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Brightness temperature – obtaining the physical properties of a non-equipartition plasma

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ABSTRACT

The limit on the intrinsic brightness temperature, attributed to 'Compton catastrophe', has been established being 10^{12} K. Somewhat lower limit of the order of $10^{11.5}$ K is implied if we assume that the radiating plasma is in equipartition with the magnetic field – the idea that explained why the observed cores of active galactic nuclei (AGNs) sustained the limit lower than the 'Compton catastrophe'. Recent observations with unprecedented high resolution by the *RadioAstron* have revealed systematic exceed in the observed brightness temperature. We propose means of estimating the degree of the non-equipartition regime in AGN cores. Coupled with the core-shift measurements, the method allows us to independently estimate the magnetic field strength and the particle number density at the core. We show that the ratio of magnetic energy to radiating plasma energy is of the order of 10^{-5}, which means the flow in the core is dominated by the particle energy. We show that the magnetic field obtained by the brightness temperature measurements may be underestimated. We propose for the relativistic jets with small viewing angles the non-uniform magnetohydrodynamic model and obtain the expression for the magnetic field amplitude about two orders higher than that for the uniform model. These magnetic field amplitudes are consistent with the limiting magnetic field suggested by the 'magnetically arrested disc' model.

Key words: radiation mechanisms: non-thermal – galaxies: active – galaxies: jets – BL Lacertae objects: general.

1 INTRODUCTION

The previous observations of active galactic nuclei (AGNs) in all radio bands have limited the core brightness temperature by 10^{12} K. This phenomenon has been explained in Kellermann & Pauliny-Toth (1969) as being an outcome of the so-called inverse Compton catastrophe.

It can be illustrated by the following argument (Kirk, Melrose & Priest 1994). Suppose we have an electron moving in the magnetic field B with the velocity v, $\beta = v/c$, where c is the speed of light, and the corresponding Lorentz factor γ. It radiates synchrotron radiation with the power (see e.g. Rybicki & Lightman 1979)

$$P_{\rm S} = \frac{4}{3}\sigma_{\rm T} c\beta^2\gamma^2 U_{\rm B}. \tag{1}$$

Here, $\sigma_{\rm T} = 8\pi/3r_0^2$ is the Thomson cross-section, where $r_0 = e^2/mc^2$ is the electron classical radius, e and m are the electron charge and mass, respectively, c is the speed of light and $U_{\rm B} = B^2/8\pi$ is the magnetic energy density. The same electron loses its energy undergoing the inverse Compton scattering of photons, the power being

$$P_{\rm C} = \frac{4}{3}\sigma_{\rm T} c\beta^2\gamma^2 U_{\rm ph}, \tag{2}$$

where the photon energy density

$$U_{\rm ph} = \int \epsilon {\rm d}n(\epsilon), \tag{3}$$

with the photon energy distribution $n(\epsilon)$. The photon energy density $U_{\rm ph}$ comprises synchrotron photons $U_{\rm ph0}$, once Comptonized photons $U_{\rm ph1}$ and so forth. The full power of Compton losses is described by Kirk et al. (1994) as follows:

$$U_{\rm ph} = U_{\rm ph0}\left[1 + \frac{U_{\rm ph0}}{U_{\rm B}} + \left(\frac{U_{\rm ph0}}{U_{\rm B}}\right)^2 + \cdots\right] = \frac{U_{\rm ph0}}{1 - U_{\rm ph0}/U_{\rm B}}. \tag{4}$$

If $U_{\rm ph0} = U_{\rm B}$, the power $P_{\rm C}$ diverges, which is referred to as the 'inverse Compton catastrophe'.

The result, obtained by Kellermann & Pauliny-Toth (1969), is the limiting brightness temperature 10^{12} K, beyond which the 'inverse Compton catastrophe' takes place. The question was how does the source 'know' this limit and sustains its brightness temperature below the limit. The answer has been proposed by

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Readhead (1994). There is another limit on the brightness temperature – the so-called equipartition temperature. If the radiating plasma and the magnetic field are in energy equipartition in the source, the corresponding temperature $T_{\rm eq}$ is just below the limiting value by the inverse Compton catastrophe one.

However, the recent observations of AGN radio cores with the high-resolution *RadioAstron* program (Kovalev et al. 2016; Lisakov et al. 2017) questioned the existence of such a limit, since there are observations that systematically show the brightness temperatures greater than not only $T_{\rm eq}$, but also 10^{12} K.

In this work, we do not address the question what is the physical process underlying such extreme brightness temperatures. We address the question of obtaining the non-equipartition physical parameters of the radiating domain such as the magnetic field B, the particle number density n and the measure of the non-equipartition Σ. This is an important issue. Indeed, the analytical and numerical modelling (Beskin & Nokhrina 2006; Komissarov et al. 2007; Tchekhovskoy, McKinney & Narayan 2008, 2009; Lyubarsky 2009) supports the idea that relativistic jets from AGNs must be in the equipartition regime. In particular, the magnetization parameter

$$\sigma = \frac{B^2}{4\pi n m c^2 \Gamma^2}, \tag{5}$$

which is the ratio of the Poynting vector flux to the plasma kinetic energy flux, must be unity. Here, Γ is the bulk Lorentz factor of a jet and n is the proper particle number density. However, it is believed that only the small portion of particles radiate. Indeed, the bulk plasma velocity, defined by the magnetohydrodynamics (MHD), is exactly the drift velocity in crossed electric and magnetic fields. So, a cold plasma, moving with the drift velocity, does not radiate. To produce the radiation, the cold plasma must be disturbed and accelerated, and the power law

$$\mathrm{d}n = k_{\rm e}\gamma^{-p}\mathrm{d}\gamma, \quad \gamma \in [\gamma_{\min}; \gamma_{\max}] \tag{6}$$

being used to describe the energy distribution of the radiating plasma. Here, $k_{\rm e}$ is the amplitude of the electron energy distribution. To model the radiation of particles in the magnetic field, we employ synchrotron emission and self-absorption. Indeed, spectral energy distributions performed for blazars (see e.g. Abdo 2011) demonstrate that at the low energy the Compton part of the radiation does not play an important role.

As to acceleration process itself, there are two main processes that may account for it. One is the particle acceleration on shocks. This process can hardly account for the observed radiation, since it has been shown (see e.g. Kirk et al. 1994) that the acceleration is not efficient for the magnetized shocks. As the possibility for a particle once accelerated to return to the shock front is suppressed, the Fermi acceleration mechanism does not work. The second process is the reconnection of the magnetic field. It accelerates about 1 per cent of particles very effectively (Sironi, Spitkovsky & Arons 2013), producing a power-law spectrum, with maximum particle energy growing with the time of numerical simulation.

2 JET PARAMETERS IN THE BLANDFORD–KÖNIGL MODEL

The following model is used to explain the properties of compact bright features observed in the radio band (see Gould 1979; Lind & Blandford 1985; Lobanov 1998; Zdziarski et al. 2015): the radiation domain is either a uniform 'plasmoid' or a uniform excited part of a continuous jet. The position of this radiating spherical (Gould 1979) domain along the jet r defines the amplitudes of the particle number density and of the magnetic field (with uniform distribution across the radiating domain) according to the Blandford–Königl model (Blandford & Königl 1979) as

$$B(r) = B_0\left(\frac{r_0}{r}\right), \quad k_{\rm e}(r) = k_{{\rm e},0}\left(\frac{r_0}{r}\right)^2. \tag{7}$$

Here, B_0 and $k_{{\rm e},0}$ are the magnetic field and the particle number density amplitude at a distance r_0, respectively.

The model has been used to obtain physical parameters of jets such as magnetic field, particle number density (Lobanov 1998; Hirotani 2005; O'Sullivan & Gabuzda 2009), and multiplicity parameter and Michel's (Michel 1969) magnetization parameter (Nokhrina et al. 2015) using the core-shift effect – the observed shift of the position of radio cores at different frequencies. All these results are based on the equipartition assumption in different formulations – either the energy densities of the magnetic field and radiating particles are equal, or the fluxes of the Poynting vector and the total particle kinetic energy are equal with the number of radiating particles consisting about 1 per cent of the total particle number density.

2.1 Magnetic field

The observed flux, or the observed brightness temperature, can be used to estimate the magnetic field in the radiating domain (Zdziarski et al. 2015). The observed spectral flux S_ν of a core at the frequency ν can be expressed, on the one hand, through the brightness temperature $T_{\rm b}$ as

$$S_\nu = \frac{2\pi\nu^2\theta^2}{c^2}k_{\rm B}T_{\rm b}, \tag{8}$$

where θ is the angular size of the radiating domain. On the other hand, the flux for the optically thick uniform source of radius R at the distance d can be expressed using the spectral photon emission rate ρ_ν and the effective absorption coefficient $æ_\nu$ as (Gould 1979)

$$S_\nu = \pi\hbar\nu\frac{\rho_\nu}{æ_\nu}\frac{R^2}{d^2}u(2Ræ_\nu), \tag{9}$$

and the function of the optical depth $u(2Ræ_\nu)$ is defined in Gould (1979). The emission and absorption coefficients for the synchrotron-self-Compton model can be expressed in a jet frame (primed), i.e. in a frame where the electric field vanishes, as (Ginzburg & Syrovatskii 1964; Blumenthal & Gould 1970; Gould 1979)

$$\rho'_{\nu'} = 4\pi\left(\frac{3}{2}\right)^{(p-1)/2} a(p)\alpha k'_{\rm e}\left(\frac{\nu'_{\rm B'}}{\nu'}\right)^{(p+1)/2}, \tag{10}$$

$$æ'_{\nu'} = c(p)r_0^2k'_{\rm e}\left(\frac{\nu_0}{\nu'}\right)\left(\frac{\nu'_{\rm B'}}{\nu'}\right)^{(p+2)/2}. \tag{11}$$

Here, $\nu'_{\rm B'} = eB'/mc$ is a gyrofrequency in the fluid frame, $\hbar$ is the Planck constant, $\alpha = e^2/\hbar c$ is the fine structure constant and the functions $a(p)$ and $c(p)$ of the electron distribution spectral index p are defined in Gould (1979).

Equations (8) and (9) are written in an observer frame. However, the spectral flux is calculated in a jet (primed) frame using (10) and (11), where it is expressed as a function of the frequency ν', magnetic field B' and particle number density amplitude $k'_{\rm e}$ in the jet frame. In order to rewrite a flux and a brightness temperature in terms of the observer frame, we use the Lorentz invariant S_ν/ν^3 (Rybicki & Lightman 1979). To express the magnetic field and particle number density in an nucleus frame, and the frequency in an observer frame, we use the following relations. The particle number density amplitude k_e' in the fluid frame correlates with its value k_e in the nucleus frame as

$$k_e' = k_e/\Gamma, \tag{12}$$

and an observed frequency transforms from the fluid frame into an observer frame as

$$\nu' = \nu_{obs} \frac{1+z}{\delta}, \tag{13}$$

and the brightness temperature $T_{b,obs} = T_b \delta/(1+z)$. Here, z is a cosmological redshift of a source and a Doppler factor of a flow $\delta = [\Gamma(1-\beta \cos\varphi)]^{-1}$. The viewing angle of a jet is φ. We assume that the toroidal component of a magnetic field dominates the jet radiating region outside the light cylinder with its position defined by $R_L = c/\Omega_F$. Indeed, the MHD analytical (Beskin 1997; Narayan, McKinney & Farmer 2007; Tchekhovskoy et al. 2008; Lyubarsky 2009; Nokhrina et al. 2015) and numerical (Tchekhovskoy & Bromberg 2016) models provide that $B_\varphi \approx B_P r/R_L$. Thus, the magnetic fields transform from the fluid frame into the nucleus frame as $B \approx B'\Gamma$.

Equating the right-hand sides of equations (8) and (9), we obtain for the magnetic field

$$B = k_0(p) \frac{m^3 c^5}{e} \frac{\Gamma\delta}{1+z} \nu_{obs} \left(k_B T_{b,obs}\right)^{-2}, \tag{14}$$

where the numerical factor k_0 depends on the electrons spectral index and is equal to

$$k_0(p) = 3.6 \times 10^{-1}, \quad p = 2. \tag{15}$$

A particular spectral index p may be found by fitting the jet spectrum for a particular source. From the theoretical point of view, it depends on the non-thermal mechanism of particle acceleration in AGNs, which is under debate. The first-order Fermi mechanism working at shocks provides $p = 2$ (Blandford & Ostriker 1978). However, the numerical simulations demonstrate that this mechanism works effectively for low magnetization flows (Sironi & Spitkovsky 2011), with $p \approx 2.5$. On the other hand, magnetic reconnection provides means for particle acceleration and formation of the power-law spectrum with the spectral index p depending on the flow magnetization (Sironi & Spitkovsky 2014) and ranging from 1.5 to 4. Keeping in mind the uncertainty of the spectral index, we choose the value $p = 2$ as a fiducial parameter characterizing the non-thermal spectrum of radiating particles. Substituting $p = 2$, we obtain

$$\left(\frac{B}{\mathrm{G}}\right) = 7.4 \times 10^{-4} \frac{\Gamma\delta}{1+z} \left(\frac{\nu_{obs}}{\mathrm{GHz}}\right) \left(\frac{T_{b,obs}}{10^{12}\,\mathrm{K}}\right)^{-2}. \tag{16}$$

The radio core is observed at the peak spectral flux, with $\nu_{obs} = \nu_{peak}$ for the magnetic field B and radiating particle number density n at the surface of the optical depth equal to unity. For each frequency, the position of this surface is different (core-shift effect, see e.g. Lobanov 1998), so the magnetic field defined by equation (16) is for the particular position r_{core} of the observed core. Equation (16) does not provide us complete information about the magnetic field amplitude, since we do not know the position of the core at the observed frequency. In order to obtain the core position, we need the measurements of the core-shift effect (Lobanov 1998; Pushkarev et al. 2011; Sokolovsky et al. 2011) as well.

2.2 Measure of equipartition

The core-shift effect is a change in the observed position of a core at different frequencies (Lobanov 1998; O'Sullivan & Gabuzda 2009). It is connected with the self-absorption of the synchrotron sources (see e.g. Gould 1979): due to absorption we observe the surface of the optical thickness equal to unity. Both the synchrotron emission rate and the absorption depend on the emitting particle number density and magnetic field magnitudes and distributions. The 'stardart' core-shift formula by Lobanov (1998) has been obtained under certain assumptions: the Blandford–Königl field and particle number density dependence on r (7) and the equipartition between the radiating plasma and magnetic field. The last assumption has been essential for the results, since measurements of the core-shift allow only to estimate the dependence of the magnetic field magnitude on the particle number density. The same equipartition assumption has been used to establish the 'equipartition brightness temperature' by Readhead (1994). However, the recent observations of the brightness temperature at high resolution provided by *RadioAstron* exceed this 'equipartition' limit, so, as indicated by Gómez et al. (2016), there is, probably, no equipartition in a jet. However, the measurements of both the brightness temperature and core shift provide us with the instrument to estimate the magnetic field and the particle number density independently (Zdziarski et al. 2015), and thus obtain the measure of 'non-equipartition'.

Let us introduce the radiation magnetization Σ – the ratio of the Poynting flux to the radiating particle energy flux. Each particle internal energy is given by $mc^2\gamma'$, where γ' is the Lorentz factor of radiating particles with respect to plasma bulk motion. The radiating particle number density n'_{rad} is given in a jet bulk motion proper frame. The amplitude k_e' is defined by the radiating particle number density n'_{rad} depending on the magnitude of the exponent p as $n'_{rad} = k_e' f(p)$ with

$$f(p) = \begin{cases} \dfrac{1}{1-p}\left(\gamma_{max}^{1-p} - \gamma_{min}^{1-p}\right), & p \neq 1, \\ \ln\dfrac{\gamma_{max}}{\gamma_{min}}, & p = 1. \end{cases} \tag{17}$$

We assume $p \in (1; 2]$, and the Lorentz factor of the plasma in a nucleus frame is defined by $\gamma = \gamma'\Gamma$. In this case, the magnetization of radiating particles is

$$\Sigma = \frac{\Gamma(2-p) B^2 f(p)}{4\pi mc^2 n_{rad}\left(\gamma_{max}^{2-p} - \gamma_{min}^{2-p}\right)} \tag{18}$$

for $p \neq 2$, and

$$\Sigma = \frac{\Gamma B^2 f(p)}{4\pi mc^2 n_{rad} \ln\frac{\gamma_{max}}{\gamma_{min}}} \tag{19}$$

for $p = 2$. Here, n_{rad} is given in the nucleus frame. For the equipartition between the magnetic field and radiating particles, $\Sigma = 1$. We will be interested in obtaining estimates for Σ from the observations. This will allow us to connect the radiation magnetization Σ with the physical properties of the radiating plasma. We will use the non-dimensional function $F_\Sigma(p)$ such as

$$\Sigma = \frac{\Gamma B^2}{mc^2 n_{rad}} F_\Sigma(p). \tag{20}$$

The expression connecting the jet physical parameters B and r_{rad} with the position of the radiating region r and the observed frequency ν_{obs} has been obtained by Lobanov (1998), Hirotani (2005) and Nokhrina et al. (2015):

$$B^{2+p} n_{rad}^2 = \nu_{obs}^{4+p} F_1^{-1} F_2^{-1} r^{-2}, \tag{21}$$

where coefficients

$$F_1 = \frac{c^2(p)(p-1)^2}{5(4+p)} \frac{e^4}{m^2c^2} \left(\frac{e}{2\pi mc}\right)^{2+p} \tag{22}$$

and

$$F_2 = \left(\frac{\delta}{\Gamma(1+z)}\right)^{4+p} \left(\frac{2\chi}{\delta \sin\varphi}\right)^2. \tag{23}$$

Here, χ is a jet half-opening angle for the conical jet. Using (20), we rewrite $n_{\rm rad}$ as a function of Σ and B, and substituting (14) into (21) we obtain the expression for the flow magnetization in a radiating domain as a function of its position r from the central source, the observed brightness temperature, the observed frequency and the geometrical and velocity factors:

$$\Sigma = 4.1 \times 10^3 \left(1.7 \times 10^2\right)^{-p} C_\Sigma(p) \frac{2\chi\Gamma^2}{\delta \sin\varphi} \left(\frac{\delta}{1+z}\right)^{p+5} \times \left(\frac{r}{\rm pc}\right) \left(\frac{\nu_{\rm obs}}{\rm GHz}\right) \left(\frac{T_{\rm b,\,obs}}{10^{12}\,{\rm K}}\right)^{-(p+6)}. \tag{24}$$

Here,

$$C_\Sigma(p) = \frac{F_\Sigma(p)}{f(p)} \frac{c(p)}{\sqrt{5(4+p)}} (2\pi)^2 \times \left[2.8(1.5)^{(p-1)/2} \frac{a(p)}{c(p)}\right]^{p+6}. \tag{25}$$

For $p = 2$, we obtain

$$\Sigma = 1.58 \times 10^{-5} \frac{2\chi\Gamma^2\delta^6}{\sin\varphi(1+z)^7} \frac{F_\Sigma(2)}{f(2)} \times \left(\frac{r}{\rm pc}\right) \left(\frac{\nu_{\rm obs}}{\rm GHz}\right) \left(\frac{T_{\rm b,\,obs}}{10^{12}\,{\rm K}}\right)^{-8}. \tag{26}$$

The above expression has been obtained assuming that (i) the radiating region has a uniform distribution of $n_{\rm rad}$ and B; (ii) the radiating domain is optically thick; (iii) the jet is conical with the half-opening angle χ, so that the jet geometrical thickness along the line of sight depends on r, χ and φ (see Hirotani 2005 for details); (iv) we observe the surface of the optical depth is approximately equal to unity at the observed frequency $\nu_{\rm obs}$. This allows us to estimate the order of Σ, assuming that r is of the order of a parsec.

However, if we additionally adopt the Blandford–Königl scalings for the magnetic field B and particle number density $n_{\rm rad}$ (7), we will be able to correlate the position of a radiating domain r with the observed frequency $\nu_{\rm obs}$. Indeed, substituting (7) into (21) one obtains the classical expression $\nu_{\rm obs} r$ proportional to the physical parameters of a jet. The last conclusion is supported by multifrequency observations by Sokolovsky et al. (2011). Thus, if we have, in addition to the measurement of the brightness temperature, the core-shift measurement, we can use it to obtain the radiating domain position. As

$$r \sin\varphi = \theta_{\rm d} \frac{D_{\rm L}}{(1+z)^2}, \tag{27}$$

where $D_{\rm L}$ is the luminosity distance, we introduce

$$\Delta\theta_{\rm d} = \Phi\left(\frac{1}{\nu_1} - \frac{1}{\nu_2}\right). \tag{28}$$

With $\Delta\theta_{\rm d}$ being measured for the two frequencies ν_1 and ν_2, we can calculate Φ in mas GHz and find the observed position of the core at a given frequency as

$$r_{\rm core} = \frac{\Phi D_{\rm L}}{\nu_{\rm obs} \sin\varphi\,(1+z)^2}. \tag{29}$$

Having the knowledge of the core shift, we can estimate the radial distance of the observed radiating domain of a jet:

$$\frac{r_{\rm obs}}{\rm pc} = \frac{4.8}{\sin\varphi(1+z)^2} \left(\frac{\nu_{\rm obs}}{\rm GHz}\right)^{-1} \left(\frac{\Phi}{\rm mas\ GHz}\right) \left(\frac{D_{\rm L}}{\rm Gpc}\right), \tag{30}$$

and, consequently, the magnetization in the observed core as

$$\Sigma = 2.1 \times 10^4 \left(1.7 \times 10^2\right)^{-p} C_\Sigma(p) \frac{2\chi\Gamma^2\delta^{p+4}}{\sin^2\varphi(1+z)^{p+7}} \times \left(\frac{D_{\rm L}}{\rm Gpc}\right) \left(\frac{\Phi}{\rm mas\ GHz}\right) \left(\frac{T_{\rm b,\,obs}}{10^{12}{\rm K}}\right)^{-(p+6)}. \tag{31}$$

For $p = 2$, the expression is

$$\Sigma = 7.7 \times 10^{-5} \frac{2\chi\Gamma^2\delta^6}{\sin^2\varphi(1+z)^9} \frac{F_\Sigma(2)}{f(2)} \times \left(\frac{D_{\rm L}}{\rm Gpc}\right) \left(\frac{\Phi}{\rm mas\ GHz}\right) \left(\frac{T_{\rm b,\,obs}}{10^{12}{\rm K}}\right)^{-8}. \tag{32}$$

2.3 Radiating particle number density

In order to obtain the radiating particle number density in a radiating domain, we substitute (14) into (21):

$$\left(\frac{n_{\rm rad}}{\rm cm^{-3}}\right) = 1.1 \times 10^{-3}(1.7 \times 10^2)^p C_{\rm n}(p) \times \frac{\Gamma \sin\varphi(1+z)^{p+3}}{2\chi\delta^{p+2}} \left(\frac{r}{\rm pc}\right)^{-1} \times \left(\frac{\nu_{\rm obs}}{\rm GHz}\right) \left(\frac{T_{\rm b,\,obs}}{10^{12}\,{\rm K}}\right)^{p+2}. \tag{33}$$

Here,

$$C_{\rm n}(p) = f(p) \frac{\sqrt{5(p+4)}}{c(p)} \left[2.8(1.5)^{(p-1)/2} \frac{a(p)}{c(p)}\right]^{-(p+2)}. \tag{34}$$

For $p = 2$, we obtain the estimate for the radiating particle number density at the region with the position r:

$$\left(\frac{n_{\rm rad}}{\rm cm^{-3}}\right) = 4 \times 10^4 \frac{\Gamma \sin\varphi(1+z)^5}{2\chi\delta^4} f(2) \times \left(\frac{r}{\rm pc}\right)^{-1} \left(\frac{\nu_{\rm obs}}{\rm GHz}\right) \left(\frac{T_{\rm b,\,obs}}{10^{12}\,{\rm K}}\right)^4. \tag{35}$$

Using (30), one can obtain the expression for $n_{\rm rad}$ as a function of the observables:

$$\left(\frac{n_{\rm rad}}{\rm cm^{-3}}\right) = 2.3 \times 10^{-4}(1.7 \times 10^2)^p C_{\rm n}(p) \times \frac{\Gamma \sin^2\varphi(1+z)^{p+5}}{2\chi\delta^{p+2}} \left(\frac{D_{\rm L}}{\rm Gpc}\right)^{-1} \left(\frac{\Phi}{\rm mas\ GHz}\right)^{-1} \times \left(\frac{\nu_{\rm obs}}{\rm GHz}\right)^2 \left(\frac{T_{\rm b,\,obs}}{10^{12}\,{\rm K}}\right)^{p+2}. \tag{36}$$

For $p = 2$,

$$\left(\frac{n_{\rm rad}}{\rm cm^{-3}}\right) = 8.2 \times 10^3 \frac{\Gamma \sin^2\varphi(1+z)^7}{2\chi\delta^4} f(2) \times \left(\frac{D_{\rm L}}{\rm Gpc}\right)^{-1} \left(\frac{\Phi}{\rm mas\ GHz}\right)^{-1} \left(\frac{\nu_{\rm obs}}{\rm GHz}\right)^2 \left(\frac{T_{\rm b,\,obs}}{10^{12}\,{\rm K}}\right)^4. \tag{37}$$

2.4 Physical parameters in the sources with extreme brightness temperatures

We can apply the above estimates to two objects with the measured brightness temperature and core shift. Equations (16), (32) and (37) permit us to obtain estimates for the radiating particle magnetization Σ, magnetic field B and radiating particle number density $n_{\rm rad}$ in the observed radio core (radiating domain) if we have precise enough measurement of the brightness temperature. On the other hand, if we have the lower limit for the brightness temperature (Lobanov 2015), these expressions provide the lower limit for the particle number density $n_{\rm rad}$ and the upper limit for the magnetic field B and the magnetization parameter Σ.

We will calculate the magnetic field B, particle number density (in nucleus frame) $n_{\rm rad}$ and magnetization (measure of equipartition) Σ for the blazar BL Lac and 3C273 based on the measurements of the core brightness temperature by Gómez et al. (2016) and Kovalev et al. (2016). The other parameters we need are the Doppler factor, the Lorentz factor of a flow, the observation angle φ, redshift z and the half-opening angle χ. We take the redshift and apparent velocity

$$\beta_{\rm app} = \frac{\beta \sin\varphi}{1 - \beta\cos\varphi} \qquad (38)$$

from Lister et al. (2013).

There are several approaches for the calculation of a Doppler factor δ from the observed jet parameters. The first one employs the relation $\varphi \approx \gamma^{-1}$ and provides $\delta_{\beta_{\rm var}} = \beta_{\rm app}$. The modelling of a probability of a source with a Doppler factor $\delta = \beta_{\rm app}$ from the flux-density-limited sample (Cohen et al. 2007) shows that this probability is peaked around unity for a large sample. Another assumption used in this method is that the pattern speed is approximately equal to the flow speed, and the results of modelling by Cohen et al. (2007) support it. The second way to estimate the jet Doppler factor is based on the assumption that the characteristic time of variability of a bright knot in a jet gives us information about the light-travel time across the knot of the observed angular size. This allows us to calculate the variability Doppler factor (Jorstad et al. 2005). It has been shown by Jorstad et al. (2005) for the set of 15 sources that $\delta_{\beta_{\rm app}}$ and $\delta_{\rm var}$ correlate with each other, following approximately the linear dependence $\delta_{\beta_{\rm app}} \approx 0.72\delta_{\rm var}$. This supports a possibility of using $\delta_{\beta_{\rm var}}$ as an estimate for the Doppler factor of each individual source. The third method used by Hovatta et al. (2009) is based on the comparison of the variability brightness temperature to the equipartition brightness temperature.

For the two sources with extreme brightness temperature, the Doppler factor can be estimated by the first two methods. We do not use the results by Hovatta et al. (2009), since they have been obtained using the equipartition assumption. For the 3C 273 source, $\delta_{\beta_{\rm app}} = 14.86$ (Lister et al. 2013) and $\delta_{\rm var} = 12.6$ (Jorstad et al. 2005). For BL Lac, $\delta_{\beta_{\rm app}} = 9.95$ (Lister et al. 2013) and $\delta_{\rm var} = 8.1$ (Jorstad et al. 2005). Here, we have chosen the maximal value for $\delta_{\rm var}$ from the set of different values for different knots. Both methods provide the estimates for the Doppler factors which are in good agreement with each other. For our purposes, we use the estimate $\delta_{\beta_{\rm app}}$, as for the sources under consideration $\delta_{\beta_{\rm app}} > \delta_{\rm var}$, thus providing the upper limit for the values of B and Σ and the lower limit for $n_{\rm rad}$ – the closest to the equipartition value limits.

The expression for the observation angle can be found using the Doppler factor definition and equation (38):

$$\varphi = {\rm atan}\left(\frac{2\beta_{\rm app}}{2\beta_{\rm app}^2 - 1}\right). \qquad (39)$$

We also use the observations of the apparent half-opening angle by Pushkarev et al. (2009). Having the knowledge of the observation angle φ and the apparent half-opening angle $\chi_{\rm app}$, one can obtain the half-opening angle

$$\chi \approx \chi_{\rm app} \sin\varphi/2. \qquad (40)$$

The luminosity distance $D_{\rm L}$ is obtained according to the Λ dark matter cosmological model with $H_0 = 71\,{\rm km\,s^{-1}\,Mpc^{-1}}$, $\Omega_{\rm m} = 0.27$ and $\Omega_\Lambda = 0.73$ (Komatsu et al. 2009).

BL Lac parameters. For this object, we use the brightness temperature measurements by Gómez et al. (2016). We choose the measurement at $\nu_{\rm obs} = 15$ GHz, as this frequency is closest to the frequencies used to estimate a core shift for this object. The lower estimate for the observed brightness temperature is 7.9×10^{12} K. We also employ the following observable parameters needed to obtain the physical properties at the core of BL Lac. They are as follows: $z = 0.069$, $\beta_{\rm app} = 9.95$ (Lister et al. 2013), $\chi_{\rm app} = 26.^\circ2$ (Pushkarev et al. 2009) and $\Phi = 0.55$ mas GHz (Pushkarev et al. 2011). From these we find $D_{\rm L} = 0.31$ Gpc, $\varphi = 0.1$, $\chi = 0.02$ and $\Gamma \approx 20$. Substituting these parameters into (16), (32) and (37), we obtain the following parameters: $B = 3.3 \times 10^{-2}$ G, $n_{\rm rad} = 3.4 \times 10^7\,{\rm cm^{-3}}$ and $\Sigma = 1.3 \times 10^{-5}$.

3C273 parameters. For this object, we take the measurements of the brightness temperature by Kovalev et al. (2016) at $\nu_{\rm obs} = 4.8$ GHz. For this object, the observed brightness temperature is $T_{\rm b,\,obs} = 13 \times 10^{12}$ K. We employ the following observable parameters for the 3C273: $z = 0.158$, $\beta_{\rm app} = 14.86$ (Lister et al. 2013), $\chi_{\rm app} = 10.^\circ0$ (Pushkarev et al. 2009) and $\Phi = 0.34$ mas GHz (Pushkarev et al. 2011). From these we calculate $D_{\rm L} = 0.75$, $\varphi = 0.067$, $\chi = 0.006$ and $\Gamma \approx 30$. For these parameters, we obtain the following physical parameters: $B = 8.1 \times 10^{-3}$ G, $n_{\rm rad} = 1.4 \times 10^7\,{\rm cm^{-3}}$ and $\Sigma = 2.9 \times 10^{-6}$.

The magnitudes of the magnetic field and particle number density in the radiation region, obtained based on the brightness temperature measurements, significantly differ from the jet parameters, obtained by Lobanov (1998), O'Sullivan & Gabuzda (2009) and Hirotani (2005) based on the equipartition assumption. From equations (16), (32) and (37), we can check the would-be observed brightness temperature for the system in equipartition and, consequently, the equipartition magnetic field $B_{\rm eq}$ and the radiating particle number density $n_{\rm rad,\,eq}$ for our sources. Setting for each source $\Sigma = 1$, which corresponds to the equipartition regime, we obtain $T_{\rm b,\,eq} = 1.9 \times 10^{12}$ K for BL Lac and $T_{\rm b,\,eq} = 6.8 \times 10^{11}$ K for 3C 273. The equipartition magnetic field and radiating particle number density in the observed core are $B_{\rm n,eq} = 0.56$ G and $n_{\rm rad,eq} = 1.2 \times 10^5\,{\rm cm^{-3}}$ for BL Lac, and $B_{\rm n,eq} = 3$ G and $n_{\rm rad,eq} = 26\,{\rm cm^{-3}}$ for 3C 273. The extreme values of the physical parameters of the radiating region are in accordance with the conclusions by Readhead (1994), who had found that even the brightness temperatures at the Compton catastrophe limit would need an extreme departure from the equipartition. Here, we want to mention that the 3C 273 source also demonstrates the extreme magnitude of Michel's magnetization, even calculated based on the equipartition assumptions (Nokhrina et al. 2015).

We have obtained the radiating particle magnetization for the two sources with extreme brightness temperatures of the order of 10^{-5}. The obtained radiating magnetization allows us to estimate the total outflow magnetization $\sigma_{\rm tot}$. Indeed, the total magnetization is defined as a function of a bulk flow magnetization $\sigma \approx 1$ for

MHD outflows and radiating magnetization $\Sigma \ll 1$:

$$\sigma_{\rm tot} = \frac{B^2}{4\pi mc^2 n\Gamma + 4\pi mc^2 n_{\rm rad} \ln \frac{\gamma_{\max}}{\gamma_{\min}} / \Gamma} = \frac{1}{1/\sigma + 1/\Sigma}. \quad (41)$$

Thus, we conclude that the radiating plasma must be highly relativistic so as to dominate the particle energy flux, at least in the radiation domain, so that the total outflow magnetization

$$\sigma_{\rm tot} \approx \Sigma \ll 1. \quad (42)$$

The non-equipartition physical parameters in the core have extreme values. Indeed, let us estimate the maximum particle number density in a jet provided that the total jet power is in the particle kinetic energy. Thus,

$$\Gamma mc^3 \int_0^{R_{\rm j}} n^{\rm lab}(r_\perp) 2\pi r_\perp {\rm d}r_\perp \le P_{\rm jet}. \quad (43)$$

For the uniform transversal number density distribution, we get

$$\left(\frac{n^{\rm lab}}{\rm cm^{-3}}\right) \le 10^4 \left(\frac{P_{\rm jet}}{10^{45}\ {\rm erg\ s^{-1}}}\right) \left(\frac{R_{\rm jet}}{0.1\ {\rm pc}}\right)^2. \quad (44)$$

Although the total jet power is not always known, the estimate based on correlation between the total jet power and the radio power (Cavagnolo et al. 2010) may be applied. This provides the values of $P_{\rm BLLac} \approx 1.2 \times 10^{44}$ erg s^{-1} and $P_{\rm 3C273} \approx 3.5 \times 10^{45}$ erg s^{-1} for both sources (Nokhrina et al. 2015). For inequality (44) to hold for the obtained values of $n_{\rm rad}$, the jet radius $R_{\rm jet}$ has to be approximately 20 and 2 pc, respectively. These values exceed the measured jet radius for M87 (Mertens et al. 2016) of the order of 0.1 pc. This may mean that we underestimate the magnetic field amplitude, or that the physical conditions in the radiating domain are very different from the conditions over the larger jet domain, so that the Blandford–Königl model is not applicable for the radiating core. In what follows, we will address the first issue of the probable underestimation of the magnetic field magnitude.

3 THE SIMPLEST NON-UNIFORM MODEL

We see that the standard approach of the Blandford–Königl model applied for the observed extreme brightness temperatures gives the small magnetic field and unphysically high particle number density. However, as pointed out by Marscher (1977), non-uniform models with the transversal structure provide the strong dependence of physical parameters of a flow on observables. In this section, we will relax the assumption of a uniform distribution of a magnetic field across the radiating domain. Here, we will employ the MHD model for the transversal jet structure in the radiating domain in order to calculate the spectral flux and thus obtain the expression for the magnetic field as a function of an observed brightness temperature. We plan to reconsider the effect of the non-uniform distribution of physical parameters on the core-shift effect in the future paper.

3.1 Model with the uniform velocity across the jet

We assume the radiation site being the part of a continuous cylindrical jet with the bulk Lorentz factor Γ with plasma excited by some process so it has a power-law energy distribution (6) in the jet frame (which in our model is also a pattern frame). We assume a radiating region of a jet being uniform along the jet axis, but with a transversal structure: the magnetic field $B(r_\perp)$ and the particle number density $n(r_\perp)$ are the functions of the radial distance from the jet axis $r_\perp$. These are discussed in the subsequent sections.

Modelling the transversal jet structure needs solving the MHD equations – the Grad–Shafranov equation together with the Bernoulli equation (see e.g. review by Beskin 2010). In general, these cannot be solved analytically, although in some special cases (self-similarity or special geometry) the solution may be obtained. The numerical MHD simulations provide a powerful instrument in constructing the jet internal structure models. In this work, we will use the obtained earlier analytical and numerical results as a simplest model for the relativistic jet transverse structure. We will model the particle number density and the toroidal magnetic field dependence on the distance from the jet axis by two domains. The first one is a jet central core, which we define as the central part of a jet with uniform n and B_φ distributions (Komissarov et al. 2007; Lyubarsky 2009; Nokhrina et al. 2015; Tchekhovskoy & Bromberg 2016). The size of the central core $R_{\rm c}$ is of the order of a few light cylinder radii. In particular, the numerical modelling by Tchekhovskoy & Bromberg (2016) gives $R_{\rm c} \approx R_{\rm L}$, and the semi-analytical modelling by Komissarov et al. (2007), Beskin & Nokhrina (2009) and Nokhrina et al. (2015) gives $R_{\rm c} \approx 5R_{\rm L}$. As these results are very close, we will use for simplicity $R_{\rm c} = R_{\rm L}$. Further, the same modelling allows us to approximate the particle number density and the poloidal and toroidal magnetic fields in a jet in the second domain by the power laws (Nokhrina et al. 2015; Tchekhovskoy & Bromberg 2016). So, we will use the following functions as an approximation for $B(r_\perp)$ and $n(r_\perp)$:

$$n_{\rm rad} = n_0 \begin{cases} 1, & r_\perp \le R_{\rm L}, \\ (R_{\rm L}/r_\perp)^2, & R_{\rm L} < r_\perp \le R_{\rm j}, \end{cases} \quad (45)$$

$$B_{\rm P} = B_0 \begin{cases} 1, & r_\perp \le R_{\rm L}, \\ (R_{\rm L}/r_\perp)^2, & R_{\rm L} < r_\perp \le R_{\rm j}, \end{cases} \quad (46)$$

$$B_\varphi = B_0 \begin{cases} r_\perp/R_{\rm L}, & r_\perp \le R_{\rm L}, \\ R_{\rm L}/r_\perp, & R_{\rm L} < r_\perp \le R_{\rm j}. \end{cases} \quad (47)$$

For the jet radiating region with the flat Lorentz factor Γ distribution across a jet, the poloidal magnetic field does not change with transformation from the jet into observer's frame, and the toroidal magnetic field transforms as

$$B'_\varphi = B_\varphi/\Gamma. \quad (48)$$

Within this model, the poloidal magnetic field dominates the toroidal for $r_\perp < \Gamma R_{\rm L}$, and we have the following scalings for the particle number density and the magnetic field in a fluid frame:

$$B' = B_0 f_{\rm B}(r_\perp) = B_0 \begin{cases} 1, & r_\perp \le R_{\rm L}, \\ (R_{\rm L}/r_\perp)^2, & R_{\rm L} < r_\perp \le \Gamma R_{\rm L}, \\ R_{\rm L}/r_\perp\Gamma, & \Gamma R_{\rm L} < r_\perp \le R_{\rm j}, \end{cases} \quad (49)$$

$$n'_{\rm rad} = \frac{n_0}{\Gamma} f_{\rm n}(r_\perp) = \frac{n_0}{\Gamma} \begin{cases} 1, & r_\perp \le R_{\rm L}, \\ (R_{\rm L}/r_\perp)^2, & R_{\rm L} < r_\perp \le R_{\rm j}. \end{cases} \quad (50)$$

Here, we also assumed that the ratio of radiating particles to all the particles in a jet is constant across the jet.

The photon emission rate (10) and the effective absorption coefficient (11) have been obtained by Ginzburg & Syrovatskii (1964) and Blumenthal & Gould (1970) for the randomly oriented magnetic field. The direction of a magnetic field in the derivation in

Blumenthal & Gould (1970) sets the possible distribution of a pitch angle α of radiating particles. In particular, for randomly oriented field, the pitch angle has a flat distribution, which gives after averaging over α the appropriate factor in terms of function $a(p)$ in (10). However, we can use expressions (10) and (11) even for the ordered magnetic field, but randomly oriented orbits of the radiating particles, provided the pitch angle α also has a flat distribution.

To obtain the numerical values, we need estimates for $R_{\rm L}$ and $R_{\rm j}$. Further we use the following dimension parameters for a central engine and an outflow angular velocity: the gravitational radius for a black hole with $M_{\rm BH} = 10^9\,{\rm M}_{\odot}$ is $r_{\rm g} = 10^{-4}$ pc. We also use the result obtained by Zamaninasab et al. (2014) for the light cylinder radius, which can be rewritten as

$$\frac{\Omega_{\rm F} r_{\rm g}}{c} = \frac{2\pi\eta}{50}, \qquad (51)$$

where $W_{\rm tot} = \eta \dot{M} c^2$. Setting $\eta = 1$, we get $R_{\rm L} \approx 10 r_{\rm g}$, the result, that we will use. As to jet radius $R_{\rm j}$, the observations of M87 provide the value $R_{\rm j} \approx 0.1$ pc (Mertens et al. 2016), so we set $R_{\rm j} = 10^2 R_{\rm L}$.

3.2 Limiting parameters

In Section 2, we have obtained that the upper limit for the particle number density in the model with the uniform particle number density distribution is approximately $10^4\,{\rm cm}^{-3}$, assuming the jet parameters of the order of $P_{\rm jet} \approx 10^{45}$ erg s^{-1} and $R_{\rm jet} \approx 0.1$ pc. For the non-uniform radial distribution (45), the upper limit on the particle number density amplitude $n_0^{\rm lab}$ is $n_0^{\rm lab} \le 10^7$, with $n^{\rm lab}(R_{\rm j})$ being of the order of $10^3\,{\rm cm}^{-3}$.

The same bounding limits can be obtained for the toroidal magnetic field – the field that in MHD models defines the Poynting flux transported by a jet:

$$\frac{c}{4\pi}\int_0^{R_{\rm j}} B_{\varphi}^2(r_{\perp}) 2\pi r_{\perp} {\rm d}r_{\perp} \le P_{\rm jet}. \qquad (52)$$

For the uniform model $B_{\varphi} \le 1$ G. For the toroidal magnetic field defined by (47), we obtain the field amplitude $B_0 \le 40$ G for the same jet parameters.

3.3 Optical depth for small viewing angles

Let us determine the optical depth

$$\tau = \int_0^{s_0'} æ'_{\nu'} {\rm d}s' \qquad (53)$$

of the radiating domain depending on n_0, B_0 and $\nu_{\rm obs}$ for the jets directed almost at the observer – the result applicable for the BL Lac and quasar-type sources. Since the optical depth is a Lorentz invariant, we will calculate it in the fluid frame. However, we express it as a function of amplitudes of the particle number density n_0, the magnetic field B_0 in the nucleus frame and the frequency $\nu_{\rm obs}$ in the observer frame, using the transformations from the jet frame into the nucleus or the observer frame (13) and (49) and (50). For small viewing angles $\varphi \ll 1$, we simplify the integration by taking ${\rm d}s' \approx {\rm d}z'$, so that

$$\tau(z, r_{\perp}) = 0.28 \frac{1}{f(p)} \left(\frac{\delta}{1+z}\right)^3 \left(\frac{n_0}{{\rm cm}^{-3}}\right) \left(\frac{B_0}{\rm G}\right)^2 \times \left(\frac{\nu_{\rm obs}}{\rm GHz}\right)^{-3} \left(\frac{z}{R_{\rm L}}\right) f_{\rm n}(r_{\perp}) f_{\rm B}^2(r_{\perp}). \qquad (54)$$

The expression for an optical depth τ (54) can be rewritten through dimensionless τ_0 that depends only on the intrinsic radiating domain parameters and the Doppler factor

$$\tau_0 = 0.28 \frac{1}{f(p)} \left(\frac{\delta}{1+z}\right)^3 \left(\frac{n_0}{{\rm cm}^{-3}}\right) \left(\frac{B_0}{\rm G}\right)^2 \left(\frac{\nu_{\rm obs}}{\rm GHz}\right)^{-3}, \qquad (55)$$

and the dimensionless ‘position’ factor, so

$$\tau = \tau_0 \frac{z}{R_{\rm L}} f_{\rm n}(r_{\perp}) f_{\rm B}^2(r_{\perp}). \qquad (56)$$

Since the jet physical parameters change significantly across the jet cross-section, the optical depth of the different domains may be greater or smaller than unity. For example, let us describe the position of a surface with an optical depth equal to unity along the jet as a function of the radial distance from the jet axis $r_{\perp}$. Let us take the observed frequency $\nu_{\rm obs} = 10$ GHz characteristic for the radio interferometric observations, and $\delta \approx 10$ and $\Gamma \approx 10$. For all reasonable parameters of a jet, n_0 and B_0, the surface $\tau = 1$ is situated at the geometrical depth z being only a small fraction of a parsec in the central part of a jet. For greater $r_{\perp}$, the geometrical depth of the surface $\tau = 1$ grows extremely fast towards the jet edges. The result strongly depends on physical parameters in a jet. For the limiting parameters (the maximal optical thickness), if we assume $n_0 = 10^7\,{\rm cm}^{-3}$ and $B_0 = 40$ G, the surface $\tau = 1$ for the whole jet cross-section remains optically thick for the radiating domain depth z greater than 10^{-3} pc.

However, for the less extreme parameters the situation is quite different. If we take $n_0 \approx 10^3\,{\rm cm}^{-3}$ and $B_0 \approx 1$ G – the equipartition parameters for the uniform model obtained by Lobanov (1998) – the position of the surface with $\tau = 1$ must be of the order of a few parsec at $r_{\perp} = \Gamma R_{\rm L}$, which means that the radiating domain is optically thick in the central jet part and optically thin at the outer jet domain.

3.4 Non-uniform jet velocity

There are some observational indications of a non-flat transversal Lorentz factor structure: the limb brightening (see e.g. Giroletti et al. 2008) and M87 observed velocity transverse profile (Mertens et al. 2016). In the latter work, superluminal velocities have been detected in the limbs as well as in the central stream. The numerical (Tchekhovskoy et al. 2008, 2009) and analytical (Beskin & Nokhrina 2006; Lyubarsky 2009) modelling show that the bulk flow Lorentz factor is not constant in the transversal jet direction. In particular, the following transverse Lorentz factor structure has been predicted by the MHD modelling:

$$\Gamma(r_{\perp}) = \gamma(r_{\perp})\sigma_{\rm M} = \begin{cases} \gamma_{\rm in} \approx 1, & r_{\perp} \le R_{\rm L}, \\ r_{\perp}/R_{\rm L}, & R_{\rm L} < r_{\perp} \le \sigma_{\rm M} R_{\rm L}, \\ \sigma_{\rm M}, & r_{\perp} > \sigma_{\rm M} R_{\rm L}. \end{cases} \qquad (57)$$

Here, $\sigma_{\rm M}$ is Michel’s magnetization parameter – the ratio of the Poynting flux to the particle kinetic energy flux at the base of an outflow. It bounds the maximum Lorentz factor as $\Gamma < \sigma_{\rm M}$. We are using the dependences (45)–(47) for a particle number density and a magnetic field.

The drift bulk velocity of a plasma has both a toroidal $v_{{\rm dr},\varphi} = v_{\rm dr} B_{\rm P}/B_{\varphi}$ and a poloidal $v_{\rm dr,P} = v_{\rm dr} B_{\varphi}/B_{\rm P}$ component. However, as outside the light cylinder $R_{\rm L} = \Omega_{\rm F}/c$ the toroidal magnetic field is much greater than the poloidal; we will neglect the latter. Thus, the poloidal magnetic field does not change with transformation from the jet into observer's frame for $r_\perp > R_{\rm L}$, and the toroidal magnetic field transforms as

$$B'_\varphi = B_\varphi / \Gamma(r_\perp). \tag{58}$$

Inside the light cylinder, we have the opposite case: we transform the poloidal magnetic field, and the toroidal field remains unchanged. So, under these assumptions in the fluid frame the toroidal magnetic field dominates the poloidal one at $r_\perp > R_{\rm L}$, and we have the following magnetic field and particle number density transversal profiles in the jet frame:

$$B' = B_0 \begin{cases} 1, \; r_\perp \le R_{\rm L}, \\ (R_{\rm L}/r_\perp)^2, \; R_{\rm L} < r_\perp \le \sigma_{\rm M} R_{\rm L}, \\ R_{\rm L}/r_\perp \sigma_{\rm M}, \; \sigma_{\rm M} R_{\rm L} < r_\perp \le R_{\rm j}, \end{cases} \tag{59}$$

$$n'_{\rm rad} = n_0 \begin{cases} 1, \; r_\perp \le R_{\rm L}, \\ (R_{\rm L}/r_\perp)^3, \; R_{\rm L} < r_\perp \le \sigma_{\rm M} R_{\rm L}, \\ (R_{\rm L}/r_\perp)^2/\sigma_{\rm M}, \; \sigma_{\rm M} R_{\rm L} < r_\perp \le R_{\rm j}. \end{cases} \tag{60}$$

The flow Doppler factor depends on the distance from the axis as well. We introduce the Doppler factor for the fastest part of a flow $\delta_0 = 1/\sigma_{\rm M}(1 - \beta(r_\perp)\cos\theta)$. As the flow is relativistic, we neglect the change in β across the flow and use

$$\delta(r_\perp) = \frac{\delta_0}{\gamma(r_\perp)}. \tag{61}$$

Due to this, the observed spectral flux will be much less homogeneous than it is suggested by mere change in B' and n' in comparison with the model with the uniform jet velocity.

The optical thickness as a function of z and $r_\perp$ is now given by

$$\begin{aligned} \tau(z, r_\perp) &= \frac{0.28}{f(p)} \left(\frac{\delta_0}{1+z}\right)^3 \left(\frac{n_0}{\rm cm^{-3}}\right) \left(\frac{B_0}{\rm G}\right)^2 \\ &\quad\times \left(\frac{\nu_{\rm obs}}{\rm GHz}\right)^{-3} \left(\frac{z}{R_{\rm L}}\right) \frac{f_{\rm n}(r_\perp) f_{\rm B}^2(r_\perp)}{\gamma^3(r_\perp)} \\ &= \tau_{0,2} \frac{z}{R_{\rm L}} \frac{f_{\rm n}(r_\perp) f_{\rm B}^2(r_\perp)}{\gamma^3(r_\perp)}. \end{aligned} \tag{62}$$

This equation coincides with (54) except for the additional factor $\gamma^3(r_\perp)$. This means that the optical depth in the central jet part is the same as in a limit of a uniform Lorentz factor. However, the position of a surface $\tau = 1$ grows much more rapidly for $r_\perp > R_{\rm L}$, and for a reasonable depth L of the radiating domain it becomes optically thin. The equation for surface $\tau = 1$ is given by

$$\frac{z(r_\perp)}{R_{\rm L}} = \frac{1}{\tau_{0,2}} \begin{cases} \sigma_{\rm M}^{-3}, \; r_\perp \le R_{\rm L}, \\ (r_\perp/R_{\rm L})^{10} \sigma_{\rm M}^{-3}, \; R_{\rm L} < r_\perp \le \sigma_{\rm M} R_{\rm L}, \\ (r_\perp/R_{\rm L})^4 \sigma_{\rm M}^3, \; \sigma_{\rm M} R_{\rm L} < r_\perp \le R_{\rm j}. \end{cases} \tag{63}$$

Although the radiating domain optical thickness declines more rapidly than in the case of a flat velocity distribution, for upper limits for n_0 and B_0 the outer part of an outflow stays optically thick for $L < 2 \times 10^{-2}$ pc.

3.5 Observed flux

Now we will calculate the observed spectral flux of a model radiating domain with the non-uniform distribution of a particle number density and a magnetic field for small viewing angles. We first determine the spectral flux in the fluid frame, where an emissivity and an effective absorption are readily calculated (Ginzburg & Syrovatskii 1964; Blumenthal & Gould 1970), and then transform it into the observer frame (Rybicki & Lightman 1979).

A spectral flux in the jet frame is defined as

$$S'_{\nu'} = \frac{1}{d^2} \int_{\Omega'} j'_{\nu'}(\nu') {\rm d}V' {\rm e}^{-\int æ'_{\nu'}(\nu'){\rm d}s'}, \tag{64}$$

where Ω' is a radiating domain. Using (10) and (11) for the synchrotron radiation, and having $j'_{\nu'}(\nu') = \hbar\nu'\rho'_{\nu'}(\nu')$, we obtain for the flux in a jet frame written through the observed frequency $\nu_{\rm obs}$ and a particle number density n and a magnetic field B in the nucleus frame the following expression:

$$S'_\nu(\nu, n_0, B_0) = 0.16 \frac{\hbar\nu}{d^2} \frac{\nu}{r_0 c} \left(\frac{\nu_{B_0}}{\nu}\right)^{-1/2} \left(\frac{1+z}{\delta}\right)^{5/2} I, \tag{65}$$

where the integral I has dimension cm^2 and is defined by

$$I = \int_0^{R_{\rm j}} \frac{1}{\sqrt{f_{\rm B}(r_\perp)}} r_\perp {\rm d}r_\perp \left[1 - {\rm e}^{-\tau_0 \frac{L}{R_{\rm L}} f_{\rm n}(r_\perp) f_{\rm B}^2(r_\perp)}\right]. \tag{66}$$

If the whole jet cross-section is optically thick, the integral can easily be calculated. In the inner domain $r_\perp \in [0, \Gamma R_{\rm L}]$, it is equal to

$$I_{\rm in} = R_{\rm L}^2 \left(\frac{1}{6} + \frac{\Gamma^3}{3}\right), \tag{67}$$

and in the outer domain $r_\perp \in (\Gamma R_{\rm L}, R_{\rm j}]$, it is equal to

$$I_{\rm out} = \frac{2}{5} R_{\rm L}^2 \left(\sqrt{\Gamma} \left(\frac{R_{\rm j}}{R_{\rm L}}\right)^{5/2} - \Gamma^3\right), \tag{68}$$

and

$$I \approx \frac{2}{5} \sqrt{\Gamma} \sqrt{\frac{R_{\rm j}}{R_{\rm L}}} R_{\rm j}^2, \tag{69}$$

the outer radiating domain provides the major part of the total flux.

In order to link the spectral flux in the jet frame with the observed brightness temperature, we use the Lorentz invariance of S_ν/ν^3 (Rybicki & Lightman 1979). Substituting (65) and (69) into (8), one obtains

$$\left(\frac{B_0}{\rm G}\right) = 6.4 \times 10^{-4} \Gamma \frac{R_{\rm j}}{R_{\rm L}} \frac{\delta}{1+z} \left(\frac{\nu_{\rm obs}}{\rm GHz}\right) \left(\frac{T_{\rm b,obs}}{10^{12}\,{\rm K}}\right)^{-2}. \tag{70}$$

Compare this result with the uniform model (16). The non-uniform model of the optically thick outflow gives for a magnetic field the amplitude of the uniform model multiplied by a 'geometrical' factor $R_{\rm j}/R_{\rm L}$, which increases the value by two orders. We see that the uniform model underestimates even the average value of a magnetic field in comparison with the non-uniform model. Indeed, B_0 is greater than the uniform magnetic field everywhere across the jet, and for $R_{\rm j}/R_{\rm L} = 10^2$ both fields become comparable only at the jet boundary.

The magnetic field estimated in the frame of a model with the non-uniform jet velocity distribution obeys the same expression (70), since the outer domain $r_\perp > \sigma_{\rm M} R_{\rm L}$ contributes most in a spectral flux, and the velocity profile in this domain is the same for two models.

For the two sources with the measured extreme brightness temperatures, the magnetic field estimated by the non-uniform model is

$$B_{\rm non-uni}^{\rm BLLac} = 3\ {\rm G}, \quad B_{\rm non-uni}^{\rm 3C273} = 0.7\ {\rm G}. \tag{71}$$

4 ASTROPHYSICAL APPLICATIONS AND DISCUSSIONS

In the frame of the Blandford–Königl model, we have rederived the expressions for a magnetic field and a particle number density used by Zdziarski et al. (2015) as a tool to estimate these physical parameters of the radiation domain in a jet independently of the equipartition assumption. However, contrary to their work, we expressed the parameters through the brightness temperature. As the values for $T_{\rm b,obs}$ obtained with high resolution exceed by two orders the equipartition temperature derived by Readhead (1994), the jet parameters differ from the ones corresponding to equipartition, the measure of equipartition being of the order of 10^{-6} to 10^{-5}. In particular, the magnetic field in the radiating domain has an order of 10^{-3}, which according to (7) provides the magnetic field at the gravitational radius $B_{\rm g}$ of the order of a few Gauss. The expression for n gives the unphysically high amount of particles of the order of $10^7\ {\rm cm}^{-3}$, since such an amount would carry energy exceeding the total jet power. However, the two sources considered in this work have core shifts smaller than typical errors, estimated in Pushkarev et al. (2011), of 0.05 mas. Thus, the results for Σ and n may be subject to large errors.

We have obtained the expression for the magnetic field amplitude B_0 that can be estimated by the measurement of a brightness temperature. The expression is applicable for blazars, since it uses the head-on model of radiation transfer for the non-uniform cylindrical optically thick radiation domain with profiles for the particle number density and the magnetic field distribution based on MHD modelling. The field amplitude characterizes the radiating core region only and may differ from the other domains along the jet. The expression for B_0 differs from the expression for the homogeneous model of the radiating domain by the factor $R_{\rm j}/R_{\rm L}$, which gives two orders of magnitude.

In the frame of MHD models, the amplitude B_0 characterizes both poloidal and toroidal magnetic fields. Thus, this amplitude provides us with an instrument of checking the electrodynamic model of the black hole energy extraction. Indeed, if we assume the unipolar inductor model for the AGNs, the total jet power is given by (Beskin 2010)

$$P_{\rm tot} = \left(\frac{\Omega r_{\rm g}}{c}\right)^2 B_{\rm g}^2 r_{\rm g}^2 c. \tag{72}$$

As we can estimate B_0 as being of the order of 1 G, using (46), we can correlate the magnetic field amplitude with the total flux crossing the gravitational radius, and thus obtain the poloidal magnetic field at the base of an outflow $B_{\rm g}$. Indeed, on the one hand,

$$\Psi_{\rm tot} = \pi B_{\rm g} r_{\rm g}^2, \tag{73}$$

and, on the other hand,

$$\Psi_{\rm tot} \approx 2\pi B_0 R_{\rm L}^2 \ln\frac{R_{\rm j}}{R_{\rm L}}. \tag{74}$$

From these equalities, we have

$$B_{\rm g} = 2B_0 \ln\frac{R_{\rm j}}{R_{\rm L}} \left(\frac{R_{\rm L}}{r_{\rm g}}\right)^2, \tag{75}$$

which gives $B_{\rm g} \approx 10^3$. It is about an order smaller than the Eddington magnetic field (see e.g. Beskin 2010)

$$B_{\rm Edd} = 10^4 \left(\frac{M_{\rm BH}}{10^9\ {\rm M}_\odot}\right)^{-1/2} {\rm G}. \tag{76}$$

Such a magnitude for $B_{\rm g}$ provides within the electrodynamical model for the total jet power an estimate $P_{\rm tot} \approx 3\times10^{43}\ {\rm erg\ s}^{-1}$.

The above expression (75) for a poloidal magnetic field magnitude is consistent with the maximum possible amount of the magnetic flux achieved by magnetically arrested discs. Indeed, for $M_{\rm BH} = 10^9\ {\rm M}_\odot$ and accretion rate with 10 per cent of the Eddington luminosity (Hawley et al. 2015), $\Phi_{\rm MAD} = 3\times10^{33}\ {\rm G\,cm}^2$, and for $B_{\rm g}$ given by (71) and (75), the magnetic flux is of the order of $10^{33}\ {\rm G\,cm}^2$.

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