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Spiral jet

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ABSTRACT

We show that a quasi-cylindrical configuration of a jet in the central region, where direct electric current flows, is confined in a radial equilibrium by a spiral wave at the periphery of a jet. A spiral wave means that in a coordinate system moving with the velocity of the matter along the axis of the jet, all quantities are proportional to $\exp\{ik_{\parallel}z + im\phi\}$, z is the longitudinal coordinate, and ϕ is the azimuthal angle. The luminosity of such a jet corresponds to observations. It is also shown that the jet slowly expands with distance z from its base by the power law, $R(z) \propto z^k$, where the exponent k varies from $\simeq$0.5 to $\simeq$1.

Key words: galaxies: active – galaxies: jets.

1 INTRODUCTION

Jets in astrophysics are observed as highly collimated flows of matter from central objects, which are either supermassive black holes in the case of active galactic nuclei (AGNs) or black holes of solar masses in microquasars. In addition, jets are observed from white dwarfs, protostars, cataclysmic variables, and powerful radio pulsars, such as the pulsar in the Crab nebula. In laboratory plasmas, jets, or directed flows, are observed in devices such as plasma focus (Beskin et al. 2017a). The main puzzle of jets is their high degree of collimation. It is not clear how hot matter of jets, ionized gas in a strong magnetic field, confined in the cylindrical radius direction, which is the direction perpendicular to the axis of the jet. Neither hot gas nor external magnetic field is usually observed in the interstellar and intergalactic environment surrounding a jet.

We will study the problem of radial equilibrium of the jet in the framework of ideal magnetohydrodynamics (MHD). This approximation is well suited for astrophysical jets. Dissipative effects are weak in hot plasma. In addition, large mean-free paths of particles, the presence of a strong magnetic field, in which cyclotron radii play the role of particle mean-free paths, and small cyclotron radii of ions in comparison with the characteristic scales of jets allow us to use ideal MHD for the description of astrophysical jets.

It has been shown before that the problem of equilibrium of the jet in the radial direction within the framework of MHD is related to electric currents flowing inside the jet. If the jet carries uncompensated current, i.e. the total current of the jet is not zero, the current can be pinched to keep the hot jet in equilibrium (Blandford & Payne 1982; Heyvaerts & Norman 1989; Sulkanen & Lovelace 1990; Li, Chiuch & Begelman 1992; Pelletier & Pudritz 1992; Sauty & Tsinganos 1994). However, the question arises how to close the uncompensated current loop. If you let the return current flow outside the jet, it is not clear where it goes since track of the return current is usually not visible. It is more convenient to assume that return current flows inside the jet on its periphery. This raises the problem of the equilibrium of the jet in the region of return current. The return current repels from the direct one, and within the framework of a purely radial structure, there is no equilibrium, if there is no contribution of the surface current at the edge of the jet (Lery et al. 1998; Beskin & Malyshkin 2000; Lyubarsky 2009; Beskin & Nokhrina 2010). Here we will explore another possibility – the emergence of spiral structure in the region of return current flow.

2 CYLINDRICAL FLOW

Let us consider stationary MHD equations in a coordinate system moving with the plasma flow along its axis with the velocity u.

$$\nabla(\rho \boldsymbol{v}) = 0; \tag{1}$$

$$(\boldsymbol{v}\nabla)\boldsymbol{v} = -\frac{1}{\rho}\nabla P(\rho) - \frac{1}{8\pi\rho}\nabla B^2 + \frac{1}{4\pi\rho}(\boldsymbol{B}\nabla)\boldsymbol{B}; \tag{2}$$

$$\mathrm{curl}[\boldsymbol{v}B] = 0, \tag{3}$$

$$\nabla \boldsymbol{B} = 0. \tag{4}$$

Here ρ is the plasma density of the jet, $\boldsymbol{v}$ is velocity of the plasma, $\boldsymbol{B}$ is the magnetic field, and $P(\rho)$ is the plasma pressure, which is a function of the density through the equation of state of the matter in the jet. Suppose first that all quantities depend only on the cylindrical radius coordinate r, that is, the jet is axisymmetric and homogeneous along axis z. In this case, the velocity and the magnetic field can be expressed as functions of r,

$$\boldsymbol{v} = \boldsymbol{e}_{\phi} v_{\phi}(r),\ \boldsymbol{B} = \boldsymbol{e}_{\phi} B_{\phi}(r) + \boldsymbol{e}_z B_p(r).$$

Velocity v_{ϕ} is the speed of the rotation of the jet, B_p is the poloidal magnetic field, and B_{ϕ} is the toroidal magnetic field. It should be

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noted that the radial velocity v_r and the radial magnetic field B_r in the case of only cylindrical radius dependence are zero by virtue of conditions $\nabla(\rho\boldsymbol{v}) = 0,\ \nabla\boldsymbol{B} = 0$ (1, 4) and finiteness condition at $r = 0$. Then the equilibrium condition in the radial direction looks as follows:

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(P + \frac{B_p^2}{8\pi} + \frac{B_\phi^2}{8\pi}\right) = \rho\frac{v_\phi^2}{r} - \frac{B_\phi^2}{4\pi r}. \quad (5)$$

Here, on the left-hand side there is the total pressure of the matter and the magnetic field, and on the right-hand side there are the centrifugal force per unit volume and the tension force per unit volume associated with the curvature of the magnetic field. Toroidal magnetic field B_ϕ enters into equilibrium twice, as the pressure on the left-hand side, and as the tension on the right-hand side, preventing expansion of plasma. Equation (5) can be transformed to the form

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(P + \frac{B_p^2}{8\pi}\right) = \rho\frac{v_\phi^2}{r} - \frac{1}{8\pi r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2 B_\phi^2\right). \quad (6)$$

If the pressure $P_p = P + B_p^2/8\pi$, which is the sum of the gas pressure and the pressure of the poloidal magnetic field, is maximal at the centre of the jet $r = 0$, then its decline to the periphery is compensated by the tension of the toroidal field $B_\phi \propto r$, created by the longitudinal electric current j_z flowing in the jet centre. Therefore, in the centre of the jet there are no problems with the plasma confinement, the longitudinal electrical current is enough to pinch plasma. If we assume that in the centre of the jet a fairly uniform direct current flows ($B_\phi \propto r$), the pressure P_p has a parabolic form in the centre ($P_p \propto (1 - r^2/R^2)$) and the rotation of the matter in the centre is almost solid ($v_\phi \propto r$), then all terms in equation (6) are proportional to r. As a result, the value of the direct current in the centre, $r < r_c$, is determined by the relation following from equation (6)

$$(j_z|_{r=0})^2 = \frac{c^2}{2\pi}\left[\rho_0\left(\left.\frac{\mathrm{d}v_\phi}{\mathrm{d}r}\right|_{r=0}\right)^2 + \left|\left.\frac{\mathrm{d}^2 P_p}{\mathrm{d}r^2}\right|_{r=0}\right|\right].$$

The pressure in the jet at the radius of the direct current can drop significantly, $P_p|_{r=r_c} \simeq P_p|_{r=0}(1 - r_c^2/R^2)$.

On the periphery, the longitudinal current j_z changes sign to close the total electrical current loop flowing in the jet. In this case, the toroidal magnetic field ceases to grow with a radius and begins to fall faster than $\propto r^{-1}$, $B_\phi = 4\pi\int_0^r j_z(r')r'\mathrm{d}r'/cr$. This means that the pressure force of the toroidal magnetic field begins to exceed its tension, and the right-hand side of equation (6) is always positive on the periphery. It is clear that at the jet boundary $r = R$ the matter density ρ and the pressure $P(\rho)$ should vanish. The speed of rotation v_ϕ should also disappear due to the fact that the rotation of the jet is associated with the rotation of the central object (black hole or star) and with the rotation of the accretion disc around the central object, and the involvement of matter into rotation falls at large distances from the axis. In addition, rotation of an ideal plasma in the poloidal magnetic field generates a radial electric field $E_r = -v_\phi B_p$. Its disappearance and disappearance of the electric charge density, $\rho_e = \mathrm{d}(rE_r)/\mathrm{d}r/4\pi r$, at the jet boundary $r = R$, result to vanishing of the rotation speed v_ϕ at the boundary. Otherwise surrounding charges outside the jet would lead to neutralization both the radial electric E_r and the charge density ρ_e. The magnetic fields B_p and B_ϕ must also disappear at the jet boundary: the toroidal one due to the complete closure of the longitudinal electric current, and the poloidal one due to its origin – this field is frozen into the jet stream and the jet captures the poloidal magnetic field at its base in the magnetospheres of black holes and neutron stars. In a word, the left-hand side of equation (6) is negative near the jet boundary, and the radial equilibrium of the axisymmetric highly collimated jet cannot be achieved unless one assumes that the pressure of the matter outside the jet exceeds pressures of gas and poloidal magnetic field inside (Beskin et al. 2017b). Here we explore another possibility of radial stabilization – the presence of a spiral structure in the jet.

3 SPIRAL WAVE

Suppose that all quantities in the jet are superpositions of a homogeneous field and a spiral wave

$$\begin{aligned}
\rho &= \rho(r) + \rho_1(r)\exp\{ik_\parallel z + im\phi\},\\
P &= P(\rho) + P_1(r)\exp\{ik_\parallel z + im\phi\},\\
\boldsymbol{v} &= \boldsymbol{e}_\phi v_\phi(r) + \boldsymbol{v}_1(r)\exp\{ik_\parallel z + im\phi\},\\
\boldsymbol{B} &= \boldsymbol{e}_\phi B_\phi(r) + \boldsymbol{e}_z B_p(r) + \boldsymbol{B}_1(r)\exp\{ik_\parallel z + im\phi\}.
\end{aligned} \quad (7)$$

Here $k_\parallel$ and m/r are the longitudinal and azimuthal wave numbers, respectively, and m is an integer. Values with index '1' must satisfy equations

$$\begin{aligned}
&\nabla(\rho_1\boldsymbol{v}) + \nabla(\rho\boldsymbol{v}_1) = 0,\\
&\nabla\boldsymbol{B}_1 = 0,\\
&(\boldsymbol{B}\nabla)\boldsymbol{v}_1 + (\boldsymbol{B}_1\nabla)\boldsymbol{v} + \boldsymbol{B}\nabla\boldsymbol{v}_1 - (\boldsymbol{v}_1\nabla)\boldsymbol{B} - (\boldsymbol{v}\nabla)\boldsymbol{B}_1 = 0,\\
&\rho\left[(\boldsymbol{v}\nabla)\boldsymbol{v}_1 + (\boldsymbol{v}_1\nabla)\boldsymbol{v}\right] - \rho_1\frac{v_\phi^2}{r}\boldsymbol{e}_r =\\
&\quad -\nabla\left(\frac{\partial P}{\partial\rho}\rho_1\right) + \frac{1}{4\pi}\left[(\boldsymbol{B}\nabla)\boldsymbol{B}_1 + (\boldsymbol{B}_1\nabla)\boldsymbol{B} - \nabla(\boldsymbol{B}\boldsymbol{B}_1)\right].
\end{aligned} \quad (8)$$

Here all terms of the equations are proportional to the first harmonic of the wave $\exp\{ik_\parallel z + im\phi\}$. We are primarily interested in the solution of equations (8) near the jet boundary $r \simeq R$, where the return longitudinal electric current flows and the equilibrium of the homogeneous jet (6) is not possible without external pressure. The quantities $B_p(r),\ B_\phi(r),\ v_\phi(r),\ \rho(r),\ P(r)$ vanish on the boundary $r = R$, therefore near the boundary they can be represented in the form

$$B_p = B_p'\xi, \quad B_\phi = B_\phi'\xi, \quad v_\phi = v_\phi'\xi, \quad \rho = \rho'\xi,$$

where $\xi = R - r$, and values with index $'$ are constants. There are no reasons to consider that near the boundary of the jet some values with the index 'prime', i.e. first derivatives, vanish, and it is necessary to take into account higher derivatives in the expansion over powers of ξ. Therefore, for the general case quantities $B_p',\ B_\phi',\ v_\phi',\ \rho' \neq 0$, although in separate cases this can happen, which require a separate investigation. In addition, the radial velocity v_{1r}, the radial magnetic field B_{1r} and the density ρ_1 in the wave must also vanish on the boundary $r = R$. Substituting these relations into system (8), we obtain

$$\rho_1\frac{\partial P}{\partial\rho} = -\frac{\xi}{4\pi B_p'}\left(B_\phi'^2 + B_p'^2\right)B_{1z}, \quad (9)$$

$$v_{1r} = -i\frac{\xi}{R}\frac{m v_\phi'}{B_p'}B_{1z}, \quad (10)$$

$$v_{1\phi} = \frac{v_\phi'}{B_p'}\frac{k_\parallel R B_p' + 2m B_\phi'}{k_\parallel R B_p' + m B_\phi'}B_{1z}, \quad (11)$$

$$v_{1z} = \frac{m v'_\phi}{k_\parallel R B'_p + m B'_\phi} B_{1z}, \tag{12}$$

$$B_{1r} = -i \frac{\xi}{R} \frac{k_\parallel R B'_p + m B'_\phi}{B'_p} B_{1z}, \tag{13}$$

$$B_{1\phi} = \frac{B'_\phi}{B'_p} B_{1z}. \tag{14}$$

In contrast to the axisymmetric solution, the radial velocity v_{1r} and the radial magnetic field B_{1r} in the spiral wave are not equal to zero inside the jet. We see that all quantities in the wave are proportional to an arbitrary amplitude, which is the value of the longitudinal magnetic field B_{1z}. We note that the longitudinal v_{1z} and the azimuthal $v_{1\phi}$ velocities in the wave formally turn into infinity (11,12) for exact resonance at the jet boundary, $k_\parallel R B'_p + m B'_\phi = 0$, when the phase of the wave $\exp\{ik_\parallel z + im\phi\}$ remains constant along the magnetic field spiral $z = R\phi B'_p/B'_\phi$. The exact resonance requires taking into account the dissipative effects which are not present in the ideal MHD. However, it is clear that the wave numbers $k_\parallel$ and m/R are approximately related as $k_\parallel R B'_p \simeq -m B'_\phi$, at which the excitation of a spiral wave is most probable.

A spiral wave with constant amplitude near the jet boundary (see equations (11, 12, 14)) affects the equilibrium in the radial direction. Indeed, the quadratic terms of ρ_1, $\boldsymbol{v}_1$ and $\mathbf{B}_1$ in the original equation (2) give an additional contribution to the radial equilibrium. On the right-hand side of equation (6), there appears a term $Q(r)$,

$$\frac{\mathrm{d}P_{\mathrm{tot}}}{\mathrm{d}r} = \rho \frac{v_\phi^2}{r} - \frac{1}{8\pi r^2} \frac{\mathrm{d}}{\mathrm{d}r}\left(r^2 B_\phi^2\right) + Q(r),$$

where $Q(r)$ is equal to

$$Q(r) = -\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{1}{4}\frac{\partial^2 P}{\partial \rho^2}|\rho_1|^2 + \frac{|\boldsymbol{B}_1|^2}{16\pi}\right) + \frac{\boldsymbol{e}_r}{8\pi}(\boldsymbol{B}_1\nabla)\boldsymbol{B}_1^* - \frac{\rho \boldsymbol{e}_r}{2}(\boldsymbol{v}_1\nabla)\boldsymbol{v}_1^* - \frac{\rho_1 \boldsymbol{e}_r}{2}\left[(\boldsymbol{v}_\phi\nabla)\boldsymbol{v}_1^* + (\boldsymbol{v}_1^*\nabla)\boldsymbol{v}_\phi\right]. \tag{15}$$

Here the index * means the complex conjugation. Substituting wave quantities (9–14) into the expression for $Q(r)$ and neglecting terms proportional to ξ, we obtain

$$Q(r) = -\frac{\mathrm{d}}{\mathrm{d}r}\frac{|B_{1\phi}|^2 + |B_{1z}|^2}{16\pi} \tag{16}$$

The physical meaning of the answer is quite simple: pressure of the spiral wave on the periphery of the jet, $|B_1|^2/16\pi$, keeps the jet from radial expansion, when the amplitude of the spiral wave is

$$|B_1|^2 = 16\pi P_p|_{r=r_c}. \tag{17}$$

In conclusion, it should be noted that the formally obtained solution of (7) is exact only if $|B_1|^2 \ll 16\pi P_p|_{r=0}$, that is, $R - r_c \ll R$ when we can restrict ourselves only to the first harmonic of $\exp\{ik_\parallel z + im\phi\}$. Otherwise, the solution will contain higher harmonics, $\mathbf{B}_l \exp\{il(k_\parallel z + m\phi)\}$, l is an integer, $l > 1$. However, it is qualitatively clear that the amplitude of the spiral wave,

$$\boldsymbol{B}_{\mathrm{sp}} = \sum_{l=1}^{\infty} \boldsymbol{B}_l \exp\{il(k_\parallel z + m\phi)\},$$

equals $|B_{sp}|^2 \simeq 16\pi P_p|_{r=r_c}$.

We have computed levels of constant values of the integral $\int|B^2|\mathrm{d}l$ (Fig. 1) for the configuration of the magnetic field (7) representing a combination of a cylindrical axisymmetric jet in the centre and spiral wave on the periphery. We assume that the radiation emission is proportional to the square of the magnetic field, and that the total intensity is the sum of emissivities from regions located along the line-of-sight $\mathbf{l}$ perpendicular to the axis of the jet. For comparison, we present observations of the jet from the 3C273 quasar in radio, optics, and X-ray ranges (Marshall et al. 2001; Fig. 2). One can see a qualitative agreement, especially in optics.

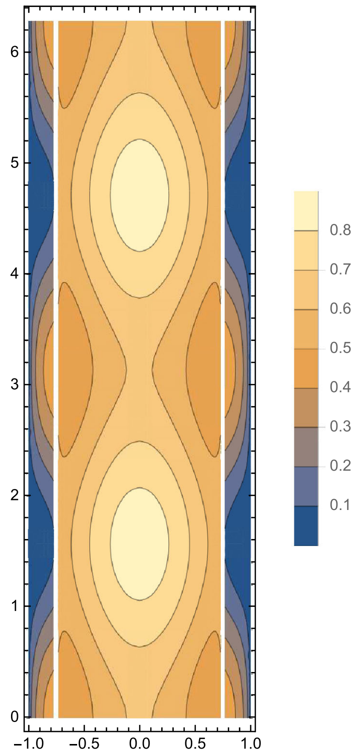

Figure 1. Levels of constant $\int|B^2|\mathrm{d}l$. The direct current radius is $r_c = 3R/4$. The pressure profile in the central part of the jet is selected in the form of a parabola, $B_p^2 \propto (1 - r^2/R^2)$. The spiral wave on the periphery of the jet, $r > 3R/4$, has the wave numbers $m = -1$, $k_\parallel R = 1$.

4 RADIAL EXPANSION OF JET

The radial equilibrium achieved as a result of balance of the gas pressure and the pressure of the poloidal magnetic field with the tension of the toroidal magnetic field in the region of the direct current, $r < r_c$, and with the magnetic field pressure of the spiral wave in the region of the return current, $r_c < r < R$, leads to the fact that the jet does not expand hydrodynamically for a short time. However, the constructed solution for the jet is a quasi-stationary in fact. The amplitude of the spiral wave B_{sp} on the edge of the jet $r = R$ has constant value, then falling to zero for $r > R$. Such profile of B_{sp} cannot be stationary. Due to dissipative effects, for example, magnetic diffusivity ν, the magnetic field must diffusely expand, increasing the radius of the jet as $R^2(t) = \nu t$, $t > R^2(z=0)/\nu$. Here time t is the time of the matter propagation from the bottom of the jet, $z = 0$, to the point $z(t)$, $\mathrm{d}z/\mathrm{d}t = u(t)$. Under the condition $R^2/\nu \gg L/u$, i.e. $L/R \ll \mathrm{Re}$, where L is the length of the jet and $\mathrm{Re} = uR/\nu \gg 1$ is the magnetic Reynolds number, the expansion of the jet is slow, and the jet remains narrow. If motion is relativistic, $u \simeq c$, the expansion law takes the form $R(z) = (\nu z/c)^{1/2}$. At large distances z, the jet slows down, the speed u becomes not relativistic, and the law of expansion becomes different. The dependence of

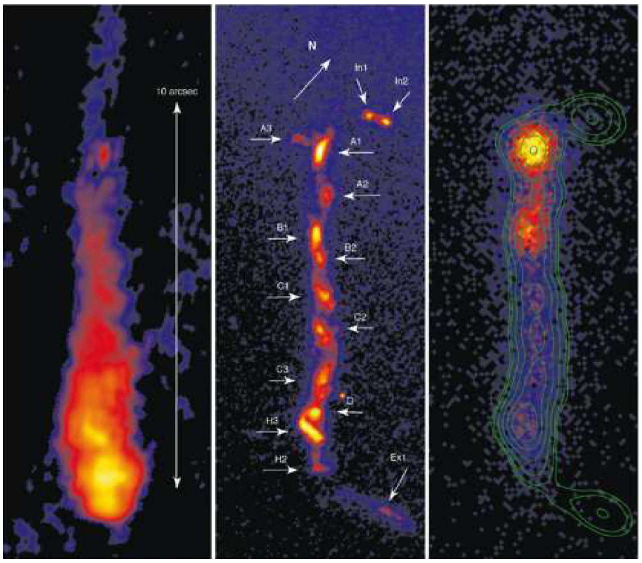

Figure 2. Images of an astrophysical jet from Quasar 3C 273 in ground-based radio (1.647 GHz), Hubble optical (617.0 nm), and *Chandra* X-ray (with optical overlay) bands. Reprinted from Marshall et al. (2001).

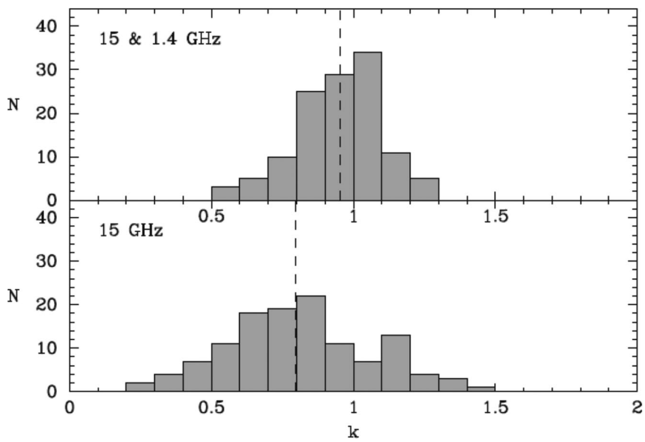

Figure 3. Histograms of the power-law index k in $R(z) \propto z^k$ fitted dependence for 122 MOJAVE-1 sources derived from the 15 GHz data only (bottom panel) and from combined 15 and 1.4 GHz measurements (top panel), with medians of 0.80 and 0.95, respectively, shown by a dashed line. Reprinted from Pushkarev et al. (2017).

$R(z)$ can be found from the following relations: $R \propto t^{1/2}$, $B_{\rm sp} \propto t^{-1/2}$, $P_{\rm tot} \propto B_{\rm sp}^2$, $P_{\rm tot} \propto \rho^\Gamma$, $\rho u R^2 = \text{const}$. From these relations the ratio $t \propto z^\Gamma$ follows. Here Γ is the polytropic exponent in the equation of state of the matter of the jet in the poloidal magnetic field $B_{\rm p}$. Thus, we obtain $R(z) \propto z^{\Gamma/2}$. It is interesting to note that if the energy of the magnetic field dominates over the energy of the particles in the jet, the exponent $\Gamma = 2$ due to the frozen-in plasma into the magnetic field, $B \propto \rho$. Then $R(z) \propto z$. On the contrary, if the energy of the plasma dominates in the jet, $\Gamma/2$ can vary from 2/3 in the case of relativistic gas to 5/6 in the case of an ideal hydrogen nonrelativistic gas. Measurements of the exponent k in the dependence $R(z) \propto z^k$ for AGNs, carried out by the MOJAVE programme (Pushkarev et al. 2017), give values close to our predictions (see Fig. 3).

5 CONCLUSIONS

We have considered the possibility of keeping the jet in radial equilibrium. Jet carries zero total electric current: direct current in the central region $r < r_c$, and the return current on the periphery $r_c < r < R$. If the jet remains axially symmetric over the entire radius, $0 < r < R$, then the radial equilibrium is not possible in the region where the return electric current is flowing. If there is a spiral wave, in which all quantities are proportional to $\exp\{ik_\parallel z + im\phi\}$, on the periphery $r_c < r < R$, then the pressure of the magnetic field of the wave makes it possible to ensure a radial force balance. In this case it is not necessary to have either gas pressure or magnetic field external to the jet. Because in a spiral wave the magnetic field oscillates along the length of the jet, the observed radiation of the jet, which is proportional to the square of the magnetic field strength $|B|^2$, will also oscillate along the axis of the jet (see Fig. 1). The resulting observational picture looks like a set of separate bright spots (Fig. 1), which gives the impression that the jet is a sequence of individual emissions. In fact, the jet is continuous, but has spiral structure. The apparent radial structure of the jet is mainly due to the fact that into the integral intensity $\propto \int |B^2| \mathrm{d}l$ there substantially contributes peripheral parts of the jet above and below the axis, where the spiral magnetic field has maxima and minima, depending on the coordinate z.

Diffusive expansion of the jet boundary due to the dissipative effects leads to a power-law dependence of the radius of the jet $R(z)$ on the distance z from its base, $R(z) \propto z^k$, where the exponent k varies from $\simeq 0.5$ to $\simeq 1$, depending on the equation of state of the matter in the jet, $P(\rho) \propto \rho^\Gamma$.

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