
A. V. Chernoglazov, V. S. Beskin, V. I. Pariev

1 Moscow Institute of Physics and Technology, Institutsky per. 9, Dolgoprudny 141700, Russia
2 P.N. Lebedev Physical Institute, Leninsky prospekt 53, Moscow 119991, Russia

Abstract

In this first paper from forthcoming series of works devoted to radio image of relativistic jets from active galactic nuclei the role of internal structure of a flow is discussed. We determine the radial profiles of all physical values for reasonable values of physical parameters such as Michel magnetisation parameter $\sigma_M$ and ambient pressure $P_{ext}$. Maps of Doppler boosting factor $\delta$ and observed directions of linear polarisation of synchrotron emission are also constructed.

Key words: galaxies: active, galaxies: jets

1 Introduction

Recent progress in high angular resolution VLBI observations of relativistic jets outflowing from active galactic nuclei (Mertens et al. 2016; Pushkarev et al. 2017) allows us to investigate directly their internal structure. In particular, the observations give us direct information about the dependence of the jet width $r_{jet}(l)$ on the distance $l$ from the "central engine".

Progress in VLBI observations allows us also to relate to more detailed information from the theory of relativistic jets. In spite of the wide variety of analytical and numerical models on jets acceleration and confinement (e.g., Vlahakis & Königl 2003; McKinney 2006; Komissarov et al. 2007; Tchekhovskoy et al. 2011; McKinney et al. 2012; Potter & Cotter 2015) considering different solutions for jets shapes, there is no common point of view on the internal structure of relativistic jets.

In a number of works (Pariev et al. 2003; Lyutikov et al. 2005; Porth et al. 2011; Fendt et al. 2014) observable maps of synchrotron polarisation and Faraday rotation for different models of relativistic jets in AGNs are suggested. The distribution of these quantities contains the information about magnetic field geometry, which is mostly toroidal. On the other hand, no predictions about Doppler boosting factor distribution have been done. The Doppler boosting factor was present in formulas, but its importance was not analysed. The ignorance of the Doppler boosting factor map was based on smallness of variation of toroidal velocity component ($v_\phi \ll c$), and it used to be impossible to measure the velocity directly. Nevertheless, it has been recently discovered (Mertens et al. 2016) via wavelet analysis that the radiating plasma inside the jet in the nearest radio galaxy M87 rotates slowly.

Another vital difference of our consideration from previous ones is the ability to consider all parts of the jet self-consistently. Below we use our model of quasi-cylindrical flow with zero total electric current inside the jet submerged into warm external gas in rest with reasonable thermal pressure: we do not assume neither external uniform magnetic field nor electromagnetic discontinuities at the jet boundary which was appearing in a bunch of papers, see (Martí 2015) figs 6,7 as example. This model gives us possibility to construct the most comprehensive radio map of FRII-type jet. The problem of the pressure-balanced jet without current sheet was also considered by Gourgouliatos et al. (2012). Their consideration is based on modification of Lundquist’s force-free solution (Lundquist 1950), and their result is that toroidal component of magnetic field is the same order of poloidal one $B_z \sim B_\phi$ instead of $B_z \ll B_\phi$ in our solution. See figs 3,5,6

This paper is the first in the forthcoming series of works where we will investigate the footprints of internal structure of relativistic jets in their radio images. For this we determine the transverse profiles of main physical parameters of relativistic jets such as the number density of particles, $n_e$, as well as toroidal and poloidal components of velocity $\mathbf{v}$ and magnetic field $\mathbf{B}$. Below we use semi-analytical 1D cylindrical approach introduced by Beskin (1997); Beskin & Malyshkin (2000). Further researches (Lery et al. 1999; Beskin & Nokhrina 2006, 2009; Lyubarsky 2009; Nokhrina et al. 2015) demonstrated that this simple approach allows...
us to describe principal properties of the internal structure of relativistic jets. For example, analytical asymptotic solutions for hydrodynamical Lorentz factor \( \gamma \)

\[
\gamma = \frac{r_\perp}{R_t}, \quad R_t > r_\perp^2 R_t \tag{1}
\]

\[
\gamma = \left( \frac{R_t}{r_\perp} \right)^{1/2}, \quad R_t < r_\perp^2 R_t \tag{2}
\]

were later reproduced numerically in 3D simulations (McKinney 2006; Komissarov et al. 2007; Tchekhovskoy et al. 2011). Here \( r_\perp \) is the distance from the rotation axis, \( R_t = \Omega/c \) is the radius of the light cylinder, and \( R_c \) is the curvature radius of magnetic surface. In addition, the image map of the Doppler boosting factor

\[
\delta = \sqrt{1 - \frac{\omega^2}{c^2}} \cos \chi / c
\]

on the cross section of the jet is given as well, where \( \chi \) is the angle between the velocity of a parcel of plasma and the line of sight. This map allow us to estimate the main parameters of the jet. Another result is map of EVP Ara (electric vector position angle) of the radiation. These functions are independent of model of high-energetic radiating particles’ distribution as well as their spectrum and are functions of MHD properties of an outflow. We also present some result of polarisation distribution in sky plane, but more detailed analysis of the radiation map and its comparison with observation will be presented in the following paper.

\section{1D CYLINDRICAL GRAD-SHAFRANOV EQUATION}

Basic equations describing the structure of relativistic axisymmetric stationary flows within Grad-Shafranov approach were formulated about forty years ago (e.g., see Aradvan 1976). This approach allows us to determine the internal structure of axisymmetric stationary jets knowing five “integrals of motion” in general case, which are energy \( E(\Psi) \) and angular momentum \( L(\Psi) \) flux, electric potential related to angular velocity \( \Omega(\Psi) \), entropy \( s(\Psi) \), and the particle-to-magnetic flux ratio \( \eta(\Psi) \). All these quantities have to be constant along magnetic surfaces \( \Psi = \text{const} \). In particular, it was shown that a jet with total zero electric current can exist only in the presence of the external media with finite pressure \( P_{\text{ext}} \). Thus, it is the ambient pressure \( P_{\text{ext}} \), which determines the transverse dimension of astrophysical jets.

As was shown by Beskin (1997); Beskin & Malyskhn (2000), for cylindrical flow it is convenient to reduce one second-order Grad-Shafranov equation to two first-order ordinary differential equations for magnetic flux \( \Psi(r_\perp) \) and poloidal Alfvénic Mach number \( M^2(r_\perp) \)

\[
M^2 = \frac{\eta_0 \mu^2}{n}. \tag{3}
\]

Here \( n \) is the number density in the comoving reference frame and \( \mu \) is relativistic enthalpy. The first equation is the relativistic Bernoulli equation \( u_p^2 = \gamma^2 - u_\perp^2 - 1 \), where \( u_p \) is the poloidal component of four-velocity \( \mathbf{u} \) and \( u_\perp \) is the toroidal component of four-velocity \( \mathbf{u} \). Replacements for \( \gamma, u_p, \) and \( u_\perp \) in the Bernoulli equation lead to the form

\[
\left( \frac{d\Psi}{dr_\perp} \right)^2 = \frac{K}{r_\perp^2 A^2} - \mu^2 \eta^2. \tag{4}
\]

Here

\[
A = 1 - \Omega_0^2 r_\perp^2 / c^2 - M^2
\]

is the Alfvénic factor,

\[
K = r_\perp^2 (e')^2 (A - \gamma^2) + M r_\perp^2 E^2 - M^2 L^2 c^2,
\]

and \( e' = E - \Omega L \). The second equation determines the Mach number \( M \)

\[
\left[ \frac{(e')^2}{\mu^2 \eta^2} - 1 + \frac{\Omega_0^2 r_\perp^2}{c^2} - A c^2 \right] \frac{dM^2}{dr_\perp} = \frac{M^4 L^2 c^2}{A r^2 \mu^2 \eta^2} + \frac{\Omega_0^2 r_\perp M^2}{c^2} \left[ 2 - \frac{(e')^2}{\mu^2 \eta^2} \right] + M^2 \left( \frac{e'}{\mu^2 \eta^2} \right) d\Psi \frac{dr_\perp}{d\Psi} \tag{7}
\]

\[
M^2 = \frac{\eta_0}{n} \frac{\partial P}{\partial n} + \left( 1 - \frac{\Omega_0^2 r_\perp^2}{c^2} \right) M^2 \frac{\partial \Psi}{\partial n} \frac{dr_\perp}{d\Psi} \frac{d\Psi}{d\Psi} \frac{d\Psi}{d\Psi}.
\]

Here \( T \) is the temperature, \( c_0 \) is the sound velocity defined as \( c_0^2 = (\partial P/\partial n)|_{n, mp} \), \( mp \) is the particle mass, \( m = mp c^2 + c_0^2 / (\Gamma - 1) \) is again relativistic enthalpy, and \( P \) is the gas pressure. As a result, Bernoulli equation (4) and equation (7) form the system of two ordinary differential equations for the Mach number \( M^2(r_\perp) \) and the magnetic flux \( \Psi(r_\perp) \) describing cylindrical relativistic jets.

As it was already stressed, the solution results from our choice of five integrals of motion. It is important to note that by determining the functions \( M^2(r_\perp) \) and \( \Psi(r_\perp) \) one can find the jet radius \( d\rho_\perp \) as well as the profile of the current \( I(r_\perp) \), the particle energy, and the toroidal component of the four-velocity from the solution of a problem under consideration. In particular, because

\[
\frac{I}{2\pi} = \frac{L - \Omega_0 r_\perp^2 E / c^2}{1 - \Omega_0^2 r_\perp^2 / c^2 - M^2}, \tag{8}
\]

the condition of the closing of the electric current within the jet \( I(\Psi_{\text{tot}}) = 0 \) can be rewritten as \( L(\Psi_{\text{tot}}) = 0 \) and \( \Omega_0 (\Psi_{\text{tot}}) = 0 \) simultaneously, where \( \Psi_{\text{tot}} \) is the given total magnetic flux in the jet.

For this reason we use the following expressions for these integrals

\[
L(\Psi) = \frac{\Omega_0 \Psi}{4\pi^2} \sqrt{1 - \frac{\Psi}{\Psi_{\text{tot}}}}, \tag{9}
\]

\[
\Omega_0 (\Psi) = \Omega_0 \sqrt{1 - \frac{\Psi}{\Psi_{\text{tot}}}}. \tag{10}
\]

In the vicinity of a rotation axis these integrals correspond to well-known analytical force-free solution for homogeneous poloidal magnetic field. On the other hand, they both vanish at the jet boundary which guarantees the fulfillment of the condition \( I(\Psi_{\text{tot}}) = 0 \).

Here we want to stress the novel point in our present work. We recall that careful matching of a solution inside the jet with the external media was produced only recently (Beskin et al. 2017). The difficulty in doing the matching is due to very low energy density of the external media in comparison with the energy density inside the relativistic jet. For
this reason, in most cases the infinitely thin current sheet was introduced at the jet boundary. Moreover, the external pressure was very often modelled by homogeneous magnetic field $B_{\text{ext}}^2/8\pi = P_{\text{ext}}$.

In contrast, Beskin et al. (2017) presented an approach which is free of the difficulties mentioned above. Following this paper we consider relativistic jet submerged into unmagnetised external media with finite gas pressure $P_{\text{ext}}$. Neither external magnetic field nor infinitely thin current sheet are assumed. We succeed in doing so due to the boundary conditions (9)–(10) at the jet boundary $r_\perp = d_{\text{jet}}$.

In addition, we assume that both magnetic field and flow velocity vanish at the jet boundary $r_\perp = d_{\text{jet}}$. As one can see from (8), our choice of $L(\Psi)$ and $\Omega_\Psi(\Psi)$ guarantees that toroidal component $B_\psi \propto 1/r$ vanishes at the jet boundary. One can find that the toroidal velocity $u_\psi$ also vanished at the jet boundary because of the following expression

$$u_\psi = \frac{1}{\mu \rho c r} \frac{(E - \Omega_\psi L) \Omega_\psi r^2/c^2 - L M^2}{1 - \Omega_\psi^2 r^2/c^2 - M^2}.$$ (11)

On the other hand, using relation $u_\psi = n B_\rho \eta$ one can conclude that the conditions $u_\psi = 0$ and $B_\rho = 0$ can be compatible with one another for finite $n$ and $\eta$. For simplicity we consider here the case

$$\eta(\Psi) = \eta = \text{const.}$$ (12)

In addition, we suppose that the flow remains supersonic up to the very boundary: $M(d_{\text{jet}}) > 1$. This supposition allows us to simplify our consideration. Indeed, in this case equation (7) has no additional singularity at the Alfvénic pressure was very often modelled by homogeneous magnetic field $B_{\text{ext}}^2/8\pi = P_{\text{ext}}$. Moreover, the external magnetic field $B_{\text{ext}}$ was introduced at the jet boundary. Also, in comparison with the derivatives in $r_\perp$ in the two-dimensional Grad-Shafranov and Bernoulli equations. This can be done for highly collimated, at least as a parabola, outflows (Nokhrina et al. 2015).

We apply here cylindrical approximation to a conical jet with small opening angle. As $v_\perp \ll v_z$ and $v_\perp \ll v_z$, one has to consider all components of velocity. We assume that jet has conical form with half opening angle $\theta \approx 2.5^\circ$. It gives

$$u_z(r_\perp) = u_p \cos \left(\frac{r_\perp}{d_{\text{jet}}}\right),$$
$$u_{\perp}(r_\perp) = u_p \sin \left(\frac{r_\perp}{d_{\text{jet}}}\right).$$ (19)

These formulae are to be valid also in the case of parabolic jet where $\theta$ is an angle between tangent and $z$-axis at the crosscut. An important formula for calculations of the Doppler boosting factor is for the angle between velocity of plasma parcel and the line-of-sight $\chi$. Introducing Cartesian coordinate system such that $\varphi = 0$ corresponds to $x$-direction and $\varphi = \pi/2$ corresponds to the $y$-direction, one obtains for line-of-sight vector

$$e = (\sin \alpha, 0, \cos \alpha),$$ (20)

where $\alpha$ is an angle between the jet axis $z$ and the line-of-sight. Hence,

$$\cos \chi = \frac{(v_x \sin \varphi + v_z \cos \varphi) \sin \alpha + v_z \cos \alpha}{\sqrt{v_x^2 + v_z^2}}.$$ (21)

3 RESULTS

As was already stressed, for given five “integrals of motion” the solution at each cross-section is fully determined by the value of ambient pressure $P_{\text{ext}}$. It concerns the jet width.
Figure 2. Poloidal magnetic field at two different cross-sections of the jet. Red solid lines correspond to magnetisation parameter $\sigma_M = 100$, and dashed blue lines to $\sigma_M = 10$. The central region is resolved.

Figure 3. Toroidal component of magnetic fields at two different cross-sections of the jet. Red solid lines correspond to magnetisation parameter $\sigma_M = 100$, and dashed blue ones to $\sigma_M = 10$. The central region is resolved.

Figure 4. Toroidal components of mass velocities at two different cross-sections of the jet. Plasma changes its rotational direction in the innermost region. Red solid lines correspond to magnetisation parameter $\sigma_M = 100$, and dashed blue lines to $\sigma_M = 10$. The central region of peak velocity is resolved.

Figure 5. Profiles of the Lorentz factors at two different cross-sections of the jet. Red solid lines correspond to magnetisation $\sigma_M = 100$, and dashed blue lines correspond to $\sigma_M = 10$. Lorentz factor at the axis $\gamma_0 = 2$.

d$_{\text{jet}}$ as well. For this reason in what follows we use the jet thickness $d_{\text{jet}}$ as a main parameter as it can be directly determined from observations.

On the other hand, as was shown by Beskin et al. (2017), our choice of “integrals of motion” allows us to express them through only one dimensionless quantity:

$$\sigma_M = \frac{\Omega_0^2 \Psi_{\text{tot}}}{8\pi^2 \mu \eta c^2},$$

(22)

i.e., through the Michel magnetisation parameter, which is the parameter of a “central engine” (it gives the maximum bulk Lorentz factor of the outflow). In addition, we use parameters of M87 black hole for calculations below. Another assumption is the value of the regular magnetic field nearby the event horizon $B = 10^4$ Gs. It is also assumed that magnetisation parameter $\sigma_M = 10 – 100$. This choice is reasonable for AGNs (see, e.g., Nokhrina et al. 2015).

The profiles of magnetic field components, velocities of particles, number density, and Lorentz factor of plasma are presented in Figs. 1–5 for two different width of the jet $d_{\text{jet}}$ and two different values of magnetisation parameter $\sigma_M$. As we show in Fig. 1, our choice of integrals of motion results in fast decrease of the number density in laboratory frame $n_e = n \gamma$ with the distance from the rotation axis, where $n$ is found via (3). As was mentioned above, number density is determined by magnetisation parameter $\sigma_M$. As we see, number density is larger for smaller $\sigma_M$. The dramatic growth of the density at the jet boundary is just dictated by pressure balance inside the thin boundary layer (Beskin et al. 2017)

$$P + \frac{B^2}{8\pi} = \text{const}$$

(23)

and by vanishing magnetic field outside the jet. Inside the jet the “cavity” is supported by large magnetic field pressure and by centrifugal force.

Further, on Figs. 2–3 we show the structure of magnetic field inside the jet. As we see, the magnetic field also forms the central core and then drops towards the jet boundary. In the narrow central part the toroidal component growth linearly as $B_\phi \propto I/r$ and $I = \pi j_p r^2$. Here $j_p$ is the current density. Outside the light cylinder $R_L$ toroidal magnetic field
On the Radio Image of Relativistic Jets — I

Figure 6. The map of the Doppler factor within a jet for inclination angle \( \alpha = 18^\circ \) for parameters of M87 and \( \sigma_M = 100, \quad \gamma_{in} = 2 \):

a) Cross-section as a whole without beaming effect. Contour lines are drawn with a step 0.01 for \( \delta \in (0, 0.8) \) and 0.3 for \( \delta \in (0.8, 4) \). b) Map of the directions of electric vector of linearly polarised radio emission. Observer is to the right of the jet axis (towards positive \( x \)). c) Map of radiation which is not depressed by relativistic beaming effect. d) Zoom in on the central core which is not resolved on plot (c). Direction toward the observer is to the right.

prevails up to the very edge \( B_\phi \gg B_p \). Since the jet radius in quasi-cylindrical part is \( d_{\text{jet}} \approx 10^2 - 10^4 R_L \), the relative size of region \( B_\phi < B_p \) is extremely small.

Besides, on Fig. 4, we show the toroidal components of hydrodynamical velocity \( v_\phi \). The differences in the magnitude are attributed to the particle-to-magnetic flux ratio which is larger for smaller magnetisation parameter (22). In any way, maximum value for toroidal velocity \( v_\phi \) cannot exceed a few tenths of speed of light \( c \). In this sense our predictions do not contradict observational data (Mertens et al. 2016). On the other hand, as one can see directly from (11), toroidal velocity \( v_\phi \) cannot be determined theoretically. The point is that two terms in the numerator have the same order of magnitude, the first one being related to the sliding along the magnetic surface, and the second one being related to the angular momentum. E.g., for monopole magnetic field (and for slow rotation) the toroidal velocity vanishes (Bogovalov 1992; Beskin & Okamoto 2000). For this reason it is not surprising that \( v_\phi \) can change sign.

Finally, as is shown on Fig. 5, the magnitude of the Lorentz factor is also determined by magnetisation parameter, its value being larger for wider jet (and, certainly, for larger magnetisation parameter \( \sigma_M \)). This fact is the illustration of the well known dependence (1). Indeed, it can be seen that in the central part of the jet Lorentz factor grows linearly as our choice of “integrals of motion” coincides with force-free choice in the vicinity of the axis.

It is also necessary to stress that there is no acceleration...
on the rotational axis because the flux of electromagnetic energy $E \propto \Omega F I$ is equal to zero there and Lorentz-factor at the axis is chosen $\gamma_m = 2$. The Lorentz factor does not also change at the boundary because $I$ is zero there too. In contrast, the Lorentz factor intensively changes at middle radii where magnetisation is initially high (see (15)).

Comparing the Lorentz factor distribution with the Doppler maps, one can see that the components with the highest Lorentz factor cannot be detected due to the relativistic beaming effect. These components may be seen in BL Lac objects only.

The maps of Doppler factors are presented in Figs. 6–10 for different inclination angles $\alpha$ as well as for different magnetisation parameters $\sigma_M$. The constraints on visible part are governed by relativistic beaming effect. If an angle between line-of-sight and velocity of plasma $\chi$ is greater than $1/\gamma$, a radiation cannot be caught. It can be seen in Fig. 6 c) that observers can see a slow outer part only. The bright core can be also detected. In the case of M87 (the black hole mass $M = 3 \times 10^9 M_\odot$, $a = 0.1$, and the distance is 17 Mpc) and inclination angle of jet $18^\circ$, the angular size of central bright core is $\sim 10^{-1}$ mas, while the peak angular resolution of VLBI is 1 mas for M87 at distance 10–100 mas from the central engine at frequency 15GHz (Yu. Kovalev, personal communication). Nevertheless, the core can be resolved with VLBI if the jet is less magnetised, e.g., Michel magnetisation parameter $\sigma_M = 10$ (see Fig. 9).

We also present results for the distributions of the Doppler factor in the case of fixed angle between rotational axis of the jet and the line-of-sight $\alpha = 18^\circ$ (as for the jet in M87) and different magnetisation parameters $\sigma_M = (10,30)$. The value $\sigma_M = 100$ was considered ear-
Figure 8. The same as at Fig. 6 but for inclination angle $\alpha = 4^\circ$. Magnetisation parameter $\sigma_M = 100$. Contour lines are drawn with step 0.1 for $\delta \in (0, 11.5)$ and 0.5 for $\delta \in (11.5, 20)$.

Relativistic motion of emitting plasma influences the direction of observed linear polarisation of synchrotron radiation. Properties of synchrotron radiation in the rest frame of plasma where only ordered magnetic field $B'$ is present and electric field $E' = 0$, are well known (e.g., chapter 5 in Ginzburg (1989)). Specifically, for highly relativistic radiating particles the electric field $E'$ of the wave is perpendicular to the local direction of the static magnetic field $B'$.

The changes in polarisation properties of polarised electromagnetic wave under Lorentz transformations was first mentioned and applied in astrophysical settings by Cocke & Holm 1972. After Lorentz boost the observed direction of the wave electric vector in observer’s reference frame is, in general, not perpendicular to the direction of the magnetic field. Calculations of polarisation properties of synchrotron radiation emitted by relativistic extragalactic jets were done in Blandford & Königl (1979); Pariev et al. (2003); Lyutikov et al. (2005). However, two dimensional distribution of polarisation over cross section of the jets was not calculated in these works. Here we calculate and draw maps of
unit vector $\mathbf{\hat{e}}$ along the electric field $\mathbf{e}$ in linearly polarised synchrotron radiation as seen by the observer. Each small patch of plasma contains isotropically distributed relativistic particles with energy spectrum $dN = K \epsilon^{-p} d\epsilon$. We do not consider circular polarisation at the moment. This approximation corresponds to ultrarelativistic energies of emitting particles. Then, degree of linear polarisation emitted by every small patch of the jets is the same $\Pi_0 = (p+1)/(p+7/3)$ (e.g., Ginzburg (1989)). We leave consideration of propagation effects in thermal plasma, such as Faraday rotation, as well as self-absorption effects for forthcoming series of works.

Under vacuum approximation the observed radiation is obtained by integration of Stokes parameters of independent incoherent emitters over the straight line of sight. Because each emitter has varying direction of the magnetic field and varying relativistic velocities, the direction of polarisation $\mathbf{\hat{e}}$ for each emitter also varies. As a result, the degree of polarisation in the total integrated radiation flux along each line of sight will be smaller than the upper limit $\Pi_0$. We leave the construction of integrated observable maps of synchrotron radiation from our theoretical models of jets for forthcoming series of works.

General expressions giving the polarisation unit vector $\mathbf{\hat{e}}$ in terms of the observed direction of magnetic field at the emitter, unit vector $\mathbf{\hat{B}}$, direction of the wavevector of the wave to the observer, unit vector $\mathbf{\hat{n}}$, and the velocity $\mathbf{v}$ of the emitter were derived in compact form in Lyutikov et al. (2003), formula (C5) in Appendix C there. We reproduce these expression here for convenience and keep speed of light $c$ in line with the notations used in the present work:

$$\mathbf{\hat{e}} = \frac{\mathbf{n} \times \mathbf{q}}{\sqrt{q^2 - (n \cdot q)^2}}, \quad \mathbf{q} = \mathbf{\hat{B}} + \mathbf{n} \times (\mathbf{v} \times \mathbf{\hat{B}})/c.$$  \tag{24}
Directions of polarisation vector $\hat{e}$ are plotted in Figs (6)–(10) on panels (b).

4 DISCUSSION AND CONCLUSION

In this paper we presented the profiles of magnetic field, velocity, and number density across cylindrical jet submerged into non-magnetised gas at rest. Another result is the map of the Doppler boosting factor at the cross-section of the jet together with the map of relativistic beaming effect under consideration.

It is shown how both the magnetic field and the number density gradually drop to external medium values close to the external boundary of the jet. We demonstrated that regardless the value of the inclination angle, only outer ring and central core can be observed. The size of central core is very small and can be measured only when the angle between the jet axis and the line-of-sight is small. It is necessary to notice that the central core must exist for MHD mechanism of acceleration since the electromagnetic energy $\Omega_F I$ at the rotation axis is zero, and Lorentz factor is conserved along the axis. The outer part may be relatively large with dramatic change of Doppler factor across the ring, but the outer part is supposed to be dim.

The multiplicity parameter calculated using results of our model is

$$\lambda = \frac{n_{\text{lab}}}{n_{\text{GJ}}} \approx 5 \times 10^{12},$$

where $n_{\text{GJ}} = \Omega_F B_\varphi / (2\pi c e)$ is Goldreich-Julian density, which is the number density of charged particles just enough to screen the longitudinal electric field. This result is in agreement with observations because $\lambda \sigma_M \approx (W_{\text{tot}}/W_a)^{1/2}$.
where $W_{\text{tot}}$ is the total energy losses of the jet, and $W_a = m_e^2c^3/e^2 \approx 10^{17}$ erg/s. It gives $\lambda\sigma_M \sim 10^{14}$, which agrees with (25) for $\sigma_M \sim 10$ to 100.

In addition, it is shown that the size of the central core actually does not depend on Michel magnetisation parameter $\sigma_M$ but is a function of the inclination angle $\alpha$. In contrast, the size of the outer ring depends on both $\sigma_M$ and the angle $\alpha$: the lower each value, the wider the ring.

The most reasonable Michel parameter to explain observations of M87 is $\sigma_M \approx 10$. This value of $\sigma_M$ is able to reproduce simultaneously the size of the outer bright core, bulk Lorentz factor $\gamma \approx 6$, the observation of super-luminal motion, and numerical simulations (see, e.g. (Porth et al. 2011)).

We have also calculated maps of observed direction of linear polarisation of synchrotron radiation emitted by a thing cross sectional layers of the jet. The effect of relativistic aberration on the polarisation was taken into account. This effect and obtained directional maps of polarisation are fundamental basics of understanding and interpreting present and future VLBI polarisation measurements of relativistic magnetised jets. In the following papers we are going to present full radio maps of the jet together with its rotation measure and polarisation of the radiation.

5 ACKNOWLEDGMENTS

We thank Y.N. Istomin for useful discussions, and especially 14th School of Modern Astrophysics SOMA-2018 (http://astrosoma.ru) for provision of fruitful ideas and discussions. This work was supported by Russian Foundation for Basic Research (Grant no. 17-02-00788).

REFERENCES

Beskin V. S., 1997, Physics Uspekhi, 40, 659
Beskin V. S., Malyskhin L. M., 2000, Astronomy Letters, 26, 208
Lundquist S., 1950, Ark. Fys., 2, 361

This paper has been typeset from a TeX/LaTeX file prepared by the author.