Radio pulsar alignment/counter-alignment: final test?

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Abstract

At present, there are theoretical models of radio pulsar evolution that predict both the alignment, i.e. evolution of the inclination angle χ between magnetic and rotational axes to 0° [1-3] and its counter-alignment, i.e. evolution to 90° [4, 5]. At the same time, all these models describe well the pulsar distribution on the P–Pdot diagram. As noone still know how to determine the evolution of the inclination angle χ for individual pulsar, the braking mechanism remains unknown till now.

Here we show that the statistics of interpulse pulsars can give us the test to solve the alignment/counter-alignment problem as the number of interpulse pulsars drastically depends on the evolution law.

Introduction

There was many attempts to resolve the alignment/counter-alignment problem by analyzing the statistical distribution of radio pulsars. In particular, it was found both directly (i.e. by an analysis of the χ distribution) and indirectly (i.e. from an analysis of the observed pulse width) that statistically the inclination angle χ decreases with period P as the dynamical age increases [6-11]. On the other hand, recently, by analyzing 45 years of observational data for the Crab pulsar, Lyne et al. [5] found that the separation between the main pulse and interpulse increases at the rate of 0.6° per century (implying a similar growth of χ). Thus, one can conclude that at present there is no common point of view on the evolution law of the inclination angle χ of radio pulsars.

Interpulse pulsars

The interpulse (the second pulse between main pulses) appears when we observe either two opposite poles (a double-pole or DP pulsar) or the same pole twice (a single-pole or SP pulsar); in the latter case, two peaks correspond to the double intersection of the hollow-cone directivity pattern [13]. For the DP case, the inclination angle χ is close to 90°, while for a SP pulsars this angle is close to 0°. For standard period dependence of the directivity pattern width $W_{\rm rad} = W_0/P^{1/2}$ (different authors give $W_0 = 2.5° – 5.7°$), most interpulse pulsars are to have rather small periods, as we just observe (see Table 1).

As the number of interpulse pulsars (both for $\chi \sim 0°$ and $\chi \sim 90°$) depends on the evolution law of inclination angle, their number can help us to resolve alignment/counter-alignment problem.

Pulsar statistics

As almost all interpulse pulsars have small periods P (see Table 1) i.e., they are young enough, one can neglect the evolution of their magnetic field. Then the pulsar population can be described by kinetic equation

$$\frac{\partial}{\partial P}(\dot{P}\,N) + \frac{\partial}{\partial \chi}(\dot{\chi}\,N) = Q \qquad (1)$$

Accordingly, the observable population looks like

$$N^{\rm obs}(P, \chi, B) = V^{\rm vis}_{\rm beam} V^{\rm vis}_{\rm lum} N(P, \chi, B). \qquad (2)$$

In particular, for non-orthogonal pulsars the beam visibility function is

$$V^{\rm vis}_{\rm beam} = \sin\chi\, W_{\rm rad}\,, \qquad (3)$$

and for ordinary pulsars we use $V^{\rm vis}_{\rm lum}(P) \propto P^{-1}$.

As was shown in [13], for two models of pulsar evolution, i.e. for MHD (alignment) [3], and BGI (counter-alignment) [4], and for P < 0.5 s the solutions of kinetic equation (3) describing well the observable number of SP interpulse pulsars are

$$\begin{aligned} N_{\rm MHD}(P,\chi) &= K_{\rm MHD}\frac{(\pi/2 - \chi - \sin\chi\cos\chi)}{\cos^3\chi}P^2, \\ N_{\rm BGI}(P,\chi) &= K_{\rm BGI}\frac{(\chi - \sin\chi\cos\chi)}{\sin^3\chi\cos^{2d-1}\chi}P^2, \end{aligned} \qquad (4)$$

As to DP interpulse pulsars, the additional consideration is necessary as we have to include into consideration the visibility function of orthogonal pulsars depending on pulsar magnetic field.

Visibility function of orthogonal pulsars

Neutron stars work as radio pulsars if they produce enough e^+e^- pairs in polar gap. The efficiency depends on potential drop ψ, and, hence, on the inclination angle χ as ψ is proportional to Goldreich-Julian charge density

$$\rho_{\rm GJ} = -\frac{\Omega \mathbf{B}}{2\pi c}\,. \qquad (5)$$

As was shown recently [12], classical Ruderman-Sutherland theory describes good enough the results of modern numerical simulation. According to [4], it this case the death line corresponds to condition $Q_{\rm BGI} = 1$, where

$$Q_{\rm BGI} \approx P^{15/14} B_{12}^{-4/7} \cos^{2d-2}\chi, \qquad (6)$$

and $d \sim 0.75$. As one can see on Fig.1, average inclination angle $\langle\chi\rangle$ of the pulsar population is to decrease with period P (and dynamical age $\tau_{\rm D}$) even if the angle χ of individual pulsar increases.

Eqn. (6) can be used if the gap height is smaller than its transverse dimension. We determine the pair production region as $H_{\rm RS} < R_{\rm min}$, where

$$H_{\rm RS} = 1.1\times 10^4\,|\cos\theta_b|^{-3/7} R_{{\rm c},7}^{2/7} P^{3/7} B_{12}^{-4/7}\,cm \qquad (7)$$

is the polar gap height [4, 12] (θ_b is the angle between $\boldsymbol{\Omega}$ and $\boldsymbol{B}$), and $R_{\rm min}$ is the distance to the nearest singularity (magnetic pole, polar cap boundary or the surface $\rho_{\rm GJ} = 0$). An example of the shape of the pair creation region and two possibilities to observe the interpulse are shown on Fig.2. Accordingly, on Fig. 3 we show the visibility function $V^{\rm vis} = 2\delta W$ (the width of the intersection region in degrees) which is to be used instead of (3).

P(s)	0.03 s − 0.5 s	0.5 s − 1.0 s	> 1 s
$\mathcal{N}_{\rm SP}$	4 ÷ 10	2 ÷ 3	0 ÷ 1
$\mathcal{N}_{\rm DP}$	10 ÷ 24	3 ÷ 5	0 ÷ 1

Table 1. The number of SP and DP interpulse pulsars. The lower values correspond to the confident classification (the same one in all catalogues) and intensity ratio IP/MP > 0.1.

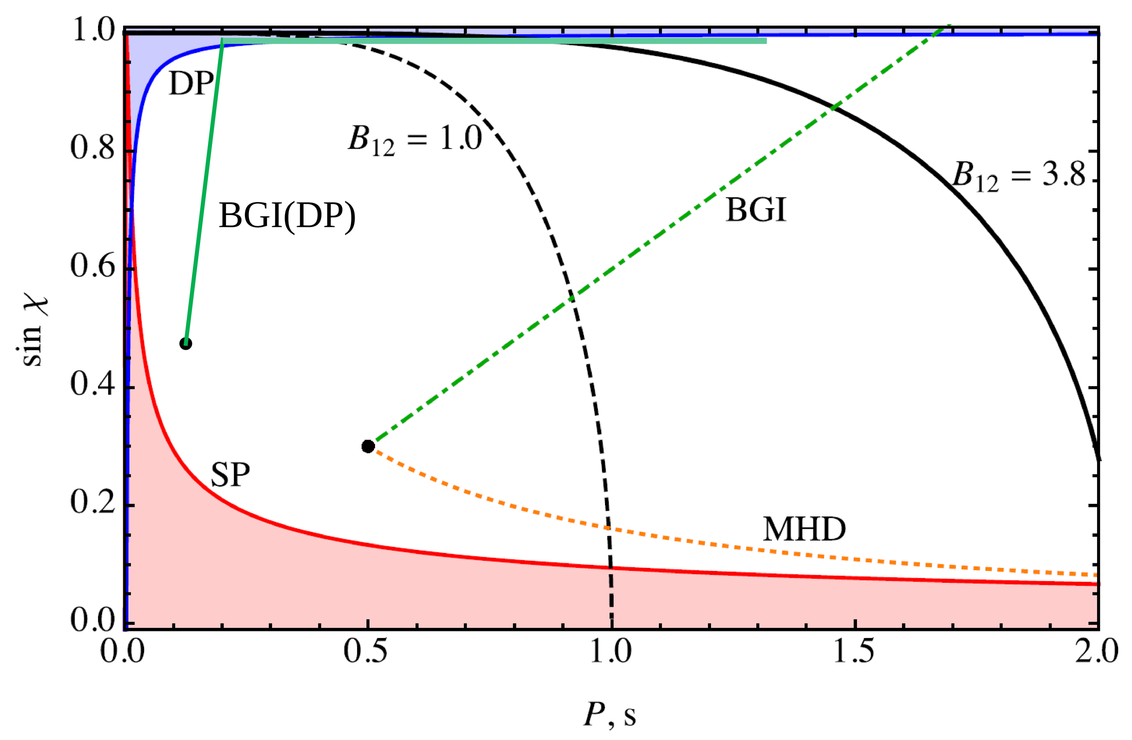

Fig. 1. Death lines on the P–sinχ diagram. Blue region corresponds to double-pole (DP) and red one to single-pole (SP) pulses. Evolution curves for BGI and MHD models are also shown.

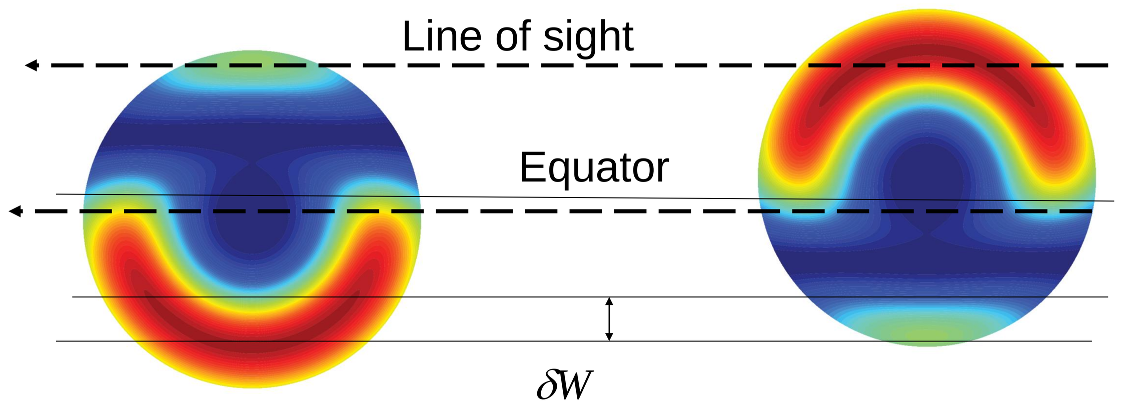

Fig. 2. Pair production regions for almost orthogonal pulsar. Two different possibilities to detect DP interpulse is also shown.

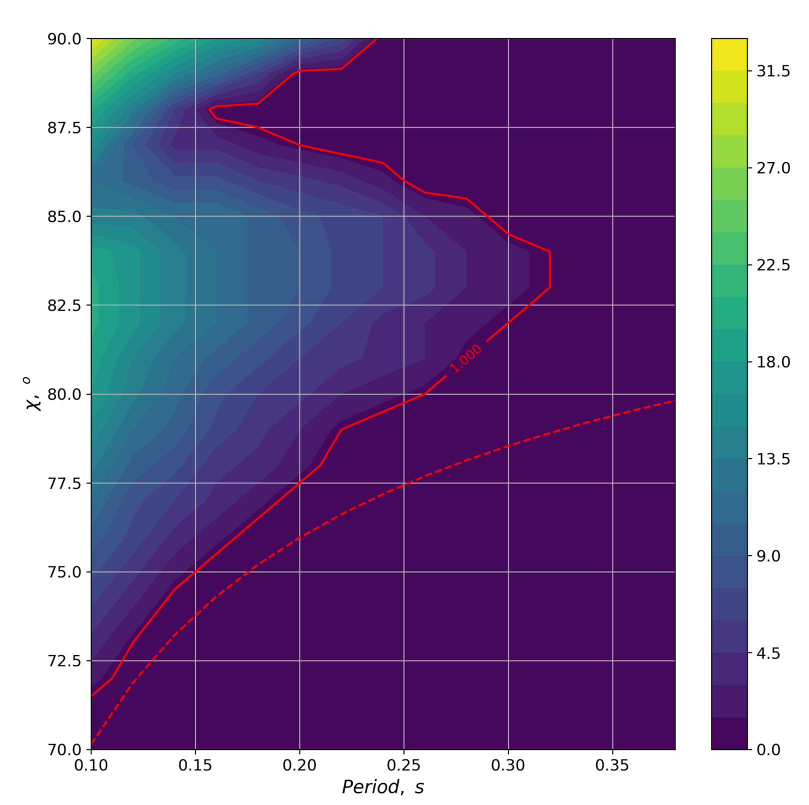

Fig. 3. Visibility function $V^{\rm vis} = 2\delta W$ for interpulse pulsars (in degrees) for $W_{\rm rad} = 5.7°/P^{1/2}$, and $B_0 = 10^{12}$ G.

Orthogonal interpulse pulsars

According to Fig. 3, one can observe orthogonal interpulse pulsars only for large enough magnetic field B_0. More exactly, for

$$B_{12} > 9\,P^2. \qquad (8)$$

Using now magnetic field distribution function $Q(B)$ obtained in [4] we found that both MHD and BGI models give only a few interpulse pulsars with $0.033 < P < 0.5$ s, i.e., approx. one order of magnitude smaller than we really observe (see Table 1).

Thus, MHD model gives too small number of orthogonal interpulse pulsars. On the other hand, as one can see on Fig. 1, BGI model predicts another group of pulsars with $\chi = 90°$ with braking law[14]

$$\dot{P}_{90} \approx 10^{-15} A(P)\,\frac{B_{12}^2}{P}\,, \quad A \approx 2\left(\frac{\Omega R}{c}\right)^{1/2}. \qquad (9)$$

Solving now kinetic equation

$$\frac{\rm d}{{\rm d}P}(\dot{P}_{90}\,N_{90}) = (N_{\rm BGI}\dot{\chi})_{\chi\to 90°} \qquad (10)$$

one can determine the number of BGI orthogonal pulsars. As is shown on Table 2, here takes place an agreement with observation data.

$P(s)$	0.03 − 0.1	0.1 − 0.2	0.2 − 0.3	0.3 − 0.4	0.4 − 0.5
obs	1 ÷ 2	7 ÷ 9	6 ÷ 8	3 ÷ 4	1 ÷ 2
BGI	3 ÷ 5	7 ÷ 9	4 ÷ 6	2 ÷ 3	1 ÷ 2

Table 2. Observed DP interpulse pulsars period distribution and BGI prediction. The scatter in the theory prediction is related to the uncertainty in (9).

Conclusions

Thus, statistics of interpulse pulsars gives the test helping us to determine the inclination angle evolution law. We show that the number of orthogonal interpulse pulsars is in agreement with counter-alignment model presented in [4]. On the other hand, MHD model gives too small number of DP interpulse pulsars.

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