# Suppression of stellar tidal disruption rates by anisotropic initial conditions

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## Abstract

We compute the rates of capture of stars by supermassive black holes, using time-dependent Fokker–Planck equation with initial conditions that have a deficit of stars on low-angular-momentum orbits. One class of initial conditions has a gap in phase space created by a binary black hole, and the other has a globally tangentially-anisotropic velocity distribution. We find that for galactic nuclei that are younger than $\sim 0.1$ relaxation times, the flux of stars into the black hole is suppressed with respect to the steady-state value. This effect may substantially reduce the number of observable tidal disruption flares in galaxies with black hole masses $M_\bullet \gtrsim 10^7\,M_\odot$. This poster is based on the article [Lezhnin & Vasiliev (2015)].

## Time-dependent Fokker–Planck equation

At a fixed energy $E$, the distribution function (DF) of stars $f(j)$ as a function of normalized angular momentum $j \equiv J/J_{\rm circ}(E)$, where $J_{\rm circ}$ is the angular momentum of a circular orbit with the same energy, satisfies the diffusion equation in a cylindrical geometry:

$$\frac{\partial f(E,j,t)}{\partial t} = \frac{\mathcal{D}}{4j}\frac{\partial}{\partial j}\left(j\frac{\partial f}{\partial j}\right). \tag{1}$$

Here $\mathcal{D}(E) \propto t_{\rm rel}^{-1}$ is the orbit-averaged diffusion coefficient, which is assumed to be independent of $j$, allowing an analytical solution to this equation. The boundary condition at $j = 1$ is of the Neumann type: $\partial f/\partial j = 0$ (zero-flux condition). The presence of the black hole creates a capture boundary at $j = j_{\rm lc}$, the angular momentum at which a star would be tidally disrupted at periapsis. As discussed in [Lightman & Shapiro(1977)], there are two limiting cases for the behaviour of $f(j)$ near $j_{\rm lc}$, depending on the ratio $q \equiv \mathcal{D}T/j_{\rm lc}^2$ between the mean-square change in $j$ due to relaxation over one orbital period $T$ and the size of the loss cone. In the general case, one may use a Robin-type boundary condition (a linear combination of the function and its derivative), which naturally interpolates between the regimes of empty ($q \ll 1$) and full ($q \gg 1$) loss cone:

$$f(j_{\rm lc}) = \frac{\alpha j_{\rm lc}}{2}\frac{\partial f(j)}{\partial j}\bigg|_{j=j_{\rm lc}}, \quad \alpha(q) \approx (q^2+q^4)^{1/4}. \tag{2}$$

## Tangentially anisotropic initial conditions

The first possible reason for the deficit is the ejection of stars by a binary SMBH that may have existed in a galaxy previously. The slingshot mechanism creates a gap in the phase space, ejecting stars with angular momenta less than some critical value, which corresponds to the periapsis radius comparable to the radius of a hard binary, $a_{\rm h} \equiv q/[4(1+q)^2]\,r_{\rm infl}$, where $q \equiv m_2/m_1 \le 1$ is the mass ratio of the binary, and $r_{\rm infl}$ is the SMBH radius of influence. In this work we adopt the definition of $r_{\rm infl}$ as the radius containing the mass of stars equal to $2(m_1+m_2)$ before the slingshot process has started.

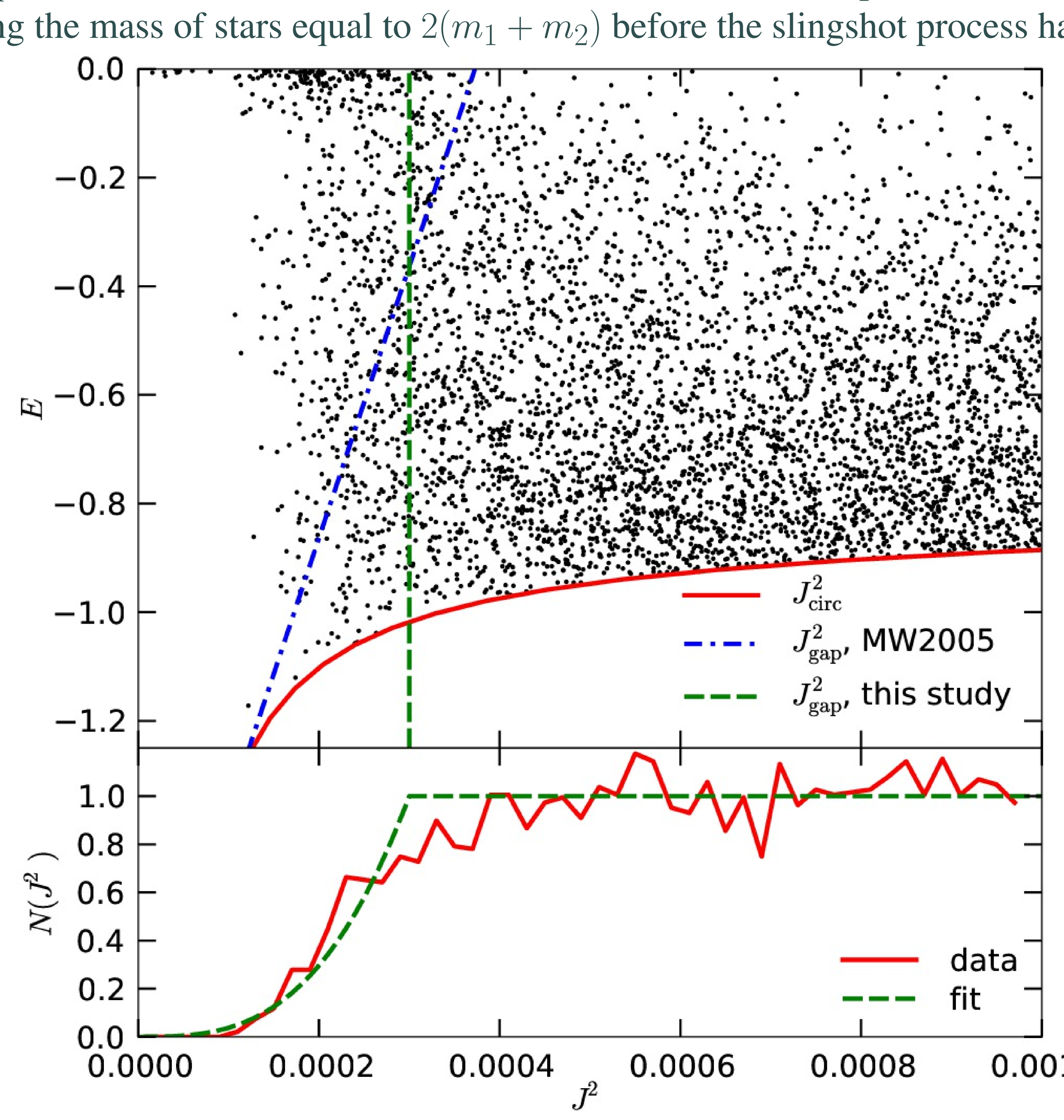


**Figure 1:** Monte Carlo simulations of a binary SMBH in a galaxy with $\gamma = 1$ Dehnen density profile. The binary mass is $10^{-2}$ of the total stellar mass, the mass ratio $q = 1$, and the binary started on a nearly-circular orbit at separation 0.2, roughly equal to the radius of influence.
*Top panel:* Phase space (squared angular momentum vs. energy) after the binary has cleared the low angular momentum region. Dashed green and dot-dashed blue lines show the definition of the gap region in this study and in [Merritt & Wang(2005)]; solid red line marks the angular momentum of a circular orbit.
*Bottom panel:* approximation of the initial distribution function (Equation 3) used in this work.

We have used a modified version of the Monte Carlo code Raga [Vasiliev(2015)] to determine the distribution of stars in angular momentum in a galaxy with a binary SMBH; our results are better fit by an energy-independent gap width, $J'_{\rm gap} \equiv \sqrt{K' G(m_1+m_2)a_{\rm h}}$, with $K' \simeq 3$, and a more gradual drop towards smaller $J$ (Figure 1):

$$f(E,J,0) = f(E)\cdot \min\left(1, \left(\frac{J}{J'_{\rm gap}}\right)^6\right). \tag{3}$$

Another possible choice of initial conditions involves a DF that has a tangential anisotropy at all $J$, not just a gap at small $J$. The simplest possibility is to consider DF in a factorized form: $f(E,J) = (1-\beta)[J/J_{\rm circ}(E)]^{-2\beta}\tilde f(E)$, where $\tilde f(E)$ is the counterpart of the usual isotropic DF and is computed from a given density profile with the method of [Cuddeford(1991)].

## Results

We have considered a set of [Dehnen(1993)] $\gamma$-models with a black hole mass $M_\bullet = 10^{-3}$ of the total mass in stars. The energy-dependent part of DF $f(E)$ or $\tilde f(E)$ and the diffusion coefficient $\mathcal{D}(E)$ are computed numerically from the given density profile, using the Eddington inversion formula or its Cuddeford's generalization [Cuddeford(1991)]. We explore several values of the power-law index $\gamma$ of the central density profile, and for each value of $\gamma$ we chose to consider a one-parameter family of models by scaling $M_\bullet$ and $r_{\rm infl}$ simultaneously, according to the following relation:

$$r_{\rm infl} = r_0\,[M_\bullet/10^8\,M_\odot]^{0.56}. \tag{4}$$

As our default normalization, we set $r_0 = 30$ pc, but we also consider values of $r_0 = 20$ and 45 pc. The ratio of the influence radius to the scale radius of Dehnen profile is $\{0.091, 0.047, 0.016\}$ for $\gamma = \{0.5, 1, 1.5\}$.

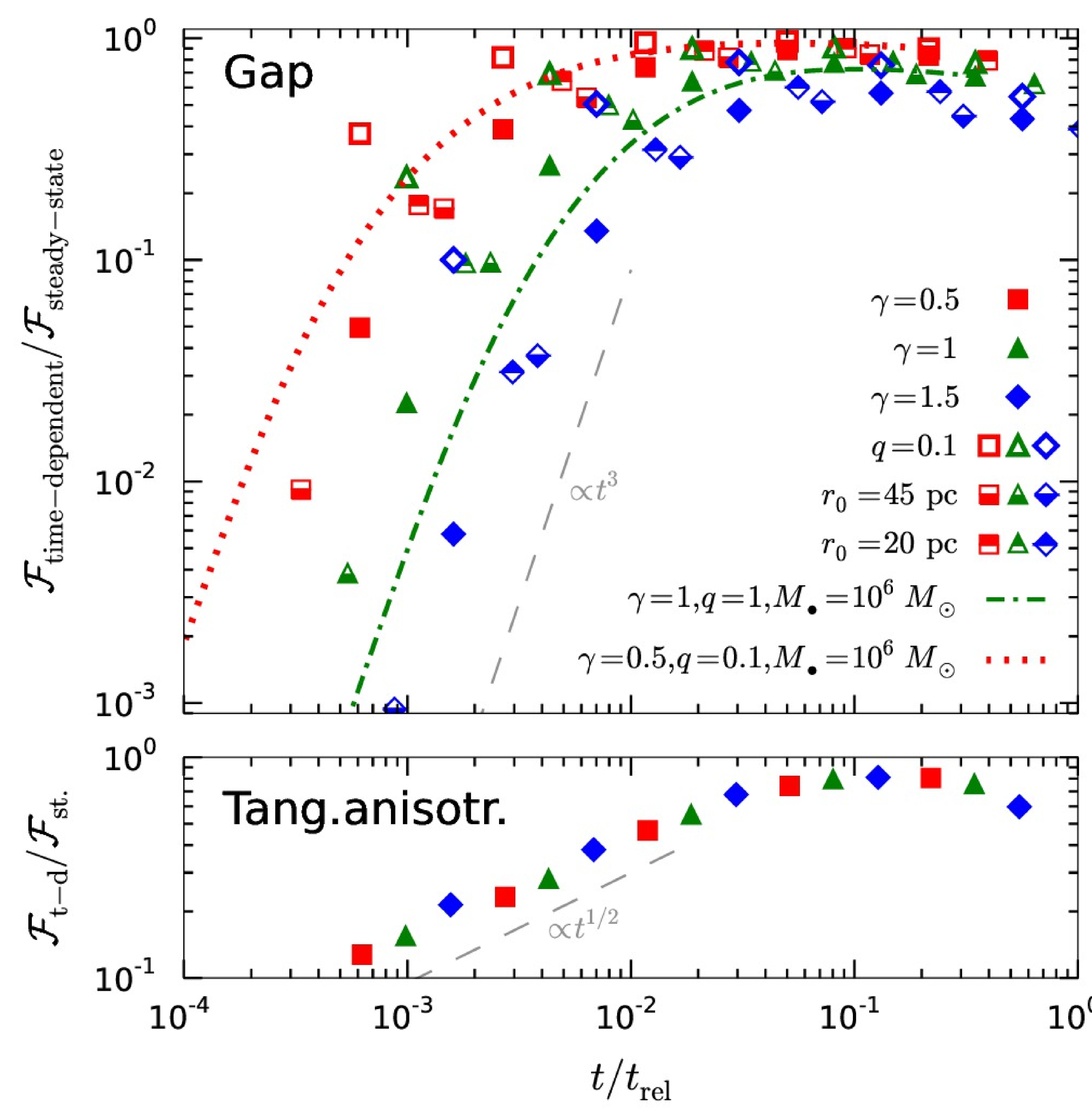


**Figure 2:** Suppression factor – the ratio of time-dependent to steady-state capture rate, as a function of time normalized to the relaxation time at the radius of influence. Individual points correspond to models with different density profiles and $M_\bullet = 10^6, 10^{6.5}, \dots, 10^8\,M_\odot$; the abscissae correspond to the Hubble time ($10^{10}$ yr) measured in units of relaxation time, thus the most massive black holes are at the left side of the plot.
*Top panel*: families of models with an initial gap in angular momentum distribution due to the action of a pre-existing binary SMBH; different symbols encode $\gamma$; open symbols are for models with smaller gap width (binary mass ratio $q = 1/10$) and filled – to equal-mass binaries; half-filled symbols are for $q = 1$ and different normalizations of $r_{\rm infl} - M_\bullet$ relation. Dotted and dot-dashed curves show examples of time-dependent flux for particular choices of parameters.
*Bottom panel*: tangentially anisotropic ($\beta = -1/2$) models for different $\gamma$.

The steady-state flux is comparable to $M_\bullet/t_{\rm rel}$. Figure 2, top panel, shows the ratio of time-dependent to steady-state capture rate, for two representative models with a gap. We also plot the same quantity measured at the time $10^{10}$ years, for various choices of $\gamma$, $M_\bullet$ and $q$.

The time time required to establish the steady-state profile at a given energy is $\sim (J_{\rm gap}/J_{\rm circ})^2\,t_{\rm rel}$; as the maximum of the total flux arrives from energies corresponding to $r_{\rm infl}$, the time to refill the gap is roughly $t_{\rm refill} \sim (a_{\rm h}/r_{\rm infl})\,t_{\rm rel}(r_{\rm infl})$, or

$$t_{\rm refill} \simeq 10^{13}\ {\rm yr} \times \frac{q}{4(1+q)^2}\left(\frac{M_\bullet}{10^8\,M_\odot}\right)^{1.28}\left(\frac{r_0}{30\,{\rm pc}}\right)^{1.5}. \tag{5}$$

**For $t \lesssim t_{\rm refill}$ the flux is reduced compared to the stationary value, which is commonly used in calculations of tidal disruption rates**. The maximum value of capture rate reached at $t \sim t_{\rm refill}$ is somewhat lower than the steady-state value, due to the fact that the $J$-averaged DF is also depleted at high binding energies (where $J_{\rm gap} \gtrsim J_{\rm circ}$) with respect to the value used in the steady-state calculation. The flux reaches 1/2 of its maximum value at $t_{1/2} \simeq 0.1\,t_{\rm refill}$. Moreover, at $t \gtrsim t_{\rm refill}$ it starts to decline in the absense of diffusion in energy. [Merritt(2015)] has performed numerical integration of two-dimensional $(E, J)$ Fokker–Planck equation restricted to the region inside $r_{\rm infl}$, also using initial conditions with a gap at $J < J_{\rm gap}$, and found a qualitatively similar behaviour if the diffusion in energy was artificially switched off. On the other hand, taking it into account modifies the solution at $t \gtrsim 0.1 t_{\rm rel}$ so that it tends to a steady-state profile. Therefore we may trust our calculations roughly up to a time when the flux reaches its maximum.

We also explore the effect of changing the normalization in $r_{\rm infl} - M_\bullet$ relation (4). This, of course, modifies both the time-dependent flux and the relaxation time at $r_{\rm infl}$, but the normalized values still stay on the same curve for each $\gamma$ and $q$.

Finally, the second class of **models with globally tangentially anisotropic initial conditions (Figure 2, bottom panel) produce a milder decline of the capture rate at $t \lesssim 0.1\,t_{\rm rel}$**.

## References

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