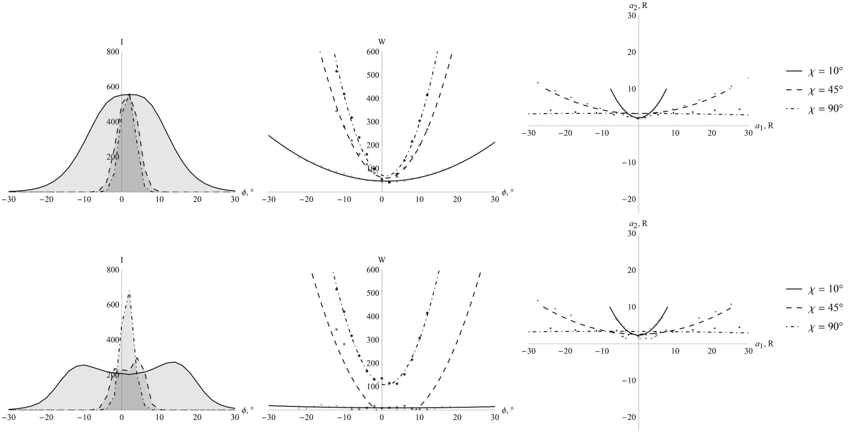


Picture for O-mode $A = 0.5$, $R_{\mathrm{m}} = 100$, $\chi = 45°$, $\gamma = 100$.

X and O modes respectively for $A = \{0.1,\ 1,\ 2\}$.

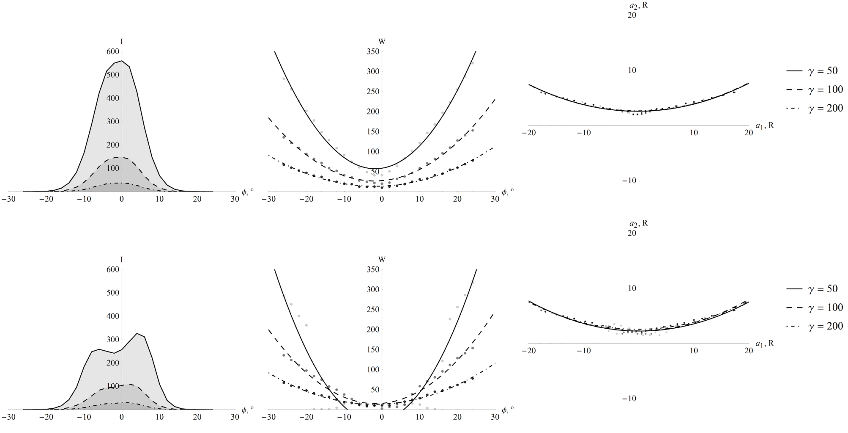


X and O modes respectively for $\chi = \{10^\circ,\ 45^\circ,\ 90^\circ\}$.



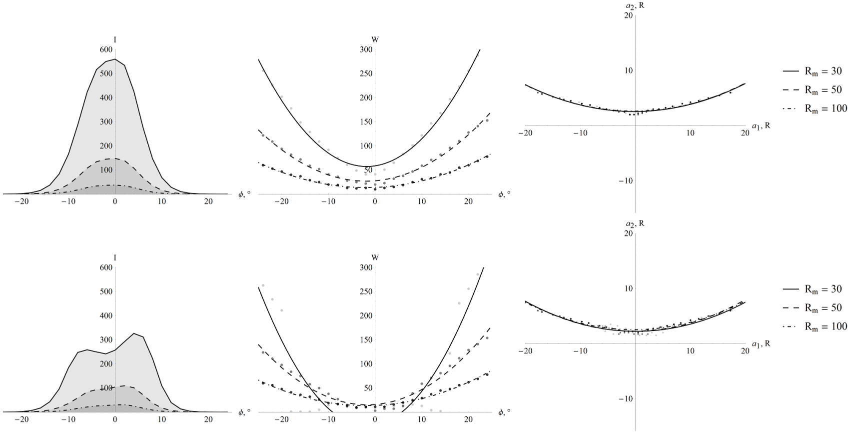
Results
Plots

X and O modes respectively for $\gamma = \{50,\ 100,\ 200\}$.



Results
Plots

X and O modes respectively for $R_{\mathrm{m}} = \{30,\ 50,\ 100\}$.



- a method was developed for obtaining pictures of radiopulsar in image surface both for O and X modes;
- images were presented for different parameters of propagation theory (A, χ, γ, R_{m});
- the dependance of the image width and its size look very similar to those studied by *[C.R. Gwinn, et al.]* using interstellar scintillations technique;
- the velocity of the astrometric motion of pulsar's emission calculated is near to that obtained by *[Ue-Li Pen, et al. 1301.7505 (2013)]* by interstellar scattering speckle pattern associated with the pulsar PSR 0834+06 ($\sim 1000\ km/s$).