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Separatrix currents as a key subject of pulsar braking

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ABSTRACT
In this letter we show that the increasing of energy losses of radio pulsars with the inclination angle χ obtained numerically by many authors connects with the separatrix currents circulating in the pulsar magnetosphere which does not outflow into pulsar wind.

Key words: stars: neutron – pulsars: general.

1 INTRODUCTION

Starting with fundamental paper by Pacini (1967) it becomes clear that electromagnetic stresses play the main role in radio pulsar evolution. It is not surprising that magneto-dipole radiation was considered for a long time as a main mechanism of the pulsar braking (Ostriker & Gunn 1969; Manchester & Taylor 1977).

But later it was shown that for zero longitudinal electric currents circulating in the pulsar magnetosphere the energy losses $W_{\rm tot}$ vanish for any inclination angle χ (Beskin et al. 1983). This effect confirmed later by Mestel et al. (1999) results from full screening of the magneto-dipole radiation by magnetospheric plasma. This implies that the pulsar braking results fully from impact of the torque K due to longitudinal currents flowing in the magnetosphere.

Nevertheless, magneto-dipole losses are still often mentioned in connection with the discussion of the mechanism of energy release of radio pulsars. Indeed, MHD simulations for inclined rotator obtained first by Spitkovsky (2006) and later confirmed in many papers (Kalapotharakos & Contopoulos 2009; Pétri 2012; Kalapotharakos et al. 2012; Tchekhovskoy et al. 2013; Philippov et al. 2014; Tchekhovskoy et al. 2016) demonstrates the following dependence on the inclination angle χ

$$W_{\rm tot}^{\rm MHD} \approx \frac{1}{4}\frac{B_0^2\Omega^4R^6}{c^3}(1+\sin^2\chi). \quad (1)$$

As we see, in addition to unity this expression has the same dependence on the angle χ as for magneto-dipole losses, indicating that the magneto-dipole contribution could still exist. Moreover, recent numerical simulations (Kalapotharakos et al. 2012; Tchekhovskoy et al. 2016) have not clarified this question as well.

Indeed, well-known analytical Bogovalov (1999) solution definitely does not contain magneto-dipole wave since with the exception of the moment of passage of the current sheet, the electromagnetic fields do not depend on time. However, as was shown by Kalapotharakos et al. (2012); Tchekhovskoy et al. (2016), in the pulsar wind there is a noticeable component depending on time. In particular, for orthogonal rotator (when the current sheet is actually absent) one can write-down (Tchekhovskoy et al. 2016)

$$B_r \approx B_0\frac{R^2}{r^2}\sin\theta\cos(\varphi-\Omega t+\Omega r/c), \quad (2)$$

$$B_\varphi = E_\theta \approx -B_0\frac{\Omega R^2}{cr}\sin^2\theta\cos(\varphi-\Omega t+\Omega r/c). \quad (3)$$

In this Letter we analyze in detail the torque acting on a rotating star and the angular distribution of the energy flux inside the light cylinder. It is shown that MHD solution (1) does not contain magneto-dipole wave. Inside the light cylinder almost 100% of the energy flux outflows through the open magnetic field domain, not quasi-isotropical, as it should have been for magneto-dipole radiation.

2 CURRENT LOSSES

2.1 Longitudinal electric currents

As is well-known (see e.g. Mestel 1973), the key role in particle motion in the pulsar magnetosphere should be played by the electric drift associated with a strong electric field produced by the rotation of a neutron star. As a result, the transverse electric current $j_\perp$ can be written in the hydrodynamical form $j_\perp = \rho_{\rm e}U_{\rm dr}$, where $U_{\rm dr} = c[E\times B]/B^2$ is the electric drift velocity.

On the other hand, for fast enough rotation when the potential drop near magnetic poles is much smaller than the maximum possible potential drop (all the numerical results obtained up to now correspond to this limit) one can write down

$$E = -\beta_{\rm R}\times B, \quad (4)$$

where $\beta_{\rm R} = [\Omega\times r]/c$. Using this relation we finally obtain

$$j = \rho_{\rm e}[\Omega\times r] + i_{\parallel}B, \quad (5)$$

where $i_{\parallel}$ a scalar function. In particular, for orthogonal wind solu-

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tion (2)–(3) we have

$$i_\parallel = -3\,\frac{\Omega}{c}\cos\theta. \qquad (6)$$

The convenience of expression (5) is due to the fact that the scalar function $i_\parallel$ must be constant along magnetic field lines. Indeed, using another Maxwell equation, written under the assumption of quasi-stationarity (Mestel 1973; Beskin et al. 1983)

$$\nabla\times[B - \beta_{\rm R}\times E] = \frac{4\pi}{c}(j - c\rho_{\rm e}\beta_{\rm R}), \qquad (7)$$

we immediately arrive to condition $B\cdot\nabla i_\parallel = 0$.

2.2 Braking torque

The general expression for energy losses of a rotating magnetized sphere (which is true not only for a neutron star surrounded by a magnetosphere filled with plasma but also for a sphere rotating in vacuum) looks like (Landau & Lifshitz 1971)

$$W_{\rm tot} = \frac{c}{4\pi}\oint[E\times B]{\rm d}s = -\Omega\cdot K, \qquad (8)$$

where according to (4)

$$K = \frac{R}{4\pi}\oint\,[n\times B]\,(B\cdot n)\,{\rm d}s. \qquad (9)$$

Here the first bracket corresponds to the surface current $J_{\rm s}$, and the second one to magnetic field in the Ampere force $F = [J_{\rm s}\times B]/c$.

It is convenient to introduce two components of the torque K parallel and perpendicular to the magnetic dipole m and connect them with dimensionless current in the polar cap zone $i = j_\parallel/j_{\rm GJ}$ separating it into symmetric and anti-symmetric contributions, $i_{\rm s}$ and $i_{\rm a}$, depending upon whether the direction of the current is the same in the north and south parts of the polar cap, or opposite. As one can easily check, $K_\parallel \propto i_{\rm s}$, and $K_\perp \propto i_{\rm a}$. Here and below we apply normalization to the 'local' Goldreich-Julian current density, $j_{\rm GJ} = |\Omega\cdot B|/2\pi$ (with scalar product). As a result, we obtain

$$W_{\rm tot} = -\Omega\,\mathcal{K}_\parallel\cos^2\chi - \Omega\,\mathcal{K}_\perp\sin^2\chi, \qquad (10)$$

where we put $K_\parallel = \mathcal{K}_\parallel\cos\chi$ and $K_\perp = \mathcal{K}_\perp\sin\chi$.

As was shown by Beskin et al. (1993), the direct action of the Ampère force on the star $K = \int[r\times[J_{\rm s}\times B]/c]\,{\rm d}s$ by surface currents $J_{\rm s}$ which close volume longitudinal electric currents circulating in the pulsar magnetosphere can be written as

$$K_\parallel^{\rm sur} \approx -\frac{B_0^2\Omega^3R^6}{c^3}i_{\rm s}, \quad K_\perp^{\rm sur} \approx -\frac{B_0^2\Omega^3R^6}{c^3}\left(\frac{\Omega R}{c}\right)i_{\rm a}. \qquad (11)$$

Thus, to satisfy MHD energy losses $W_{\rm tot}$ (1) for $\chi = 90^\circ$ the anti-symmetric current $i_{\rm a}$ is to be large enough: $i_{\rm a}\approx(\Omega R/c)^{-1}$. On the other hand, comparing the total currents (6) flowing through the upper hemisphere of the orthogonal wind and through the northern part of the polar cap on the surface of the star, one can obtain

$$i_{\rm a}\approx\left(\frac{\Omega R}{c}\right)^{-1/2}. \qquad (12)$$

Thus, volume currents are too small to explain MHD energy losses.

2.3 Additional torque: rotating magnetized sphere

To clarify the braking mechanism responsible for MHD solution, let us consider uniformly magnetized sphere rotating in a vacuum. Surprisingly, but even in this long-settled question there is nontrivial point. Indeed, the total energy losses $W_{\rm tot} = -\Omega\cdot K = (2/3)m^2\Omega^4$ depend only on the magnetic moment of the pulsar m. However, the appropriate surface currents depend strongly on the fine structure of the magnetic field on the sphere surface.

To show this, we note first of all that the braking torque K (9) must be proportional to the third power of the angular velocity Ω. In other words, it must correspond to the third power of the expansion with respect to the small parameter $\varepsilon = \Omega R/c$. Further, one can show that the first order terms $B^{(1)}$ turn out to be zero (see, e.g., Beskin & Zheltoukhov 2014). As a result, the general expression for the braking torque can be written-down as

$$K = \frac{R}{4\pi}\oint\{[n\times B^{(3)}](B^{(0)}\cdot n) + [n\times B^{(0)}](B^{(3)}\cdot n)\}{\rm d}s, \qquad (13)$$

where the indices (0) and (3) correspond to the expansion powers in small parameter ε.

Recall now that cornerstone Deutsch (1955) solution was constructed under the assumption that the normal component of the magnetic field exactly coincides with the field of the magnetic dipole. This implies that $B_n^{(3)} = 0$, so that the only contribution to the expression (13) for the braking torque is given by the first term.

However, if we now use the Landau & Lifshitz (1971) solution for the point orthogonal dipole

$$B_r^\perp = \frac{|m|}{r^3}\sin\theta\,{\rm Re}\left(2 - 2i\frac{\Omega r}{c}\right)\exp\left(i\frac{\Omega r}{c} + i\varphi - i\Omega t\right),$$

$$B_\theta^\perp = \frac{|m|}{r^3}\cos\theta\,{\rm Re}\left(-1 + i\frac{\Omega r}{c} + \frac{\Omega^2 r^2}{c^2}\right)\exp\left(i\frac{\Omega r}{c} + i\varphi - i\Omega t\right),$$

$$B_\varphi^\perp = \frac{|m|}{r^3}\,{\rm Re}\left(-i - \frac{\Omega r}{c} + i\frac{\Omega^2 r^2}{c^2}\right)\exp\left(i\frac{\Omega r}{c} + i\varphi - i\Omega t\right), \qquad (14)$$

we find that only two thirds of the losses will still be determined by the first term in (13), while one third with the second one. Certainly, the total energy losses and the direction of the evolution of the inclination angle χ do not depend on the choice of the solution.

The source of this discrepancy can easily be found. "Landau-Lifshitz solution" differs from the Deutsch solution by adding an additional magnetic dipole $\delta m/|m| = (\varepsilon^3/3)e_{y'}$ generating by third order homogeneous magnetic field

$$B^{(3)} = -\frac{2}{3}\frac{|m|}{R^3}\left(\frac{\Omega R}{c}\right)^3 e_{y'}. \qquad (15)$$

It is clear that appropriate electromagnetic losses are much less than even the electric quadrupole losses associated with the inevitable redistribution of charges inside the sphere. However, the very structure of the decelerating currents changes radically.

Note, by the way, that braking torque (13) does not depend on whether the zeroth-ordering currents are concentrated on the surface of the star or at its center. This is due to the fact that, as in expression (15), the braking torque K does not depend on the radius of the sphere R for a given magnetic dipole m.

Thus, in addition to current losses mentioned above, the star braking can be connected with the perturbation of the normal component of the magnetic field B_n. In this case, it is necessary to take into account the perturbation of the magnetic field over the entire surface of the neutron star, not only within polar cap. Such additional contribution could be related to the violation of the exact mutual compensation between the magneto-dipole radiation of the central star and the radiation of the magnetosphere taking place for zero longitudinal current.

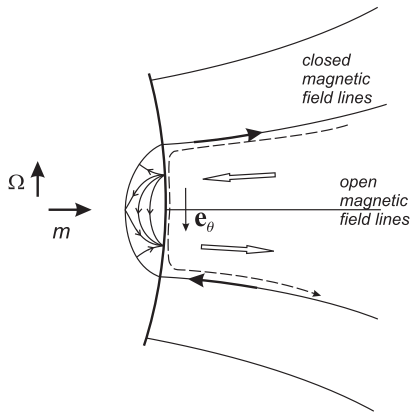

Figure 1. The structure of the volume (contour arrows), separatrix (fat arrows) and surface (thin arrows) currents near the polar cap of the orthogonal rotator. The additional separatrix current is shown by a dashed curve.

2.4 Additional torque: separatrix currents

To discuss another possible additional torque, let us consider in more detail the braking of an orthogonal rotator. It is convenient to rewrite the braking torque in the original form (do not forget that the radio pulsar has two poles!)

$$K = 2 \int r \times \frac{[J_\mathrm{s} \times B]}{c} \,\mathrm{d}s. \tag{16}$$

Accordingly, the total energy losses $W_\mathrm{tot} = -\Omega \cdot K$ can be written as

$$W_\mathrm{tot} = 2\,\frac{\Omega R}{c} \int J_\theta B_n \mathrm{d}s. \tag{17}$$

Analyzing expression (16) into the forehead, one can arrive at the erroneous conclusion that for a local GJ current ($i_\mathrm{s} = i_\mathrm{a} = 1$), the braking torque should not strongly depend on the inclination angle χ. Indeed, as the angle χ increases, the surface current J_s decrease as $\cos\chi$. But the characteristic distance from the axis to the polar cap r, on the contrary, increase as $\sin\chi$.

However, as an accurate analysis shows (Beskin et al. 1993), in this, at first glance, obvious reasoning does not take into account the real structure of the surface currents within polar cap region. As shown in Figure 1, surface currents should in fact be arranged in such a way that the current $< J_\theta >$ averaged over the polar cap surface (namely, this component, as we saw, determines the energy loss of the neutron star) would be zero. Therefore, in determining the energy losses, it was necessary to take into account the effects of a higher order in the parameter $\varepsilon = \Omega R/c$.

But if the surface averaged surface current $< J_\theta >$ is equal to zero, then, as shown in Figure 1, along the separatrix separating open and closed magnetic field line domains, must flow surface current comparable to the total current flowing open region. For example, for a round polar cap and for a local GJ current (when the answer can be obtained analytically) the reverse current must be 3/4 from the volume current (Beskin 2010).

Here, however, one very important remark should be made. The above conclusion about energy losses was based on the assumption that there are no longitudinal currents in the closed magnetosphere region. And that the surface currents that close the volume currents exist only in the region of the polar cap, without leaving it (Beskin & Nokhrina 2004). If we do not make these assumptions, then the problem of deceleration of the neutron star became uncertain, since it was not possible to determine the magnitude of the additional current circulating in the magnetosphere, but not outgoing into the pulsar wind region. Indeed, as shown in Figure 1, additional separatrix currents must inevitably lead to a nonzero average surface current $< J_\theta > \neq 0$ on the polar cap, and, consequently, to additional energy losses.

Thus, it was not possible to achieve definitive clarity in the theoretical analysis here. Some clarification became possible only after the results of numerical modeling for the magnetosphere of the inclined rotator were obtained. As a result, it was shown that volume currents in a closed magnetosphere are really absent. A remarkable event was also the fact that reverse currents along the separatrix were also discovered (Bai & Spitkovsky 2010). True, the reverse current was only 20% of the volume current. Of course, this discrepancy could be explained by the fact that in the calculations carried out, the star radius R was two to three times smaller the light cylinder radius R_L. However, a significant difference in these quantities could also be associated with additional separatrix currents that were not taken into account in the previous analysis.

3 RESULTS

Returning now to the discussion of the energy losses of a neutron star, we recall first of all that the current losses (11) discussed above correspond only to the first term in the expansion (13). They are caused by surface currents that close the volume currents flowing in the magnetosphere. Therefore, the action of such a braking torque is concentrated only in the area of the polar cap. The magnetic field here corresponds to the magnetic field of the zeroth approximation. In this case, as we have seen, volume current losses can not explain the energy release of $W_\mathrm{tot}^\mathrm{MHD}$ (1) for the MHD solution.

On the other hand, as was shown above, there are two more possible causes of the deceleration of radio pulsars. First of all, the example of vacuum losses shows that in general case the second term in the expansion (13) can be important. In this case in the expression $J_\mathrm{s} \times B$, the surface current J_s corresponds to the zero-order current, and the magnetic field is related to the third-order perturbation induced by the rotation. Another possible cause may be associated with additional separatrix currents closing along the surface of the polar cap; they will still correspond to the first term in the expansion (13).

Up to now, not only to determine analytically, but even to evaluate corresponding contributions was not possible. Therefore, it is not surprising that previously there was practically nothing known about such losses. However, now we can get an answer by analyzing directly the results of numerical simulation. Figure 2 shows the dependence of the flux of electromagnetic energy on the radius r for the inclination angle 60° through the region of closed (lower curve) and open (middle curve) field lines. The size of the star is 50 cells, and the light cylinder is 500 cells ($\varepsilon = 0.1$).

As we see, practically all the energy flux is concentrated within open magnetic field region. This implies that the energy losses of the inclined rotator are associated with additional separatrix currents. There are no energy flux from the closed region which should have occurred if the second tern in (13) was important. It should not be said that the above results do not leave any room for magneto-dipole losses (in this case, the energy flux should also have been distributed quasi-homogeneously, and would not be concentrated only in the open magnetosphere).

Now using relation (17), one can obtain an expression for the surface current averaged over the polar cap $< J_\theta >$, which provides

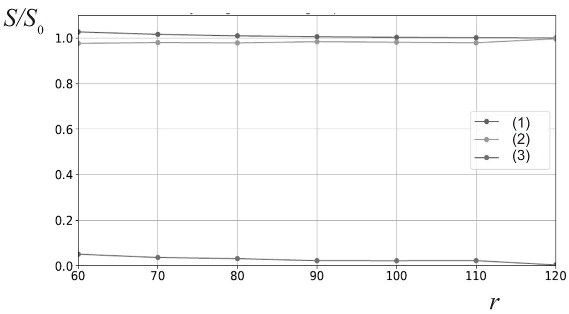

Figure 2. Electromagnetic energy flux from the region of closed (lower curve) and open (middle curve) magnetic field lines. The upper curve corresponds to the total energy loss. The radius of the neutron star (50 cells) is 10 times smaller than the radius of the light cylinder.

the MHD loss $W_{\rm tot}^{\rm (MHD)}$ (1) for the orthogonal rotator

$$< J_\theta >= \frac{c}{4\pi f_*} B_0 \left(\frac{\Omega R}{c}\right)^2 . \tag{18}$$

Accordingly, the averaged toroidal magnetic field $< B_\varphi >$ within the open field lines at the distance r is to equal to

$$< B_\varphi >= \frac{1}{f_*} B_0 \left(\frac{\Omega R}{c}\right)^2 \frac{R}{r}. \tag{19}$$

It can be easily obtained by determining the flux of the Poynting vector through the corresponding area $s(r) = f_*\pi(\Omega/c)^{1/2}r^{3/2}$. As we see, toroidal magnetic fiesd B_φ behaves in the same way as in a magneto-dipole wave. Finally, we note that the total additional separatrix current must be $(\Omega R/c)^{1/2}$ times smaller than the total current circulating in the magnetosphere (as it was already emphasized, it weakly depends on the MHD model From the angle of inclination of the axes χ).

Finally, let us write down the additional braking torque in general form as

$$K_\perp^{\rm add} = -A \frac{B_0^2 \Omega^3 R^6}{c^3} i_{\rm a}, \tag{20}$$

and try to estimate the dimensionless constant A from the results of numerical modeling. Since, as was shown, in this case the dimensionless longitudinal current is estimated as $i_{\rm a} \sim (\Omega R/c)^{-1/2}$, then the coefficient A turns out to be equal to

$$A \sim (\Omega R/c)^{1/2}. \tag{21}$$

For such a small value $A \ll 1$, one can neglect the magnetospheric contribution $K_\perp^{\rm mag}$ (20) for the local GJ current $i_{\rm a}^{\rm A} \sim 1$, which, in fact, was done within the framework of BGI model (Beskin et al. 1993).

4 CONCLUSION

Thus, direct analysis of the energy flux angular distribution inside the light cylinder unambiguously indicate the absence of magneto-dipole losses. This implies that pulsar wind should be considered as an example of a relativistic magneto-hydrodynamical wave, which unusual properties was previously unknown. For example, the angular distribution of the energy flux varies from $\sin^2\theta$ for axisymmetrical case to $\sin^4\theta$ for the orthogonal rotator. Already in this point there is a significant difference from magneto-dipole losses, for which the losses are proportional to $(1 + \cos^2\theta)$ (Landau & Lifshitz 1971). Absence of energy flux along the axis of rotation (i.e., at $\theta = 0$) for the inclination angle $\chi = 60^\circ$ was already noted by Beskin et al. (2013) where the results of numerical simulation performed by Spitkovsky (2006) were directly analyzed.

Further, we have shown the importance of an additional separatrix currents circulating in the pulsar magnetosphere which provide the most energy losses for incline rotator. It is important that this currents connect both magnetic poles of a rotating neutron star. As is well-known, indications on such a connection were already widely discussed.

Finally, we would like to draw attention to the following circumstance. Starting from Shitov (1983), the additional bending of magnetic field lines $\delta\varphi_{\rm rot} = 1.2(r/R_{\rm L})^3 \sin^2\chi$ was widely considered in the analysis of mean profiles of radio pulsars (see, e.g., Lyne & Graham-Smith 2006). But this value was obtained for magneto-dipole mechanism of pulsar braking when the disturbance of toroidal magnetic field (15) is small enough. As was demonstrated above, radiative toroidal magnetic field (19) is much larger. Accordingly, much larger is to be the bending angle

$$\delta\varphi_{\rm rot} \approx \left(\frac{r}{R_{\rm L}}\right)^2 \sin^2\chi. \tag{22}$$

It is this expression that is to be used in analysis of mean profiles of radio pulsars.

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