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So how do radio pulsars slow-down?

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Abstract. We show that the increasing of energy losses of radio pulsars $W_{\rm tot}$ with the inclination angle χ obtained numerically by many authors can be explained by the separatrix currents circulating in the pulsar magnetosphere which do not outflow into pulsar wind.

1. Introduction

From the very beginning magneto-dipole radiation was considered as a main mechanism of the pulsar braking [1, 2]. But later it was shown that if the pulsar magnetospheric fills fully with plasma so that $E_{\parallel} = 0$, for zero longitudinal electric currents circulating in the pulsar magnetosphere the energy losses $W_{\rm tot}$ vanish for any inclination angle χ [3]. This effect confirmed later in [4] results from full screening of the magneto-dipole radiation of neutron star by magnetospheric plasma. This implies that the pulsar braking results fully from the impact of the torque $\mathbf{K}$ due to longitudinal currents flowing in the magnetosphere.

Nevertheless, magneto-dipole losses are still often mentioned in connection with energy release mechanism. On the other hand, MHD simulations obtained first in [5] and later confirmed in many papers [6, 7, 8, 9] give the following dependence $W_{\rm tot}$ on the inclination angle χ

$$W_{\rm tot}^{\rm MHD} \approx \frac{1}{4}\,\frac{B_0^2\Omega^4R^6}{c^3}(1+\sin^2\chi). \tag{1}$$

As we see, in addition to unity this expression has the same $\sin^2\chi$ dependence as for magneto-dipole losses. Recent numerical simulations have not clarified this question as well. Indeed, as was shown in [9, 10], in the pulsar wind there is a noticeable component depending on time. In particular, for orthogonal rotator one can write-down [9]

$$B_r \approx B_0\frac{R^2}{r^2}\sin\theta\cos(\varphi-\Omega t+\Omega r/c); \qquad B_{\varphi} = E_{\theta} \approx -B_0\,\frac{\Omega R^2}{cr}\,\sin^2\theta\,\cos(\varphi-\Omega t+\Omega r/c). \tag{2}$$

Below we analyze the torque acting on a rotating star and the angular distribution of the energy flux inside the light cylinder. It is shown that MHD solution (1) does not contain magneto-dipole wave. Inside the light cylinder almost 100% of the energy flux outflows through very narrow angular domain of open magnetic field region, not quasi-isotropical, as it should have been for magneto-dipole radiation (e.g., in Deutsch solution [11]).

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2. Current losses

As is well-known, the key role in particle motion in the pulsar magnetosphere should be played by the electric drift associated with a strong electric field produced by the rotation of a neutron star. As a result, the transverse electric current $\mathbf{j}_\perp$ can be written in the hydrodynamical form $\mathbf{j}_\perp = \rho_\mathrm{e}\mathbf{U}_\mathrm{dr}$, where $\mathbf{U}_\mathrm{dr} = c[\mathbf{E}\times\mathbf{B}]/B^2$ is the electric drift velocity. On the other hand, for fast enough rotation when the potential drop near magnetic poles is much smaller than the maximum possible value one can write down $\mathbf{E} = -\beta_\mathrm{R}\times\mathbf{B}$, where $\beta_\mathrm{R} = [\boldsymbol{\Omega}\times\mathbf{r}]/c$. Using this relation we finally obtain

$$\mathbf{j} = \rho_\mathrm{e}[\boldsymbol{\Omega}\times\mathbf{r}] + i_\|\mathbf{B}, \tag{3}$$

where $i_\|$ a scalar function. In particular, for orthogonal wind solution (2) we have

$$i_\| = -3\,\frac{\Omega}{c}\cos\theta. \tag{4}$$

The convenience of expression (3) is due to the fact that the scalar function $i_\|$ must be constant along magnetic field lines [12].

Further, general expression for energy losses of a rotating magnetized sphere looks like [13]

$$W_\mathrm{tot} = \frac{c}{4\pi}\oint[\mathbf{E}\times\mathbf{B}]\mathrm{d}s = -\boldsymbol{\Omega}\cdot\mathbf{K}; \quad \mathbf{K} = \frac{R}{4\pi}\oint[\mathbf{n}\times\mathbf{B}]\,(\mathbf{B}\cdot\mathbf{n})\,\mathrm{d}s. \tag{5}$$

Here the first bracket in the relation for the torque $\mathbf{K}$ corresponds to the surface current $\mathbf{J}_\mathrm{s}$, and the second one to magnetic field $\mathbf{B}$ in the Ampere force $\mathbf{F} = [\mathbf{J}_\mathrm{s}\times\mathbf{B}]/c$.

It is convenient to introduce two components of the torque $\mathbf{K}$ parallel and perpendicular to the magnetic dipole $\mathbf{m}$ and connect them with dimensionless current in the polar cap zone $i = j_\|/j_\mathrm{GJ}$ separating it into symmetric and anti-symmetric contributions, i_s and i_a, depending upon whether the direction of the current is the same in the north and south parts of the polar cap, or opposite. As one can easily check, $K_\| \propto i_\mathrm{s}$, and $K_\perp \propto i_\mathrm{a}$. Here and below we apply normalization to the 'local' Goldreich-Julian current density, $j_\mathrm{GJ} = |\boldsymbol{\Omega}\cdot\mathbf{B}|/2\pi$ (with scalar product). As a result, we obtain

$$W_\mathrm{tot} = -\Omega\,\mathcal{K}_\|\cos^2\chi - \Omega\,\mathcal{K}_\perp\sin^2\chi, \tag{6}$$

where we put $K_\| = \mathcal{K}_\|\cos\chi$ and $K_\perp = \mathcal{K}_\perp\sin\chi$.

As was shown in [14], the direct action of the Ampère force on the star $\mathbf{K} = \int[\mathbf{r}\times[\mathbf{J}_\mathrm{s}\times\mathbf{B}]/c]\,\mathrm{d}\mathbf{s}$ by surface currents $\mathbf{J}_\mathrm{s}$ which close volume longitudinal electric currents circulating in the pulsar magnetosphere can be written as

$$K_\|^\mathrm{sur} \approx -\frac{B_0^2\Omega^3R^6}{c^3}i_\mathrm{s}, \quad K_\perp^\mathrm{sur} \approx -\frac{B_0^2\Omega^3R^6}{c^3}\left(\frac{\Omega R}{c}\right)i_\mathrm{a}. \tag{7}$$

Thus, to satisfy MHD energy losses W_tot (1) for $\chi = 90^\circ$ the anti-symmetric current i_a is to be large enough: $i_\mathrm{a} \approx (\Omega R/c)^{-1}$. On the other hand, equating the total current $I = \int i_\|\mathbf{B}\mathbf{ds}$ with $i_\|$ from (4) flowing through the upper hemisphere of the orthogonal wind (2) and the total current flowing through the northern part of the polar cap on the star surface (and remembering that $i_\|$ is constant along magnetic field lines), one can obtain

$$i_\mathrm{a} \approx \left(\frac{\Omega R}{c}\right)^{-1/2}. \tag{8}$$

Thus, antisymmetric current itself is too small to explain MHD energy losses.

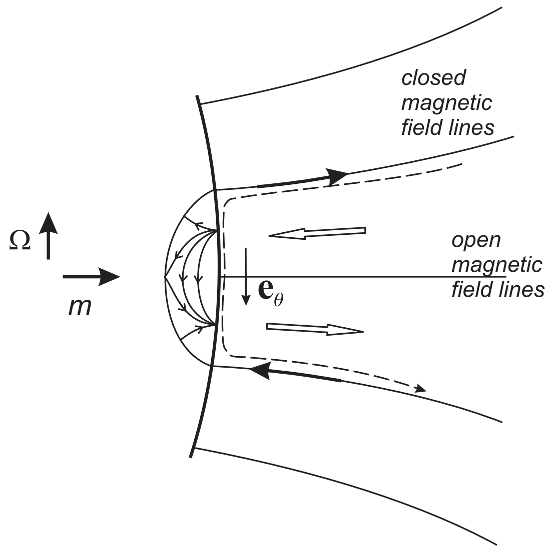

Figure 1. The structure of the volume (contour arrows), separatrix (fat arrows) and surface (thin arrows) currents near the polar cap of the orthogonal rotator. The additional separatrix current is shown by a dashed curve.

3. Additional torques

To find another possible braking mechanisms responsible for MHD solution, let us consider arbitrarily magnetized sphere rotating in a vacuum. According to (7), braking torque $\mathbf{K}$ must correspond to the third power of the expansion with respect to the small parameter $\varepsilon = \Omega R/c$. On the other hand, one can show that the first order terms $B^{(1)}$ turn out to be zero (see, e.g., [15]). As a result, general expression for the braking torque can be written down as

$$\mathbf{K} = \frac{R}{4\pi} \oint \{[\mathbf{n} \times \mathbf{B}^{(3)}](\mathbf{B}^{(0)} \cdot \mathbf{n}) + [\mathbf{n} \times \mathbf{B}^{(0)}](\mathbf{B}^{(3)} \cdot \mathbf{n})\} \mathrm{d}s. \tag{9}$$

In particular, in Deutsch [11] solution ($B_n^{(3)} = 0$) the only contribution is given by the first term. On the other hand, for the point orthogonal dipole [13] only two thirds of the losses will still be determined by the first term, while one third with the second one. Certainly, the total energy losses and the inclination angle evolution do not depend on the choice of the solution.

Thus, in addition to current losses (7) mentioned above, pulsar braking can be connected with the disturbance of normal component of the magnetic field B_n. It is important that this disturbance covers the entire surface of the neutron star. Such additional contribution could be related to the violation of the exact mutual compensation between the magneto-dipole radiation of a central star and the radiation of the magnetosphere itself resulting from longitudinal currents circulating in the magnetosphere (e.g., due to the changing of a shape of closed magnetic field lines region). For zero longitudinal currents exact mutual compensation takes place.

To discuss another possible additional torque, let us consider in more detail the braking of an orthogonal rotator. It is convenient to rewrite the energy losses $W_{\rm tot}$ (5) in the form (do not forget that the radio pulsar has two poles!)

$$W_{\rm tot} = 2\,\frac{\Omega R}{c} \int J_\theta B_n \mathrm{d}s. \tag{10}$$

An accurate analysis shows [14] that surface currents should in fact be arranged in such a way that the current $< J_\theta >$ averaged over the polar cap surface would be zero. In this case, as

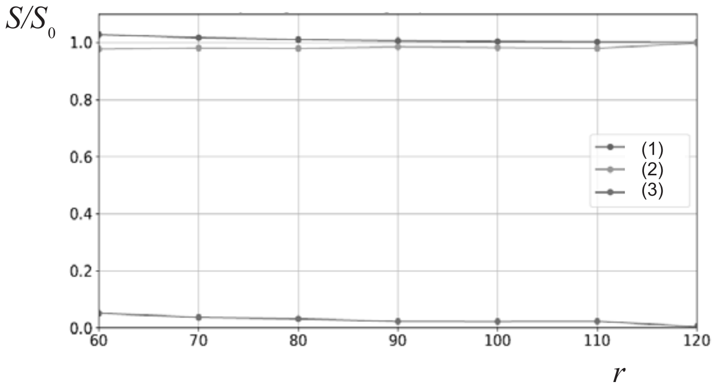

Figure 2. Electromagnetic energy flux from the region of closed (lower curve) and open (middle curve) magnetic field lines. The upper curve corresponds to the total energy loss. The radius of the neutron star (40 cells) is 10 times smaller than the radius of the light cylinder.

shown in Figure 1, along the separatrix separating open and closed magnetic field line domains must flow surface current $I_{\rm sep}$ comparable to the total volume current $I_{\rm vol}$ circulating in the magnetosphere, but flowing in the opposite direction. E.g., for a round polar cap and for a local GJ current the reverse current must be 3/4 from the volume current ($I_{\rm sep}/I_{\rm vol} \approx 0.75$) [16]. Such a reverse current was later confirmed in numerical simulations, but its value turned out to be smaller: $I_{\rm sep}/I_{\rm vol} \approx 0.2$ [17].

Here, however, one very important remark should be made. The above conclusion was based on the assumption that there is no additional current circulating in the magnetosphere, but not outgoing into the pulsar wind region. Indeed, as shown in Figure 1, additional separatrix currents must inevitably lead to a nonzero average surface current $< J_\theta > \neq 0$ on the polar cap, and, consequently, to additional energy losses.

4. Results

Thus, there are two possibilities to explain large enough energy losses of the orthogonal rotator. Previously not only to determine analytically, but even evaluate corresponding contributions was not possible. Now we can get an answer by analyzing directly the results of numerical simulation. Figure 2 shows the dependence of the flux of electromagnetic energy on the radius r through the region of closed (lower curve) and open (middle curve) field lines for the inclination angle 60°. The size of the star is 40 cells, and the light cylinder is 400 cells ($\varepsilon = 0.1$).

As we see, practically all the energy flux is concentrated within open magnetic field region. Hence, the energy losses of orthogonal rotator are associated with additional separatrix currents. There are no energy flux from the closed region which should have occurred if the second tern in (9) was important. This result does not leave any room for magneto-dipole losses as well.

Now using relation (10), one can obtain an expression for the surface current averaged over the polar cap $< J_\theta >$, which provides the MHD loss $W_{\rm tot}$ (1) for the orthogonal rotator

$$< J_\theta > = \frac{c}{4\pi f_*} B_0 \left(\frac{\Omega R}{c}\right)^2. \tag{11}$$

Accordingly, the averaged toroidal magnetic field $< B_\varphi >$ within the open field lines at the distance r is to equal to

$$< B_\varphi > = \frac{1}{f_*} B_0 \left(\frac{\Omega R}{c}\right)^2 \frac{R}{r}. \tag{12}$$

It can be easily obtained by determining the flux of the Poynting vector through the corresponding area $s(r) = f_{*}\pi(\Omega/c)^{1/2}r^{3/2}$ (f_{*} is a dimensionless polar cap area). As we see, toroidal magnetic field B_{φ} behaves in the same way as in a magneto-dipole wave. Finally, we note that the total additional separatrix current must be $(\Omega R/c)^{1/2}$ times smaller than the total current circulating in the magnetosphere.

Finally, let us write down the additional braking torque in general form as

$$K_{\perp}^{\rm add} = -A\,\frac{B_0^2\Omega^3R^6}{c^3}\,i_{\rm a}, \tag{13}$$

and try to estimate the dimensionless constant A from the results of numerical modeling. Since, as was shown, in this case the dimensionless longitudinal current is estimated as $i_{\rm a} \sim (\Omega R/c)^{-1/2}$, then the coefficient A turns out to be equal to

$$A \sim (\Omega R/c)^{1/2}. \tag{14}$$

For such a small value $A \ll 1$, one can neglect the magnetospheric contribution $K_{\perp}^{\rm add}$ (13) for the local GJ current $i_{\rm a}^{\rm A} \sim 1$, which, in fact, was done within the framework of BGI model [14].

5. Conclusion

Thus, direct analysis of the energy flux angular distribution inside the light cylinder unambiguously indicate the absence of magneto-dipole losses. This implies that pulsar wind should be considered as an example of a relativistic magneto-hydrodynamical wave. For example, the angular distribution of the energy flux varies from $\sin^2\theta$ for axisymmetrical case to $\sin^4\theta$ for the orthogonal rotator. Already in this point there is a significant difference from magneto-dipole losses for which the losses are proportional to $(1+\cos^2\theta)$ [13].

Besides, we recognized the importance of additional separatrix current providing the most energy losses for orthogonal rotator. The amplitude of this current is to be much smaller than the total current circulating in the magnetosphere. For this reason the structure of this current near the light cylinder is unclear. Numerical simulations give us only the qualitative indication on its existence; quantitative consideration was not produced yet.

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