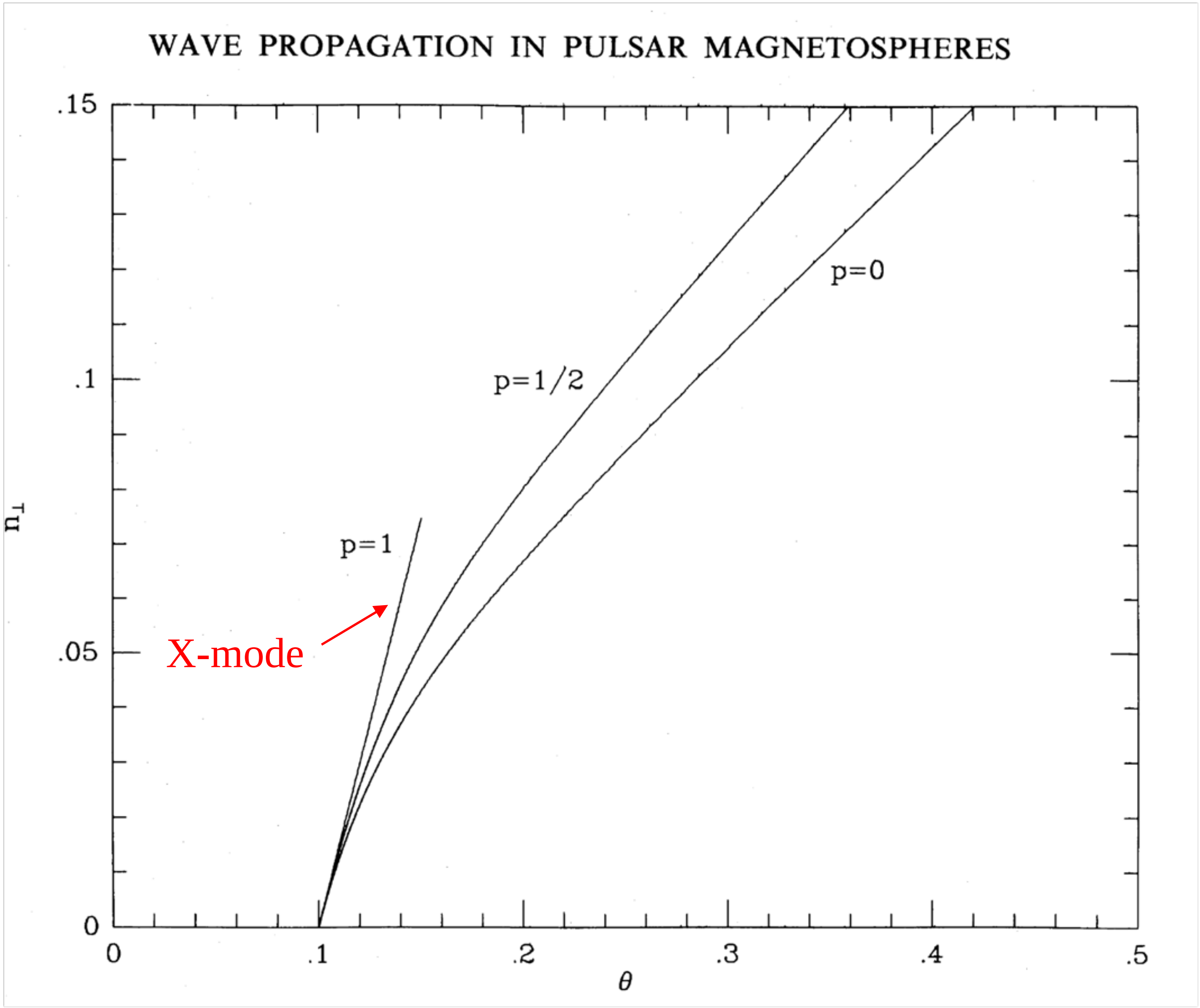
Аномальный момент, действующий на вращающийся шар в вакууме, и другие простые задачи

Бескин В.С., Желтоухов А.А.

ФИАН, МФТИ

Refraction

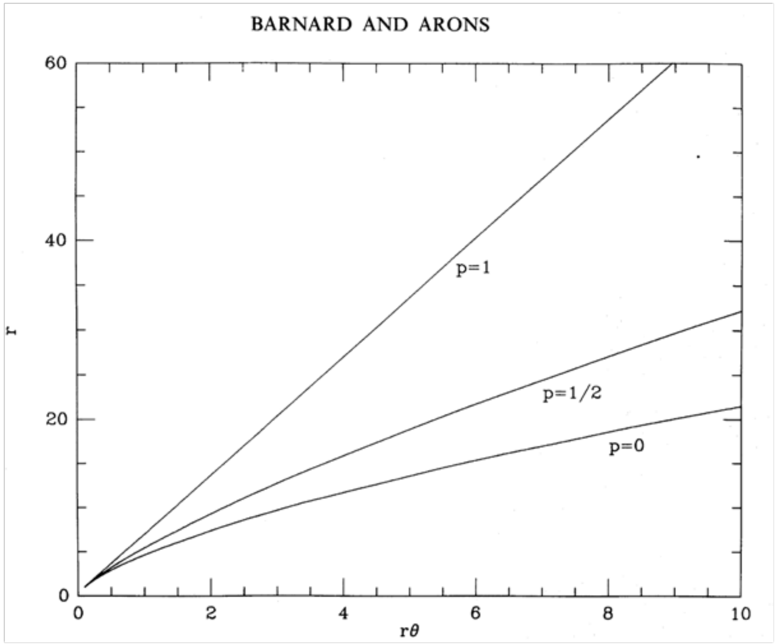
J.Arons, J.Barnard. ApJ, **302**, 120 (1986)

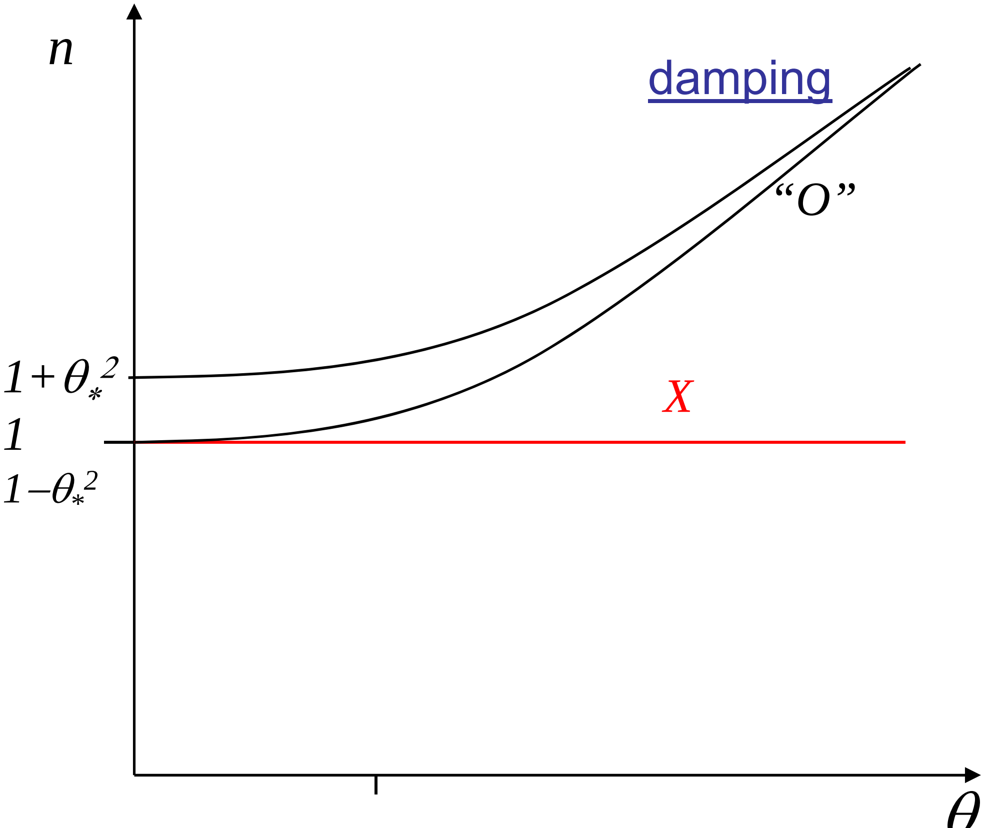


Refraction

J.Barnard, J.Arons, ApJ, **302**, 138 (1986)

$$\varepsilon_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 - < \frac{\omega_p^2}{\varpi^2 \gamma^3} > \end{pmatrix}$$





$$\theta_* = < \left(\frac{\omega_p^2}{\omega^2 \gamma^3} \right)^{1/4} >$$

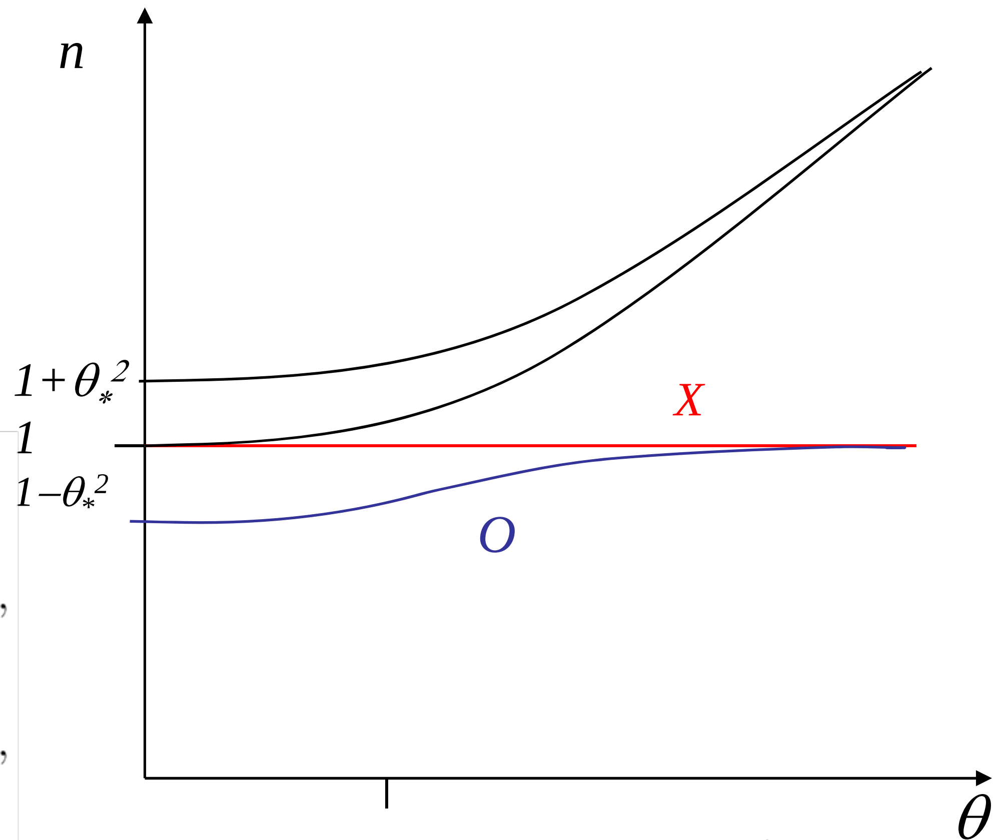
Refraction

VB, A.V.Gurevich & Ya.N.Istomin, Ap&SS, **146,** 205 (1988)

$$\varepsilon_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1- < \frac{\omega_p^2}{\varpi^2\gamma^3} > \end{pmatrix}$$

If $$A_{\rm p} = \frac{\omega_p^2}{\omega^2} < \gamma > \gg 1$$

$$\begin{aligned} n_1 &= 1, \\ n_2 &= 1 + \frac{\theta^2}{4} - \left(\frac{\omega_p^2}{\omega^2} < \frac{1}{\gamma^3} > + \frac{\theta^4}{16}\right)^{1/2}, \\ n_3 &= 1 + \frac{\theta^2}{4} + \left(\frac{\omega_p^2}{\omega^2} < \frac{1}{\gamma^3} > + \frac{\theta^4}{16}\right)^{1/2}, \\ n_4 &= \frac{1}{\cos\theta}. \end{aligned}$$



$$\theta_* = < \left(\frac{\omega_p^2}{\omega^2\gamma^3}\right)^{1/4} >$$

Propagation

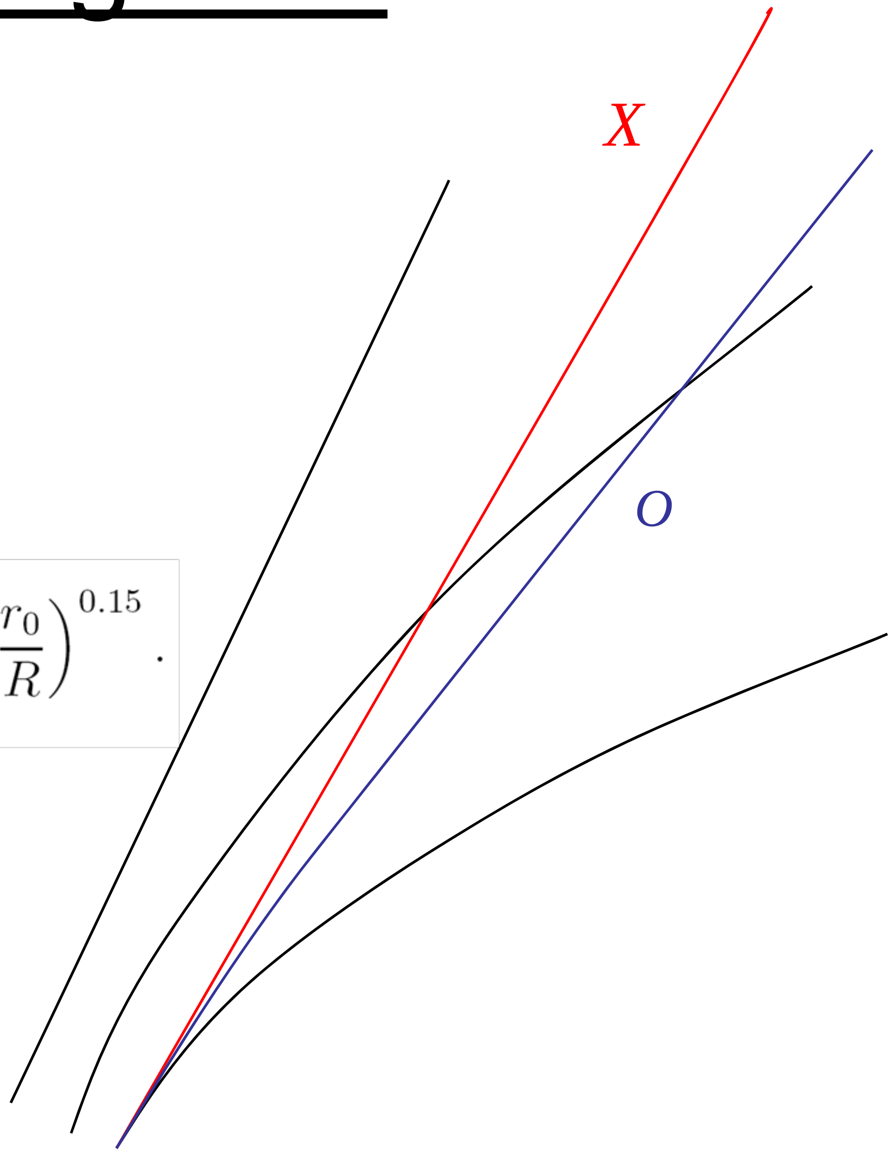
$$\frac{\mathrm{d}r_\perp}{\mathrm{d}l} = \frac{\partial}{\partial k_\perp}\left(\frac{k}{n_j}\right),$$

$$\frac{\mathrm{d}k_\perp}{\mathrm{d}l} = -\frac{\partial}{\partial r_\perp}\left(\frac{k}{n_j}\right)$$

$$W \approx \left(\frac{\Omega R}{c}\right)^{0.36} \left(\frac{\omega_{p0}}{\omega}\right)^{0.14} < \gamma^{-3} >^{0.07} \left(\frac{r_0}{R}\right)^{0.15}.$$

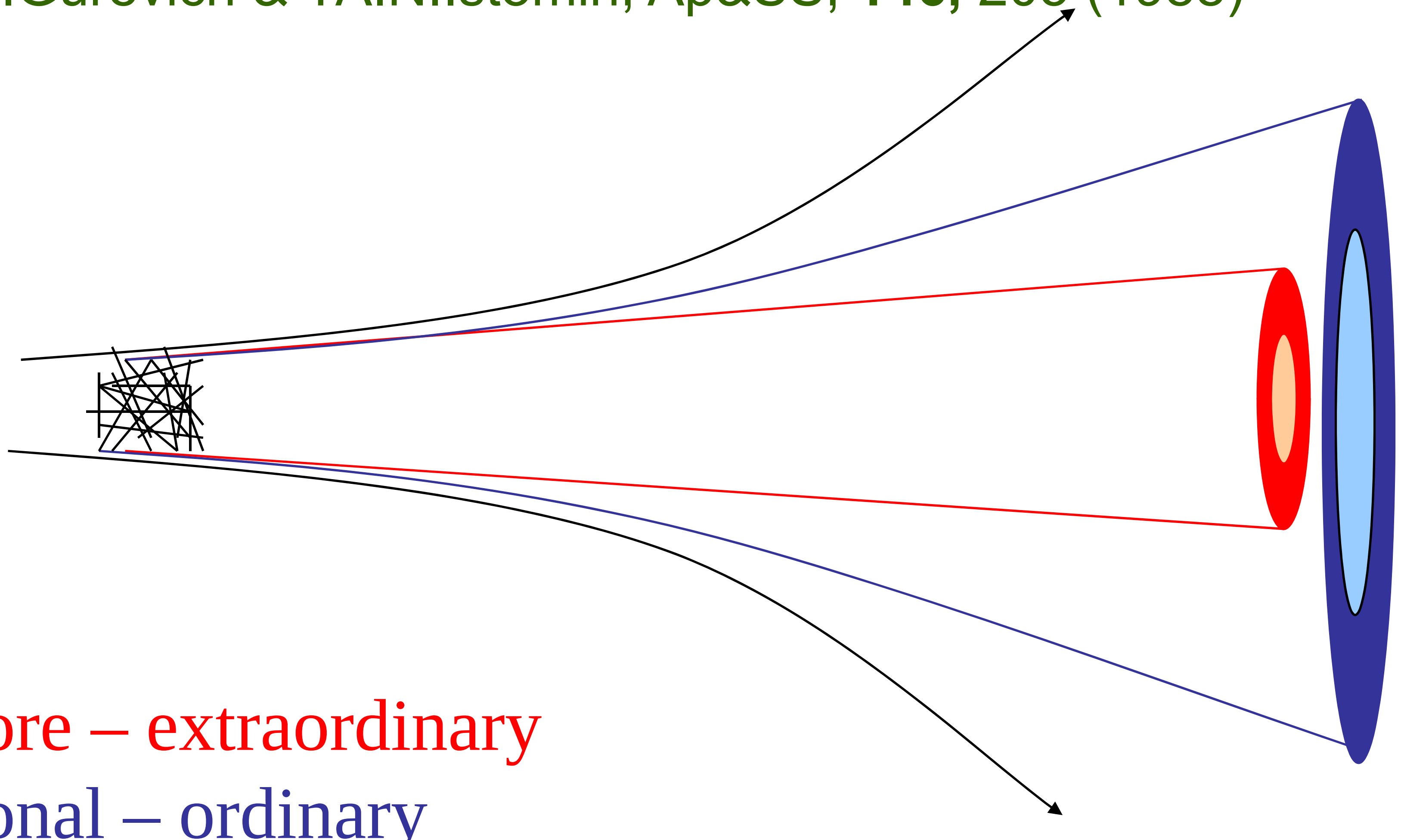
On can know $r_0(\nu)$

Yu. Lyubarsky, S.Petrova
D.Lai et al, VB et al



Core & Conal

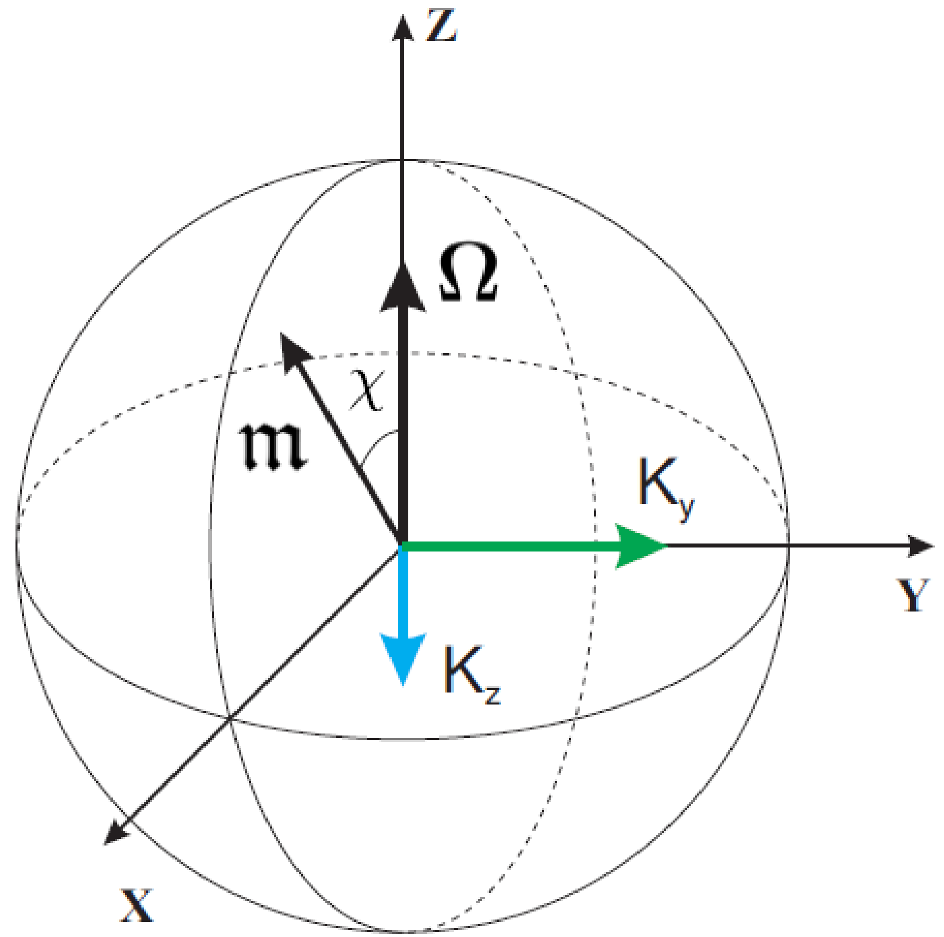
VB, A.V.Gurevich & YA.N.Istomin, Ap&SS, **146,** 205 (1988)



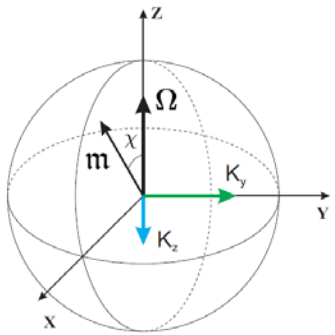
Core – extraordinary

Conal – ordinary

Anomalous torque



Anomalous torque



$$K_{z'} = \frac{2}{3}\frac{\mathfrak{m}^2}{R^3}\left(\frac{\Omega R}{c}\right)^3 \sin^2\chi$$

$$K_{x'} = \frac{2}{3}\frac{\mathfrak{m}^2}{R^3}\left(\frac{\Omega R}{c}\right)^3 \sin\chi\cos\chi$$

$$\boxed{K_{y'} = \xi\frac{\mathfrak{m}^2}{R^3}\left(\frac{\Omega R}{c}\right)^2 \sin\chi\cos\chi}$$

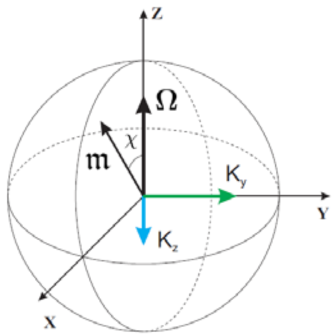
$\xi = 1$ (Davis & Goldstein 1970, Goldreich 1970)

$\xi = 1/5$ (Good&Ng 1985)

$\xi = 0$ (Michel 1991, Istomin 2005)

$\xi = 3/5$ (Melatos 2000)

Anomalous torque



Main assumptions

- Rotation in vacuum
- Freezing-in condition inside the body

$$\mathbf{E} + \beta_{\mathrm{R}} \times \mathbf{B} = 0$$

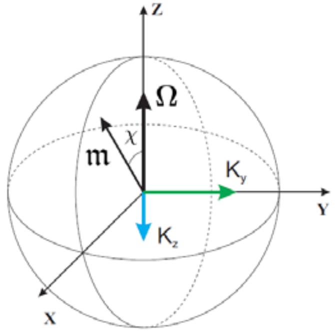
$$\beta_{\mathrm{R}} = \mathbf{\Omega} \times \mathbf{r}/c$$

- Corotation currents $\mathbf{j} = c\rho_{\mathrm{e}}\beta_{\mathrm{R}}$ only (hence, zero volume force)

$$\mathrm{d}\mathbf{F} = \rho_{\mathrm{e}}\mathbf{E}\,\mathrm{d}V + [\mathbf{j} \times \mathbf{B}]/c\,\mathrm{d}V$$

- Spherical body

Anomalous torque



Basic equation

$$\frac{\mathrm{d}\,\mathbf{L}_{\mathrm{field}}}{\mathrm{d}t} + \mathbf{K}^M + \int [\mathbf{r} \times \mathbf{F}]\mathrm{d}V = 0$$

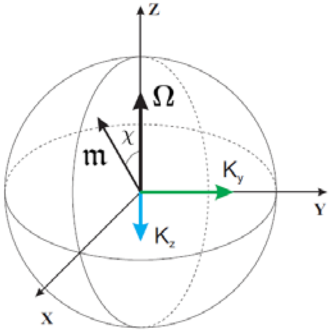
Electromagnetic angular momentum

$$\mathbf{L}_{\mathrm{field}} = \int \frac{[\mathbf{r} \times [\mathbf{E} \times \mathbf{B}]]}{4\pi c}\mathrm{d}V$$

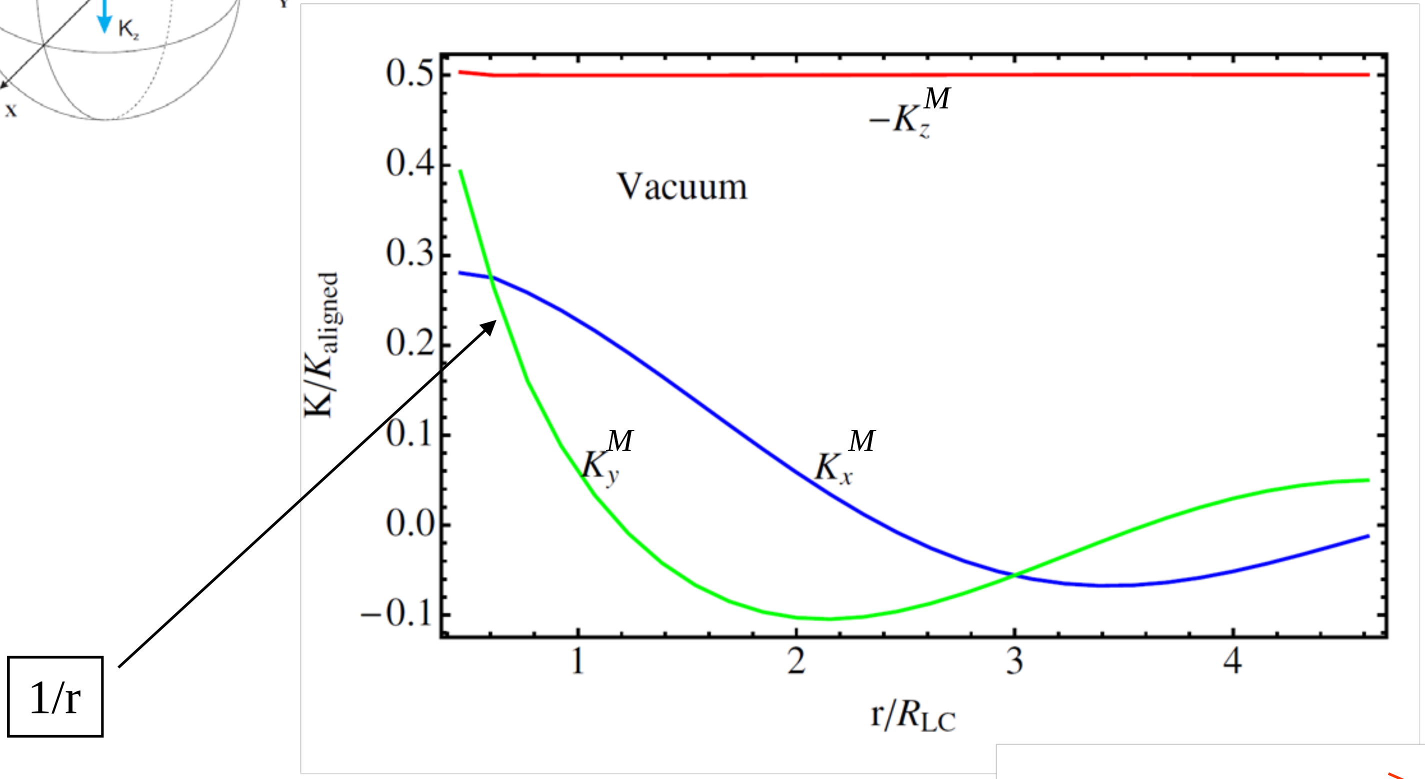
Flux $K_i^M = -\int \varepsilon_{ijk} r_j T_{kl} \mathrm{d}S_l$ (spherical surface)

$$\mathbf{K}_{y'}^M = \frac{R^3}{4\pi} \int \Big([\mathbf{n} \times \mathbf{B}]_{y'} (\mathbf{Bn}) + [\mathbf{n} \times \mathbf{E}]_{y'} (\mathbf{En}) \Big) \mathrm{d}o$$

Anomalous torque

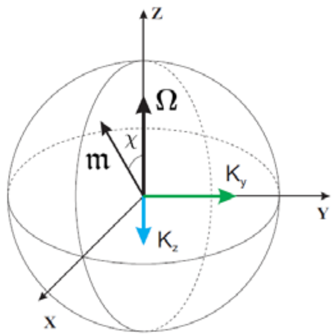


A.Philippov, A.Tchekhovskoy, J.Li
http://arxiv.org/abs/1311.1513



$$\frac{\mathrm{d}\,\mathbf{L}_{\rm field}}{\mathrm{d}t} + \mathbf{K}^M + \cancel{\int [\mathbf{r}\times\mathbf{F}]\mathrm{d}V} = 0$$

Anomalous torque



Torque, acting on the body

$$\frac{\mathrm{d}\,\mathbf{L}_{\mathrm{mat}}}{\mathrm{d}t} = \int [\mathbf{r} \times \mathbf{F}]\mathrm{d}V$$

$$\mathrm{d}\mathbf{F} = \sigma_{\mathrm{e}}\mathbf{E}\,\mathrm{d}S + [\mathbf{I}_{\mathrm{S}} \times \mathbf{B}]/c\,\mathrm{d}S$$

$$\mathbf{K} = \int \mathbf{r} \times \mathrm{d}\mathbf{F} = \frac{R^3}{4\pi}\int \Big([\mathbf{n} \times \{\mathbf{B}\}]\,(\mathbf{B}\mathbf{n}) + [\mathbf{n} \times \mathbf{E}]\,(\{\mathbf{E}\}\,\mathbf{n}) \Big)\mathrm{d}o$$

- for spherical body
- for highly conducting interior
- for corotation currents only

Anomalous torque

Calculations

- small parameter $$\varepsilon = \frac{\Omega R}{c}$$
- quasi-stationary $$\varphi - \Omega t$$

$$\begin{aligned} \nabla \times (\mathbf{E} + \beta_{\rm R} \times \mathbf{B}) &= 0, \\ \nabla \times (\mathbf{B} - \beta_{\rm R} \times \mathbf{E}) &= \frac{4\pi}{c}\mathbf{j} - 4\pi\rho_e\beta_{\rm R} \end{aligned}$$

$$\beta_{\rm R} = \mathbf{\Omega} \times \mathbf{r}/c$$

$$\mathbf{j} = c\rho_{\rm e}\beta_{\rm R}$$

$$\begin{aligned} \mathbf{E} + \beta_{\rm R} \times \mathbf{B} &= -\nabla\psi, \\ \mathbf{B} - \beta_{\rm R} \times \mathbf{E} &= \nabla h \end{aligned}$$

$\psi \longrightarrow \psi^{(1)} \longrightarrow E^{(1)}$

$h \longrightarrow h^{(2)} \longrightarrow B^{(2)}$

Anomalous torque

$$
\begin{aligned}
B_r^{\perp} &= \frac{|\mathfrak{m}|}{r^3} \sin\theta \,\mathrm{Re}\left(2 - 2i\frac{\Omega r}{c}\right) \exp\left(i\frac{\Omega r}{c} + i\varphi - i\Omega t\right), \\
B_\theta^{\perp} &= \frac{|\mathfrak{m}|}{r^3} \cos\theta \,\mathrm{Re}\left(-1 + i\frac{\Omega r}{c} + \frac{\Omega^2 r^2}{c^2}\right) \exp\left(i\frac{\Omega r}{c} + i\varphi - i\Omega t\right), \\
B_\varphi^{\perp} &= \frac{|\mathfrak{m}|}{r^3} \mathrm{Re}\left(-i - \frac{\Omega r}{c} + i\frac{\Omega^2 r^2}{c^2}\right) \exp\left(i\frac{\Omega r}{c} + i\varphi - i\Omega t\right), \\
E_r^{\perp} &= 0, \\
E_\theta^{\perp} &= \frac{|\mathfrak{m}|\Omega}{r^2 c} \mathrm{Re}\left(-1 + i\frac{\Omega r}{c}\right) \exp\left(i\frac{\Omega r}{c} + i\varphi - i\Omega t\right), \\
E_\varphi^{\perp} &= \frac{|\mathfrak{m}|\Omega}{r^2 c} \cos\theta \,\mathrm{Re}\left(-i - \frac{\Omega r}{c}\right) \exp\left(i\frac{\Omega r}{c} + i\varphi - i\Omega t\right).
\end{aligned}
$$

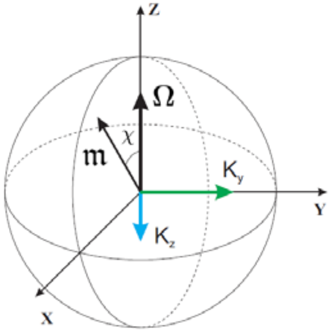
Anomalous torque

$$\mathbf{E} + \beta_{\mathrm{R}} \times \mathbf{B} = -\nabla\psi,$$
$$\mathbf{B} - \beta_{\mathrm{R}} \times \mathbf{E} = \nabla h$$

Calculations

- Freezing-in condition $\psi^{(\mathrm{In})} = 0$
- Hence, no freedom in $E^{(1)}$
- Exactly corresponds to vacuum solution
- Freedom in $B^{(2)}$ (free functions $h^{(2)}$)
- Result DOES NOT depend on this freedom

Anomalous torque



Torque, acting on the body

$$\mathbf{K} = \int \mathbf{r} \times \mathrm{d}\mathbf{F} = \frac{R^3}{4\pi} \int \Big([\mathbf{n} \times \{\mathbf{B}\}] \, (\mathbf{B}\mathbf{n}) + [\mathbf{n} \times \mathbf{E}] \, (\{\mathbf{E}\} \, \mathbf{n}) \Big) \mathrm{d}o$$

$$\boxed{K = K^M(R+0) - K^M(R-0)}$$

$$\xi = \frac{3}{5}$$

$$\mathbf{K}^M = -\frac{\mathrm{d}\,\mathbf{L}_{\mathrm{field}}}{\mathrm{d}t}$$

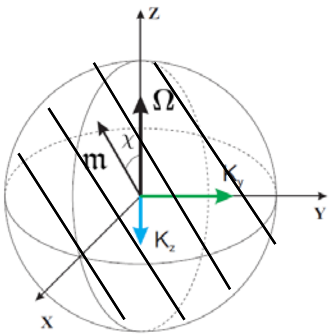
$$\mathbf{L}_{\mathrm{field}} = \int \frac{[\mathbf{r} \times [\mathbf{E} \times \mathbf{B}]]}{4\pi c} \mathrm{d}V$$

- Does not depend on the internal structure and freedom in $h^{(2)}$
- Obtained by A.Melatos (2000) for Deutsch solution

Depends on the internal structure

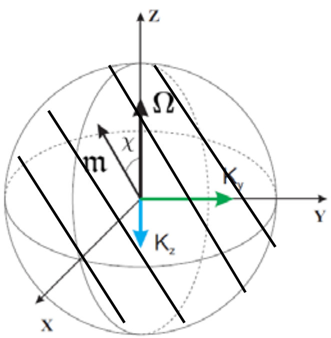
Anomalous torque

Three cases



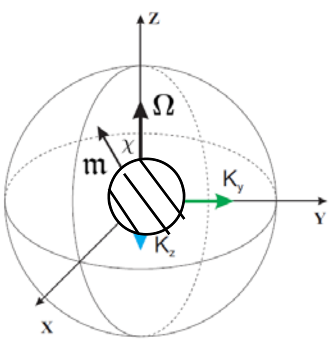
- Homogeneously magnetized sphere

$$K_{y'} = \frac{1}{3}\frac{\mathfrak{m}^2}{R^3}\left(\frac{\Omega R}{c}\right)^2 \sin\chi\cos\chi$$



- Homogeneously magnetized hollow envelope

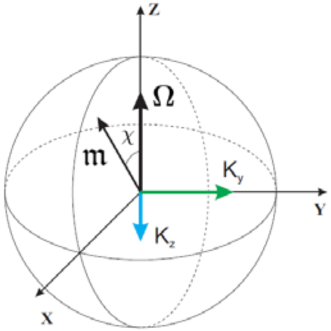
$$K_{y'} = \frac{31}{45}\frac{\mathfrak{m}^2}{R^3}\left(\frac{\Omega R}{c}\right)^2 \sin\chi\cos\chi$$



- Homogeneously magnetized core $R_{\text{in}} < R$

$$K_{y'} = \left(\frac{8}{15} - \frac{1}{5}\frac{R}{R_{\text{in}}}\right)\frac{\mathfrak{m}^2}{R^3}\left(\frac{\Omega R}{c}\right)^2 \sin\chi\cos\chi$$

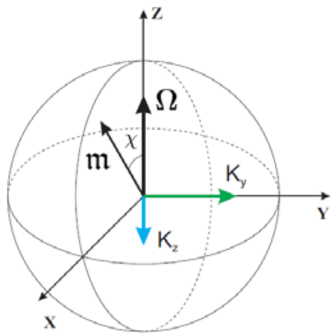
Anomalous torque



Main conclusion

Anomalous torque acting on the body depends on the internal structure of the electromagnetic field inside the body:

Anomalous torque



More conclusions

- Nonzero angular momentum L_{field} of the electromagnetic field exist only in Ω^2 order.
- In Ω^3 order it vanishes. It implies no problem in calculating the braking torque.
- Hence, energy losses can be determined via the flux $K_i = -\int \varepsilon_{ijk} r_j T_{kl} \mathrm{d}S_l$